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Research on Practical Determination of Cable Tension

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ABSTRACT— ***In the process of identifying the tension of cables, the frequency errors have a great influence on them. The paper puts forward a method of modifying the cable tension by calculating the corrections of the frequency according to the principles of Least Squares; and it also analyses accuracy of the tension of cables. The results show that the correction technique of the natural frequency is right, its accuracy is better, and the cables are safe.***

Keywords— accuracy evaluation; frequency; principles of Least Squares; tension

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1. INTRODUCTION

In the process of hoisting steel tube arch and to obtain accurately each segment of cable tension is an important guarantee for hoisting bridge safely and bridge alignment. The control of frequency error is the linchpin in a method of measuring cable tension. This paper established a cable tension based condition adjustment model based on the least square principle, to calculate the cable force through solving each order frequency correction.

At present, among the many methods of measuring cable tension, the vibration test method is widely used for less equipment, short cycle, easy implementation, and high precision. Among them[1-5], about the calculation of cable tension, the scholars mainly embarksupon bending stiffness, additional mass and temperature of cable, when revise the calculation model of cable tension, and calculated the size of cable tension. Zhao et al.[6] introduced the balanced state of cable under the action of temperature variation, by solving the corresponding equilibrium equations and the equation of motion, and obtained the relational expression between cable tension and frequency when temperature changes. Zhang et al.[7] analysis on frequency and force of stay cable considering bending rigidity, and study the rule of frequency and vibration type when the cable force change by The Finite Element Method. They proposed to use the frequency difference method to determine the order of frequency measured, and use the second frequency for cable force calculation. Li et al.[8] did a research on cable-stayed bridge model based on measurements of cable tensions, get the major and minor corrected parameters of cable-stayed bridge model, and calculate cable tension of each condition by using the modified finite element model. As the hyperbolic function for frequency equation item may cause data overflowing, Liao et al.[9] derived approximate frequency calculation method for clamped cables, by processing hyperbolic function approximatively. He et al.[10], derived an implicit expression between suspender tension which considering elastic support on both ends of the hanger rod and of the effect of added mass, and its vibration frequency. Guo Liangyou derived the conclusion that without considering the bending stiffness of the circumstances. Article K order frequency of rope and cable is K times of the first order natural vibration frequency, namely the arbitrary order two adjacent natural vibration frequency difference equal to the value of the natural vibration frequency of first order[11].

The study shows: Frequency error is a combined effect, each factor which affects the frequency are difficult to precisely processing on volume. When calculate cable tension after picking each frequency, whether use nonlinear numerical method for processing or finite element model updating, the whole solving process is complex and error significantly. Thus, the method which is put forward in this paper, according to the errors of cable force vibration testing method of testing and measuring accuracy, makes the frequency better corrected, and can guarantee the accuracy of calculation of cable force well matched with the reality.

2. BASIC PRINCIPLE

Least Square Principle: The least square principle is a basic method which deal with all kinds of observation data of surveying adjustment. It has been widely used in industrial technology and other scientific work fields[12]. To obtain the correction of observed values that redundant observations about a value and according to the conditions of function model which a measurement result is uncertain. It can make the calculation values are close to the true value. It accords with ΔtD-1Δ=min[13], Where Δ is random error vector of observation, D is a prior covariance matrix of observation vector.

Cable Tension Characteristic Equation: When the homogeneous cable vibrate transversely, cable tension[14], is:

$$T_K = \frac{\rho A L^2 W_K^2}{K^2 \pi^2} - EI(\frac{K\pi}{L})^2 \quad (1)$$

Where, T_K: the corresponding cable tension of the K order frequency; EI: the bending rigidity of a cable; ρA: The quality of the unit length of a cable; L: cable length; W: the K order natural frequency of a cable.

Given two any order frequency. For $W = 2\pi f_k$ and calculation of cable tension turn Eq.(1) into:

$$T = (\frac{n^2}{k^2} \cdot f_k^2 - \frac{k^2}{n^2} \cdot f_n^2) \cdot \frac{1}{n^2 - k^2} \cdot (4\rho A L^2) \quad (2)$$

Where, f_k and f_n are the K and N order frequency. Eq.(2) show that if given two any order frequency, the cable tension can be calculated.

3. THE LEAST SQUARE CALCULATION OF CABLE TENSION

Condition Adjustment Model Cable: internal force is recognized mainly by measuring its natural frequency. When the cable is not too long, the bending stiffness influence is large on high order modes. Therefore, in the process of measurement, should pick up the low order natural frequency (usually 4 order) to calculate[15,16].

According least square principle, there are two observation equation of excess condition:

$$\hat{T}_{1,2} - \hat{T}_{2,3} = 0 \quad (3)$$

$$\hat{T}_{1,2} - \hat{T}_{1,4} = 0 \quad (4)$$

Where $\hat{T}_{i,j}$ is the theoretical value of cable tension in the *I* and *j* order frequency. Let $T_{i,j}$ be the calculated value of cable tension in the corresponding frequency, and its correction number is $\Delta_{i,j}$. The Eq. (3) and Eq. (4) can be reduced to

$$\Delta_{1,2} - \Delta_{2,3} - W_1 = 0 \quad (5)$$

$$\Delta_{1,2} - \Delta_{2,4} - W_2 = 0 \quad (6)$$

Where: $W_1 = -(T_{1,2} - T_{2,3})$, $W_2 = -(T_{1,2} - T_{1,4})$.

From Eq.(5), Eq.(6) and Eq.(2), for solving the correction number $\Delta_{i,j}$ of cable tension is actually solving the correction number Δ of frequency *f*. Eq.(2) is processed by perfect differential, and combine with Eq.(5) and Eq.(6) to

$$a_1\Delta_1 + a_2\Delta_2 + a_3\Delta_3 - W_1 = 0 \quad (7)$$

$$b_1\Delta_1 + b_2\Delta_2 + b_4\Delta_4 - W_2 = 0 \quad (8)$$

Where a_i and b_i are corresponding coefficients(*i*=1,2,3,4). For Eq.(7) and Eq.(8), the equation can be written as

$$A \cdot \Delta - W = 0 \quad (9)$$

Where: $A = \begin{bmatrix} a_1\, a_2\, a_3\ 0 \\ b_1\, b_2\ 0\ b_4 \end{bmatrix}$, $\Delta = (\Delta_1 \quad \Delta_2 \quad \Delta_3 \quad \Delta_4)^T$, $W = (W_1 \quad W_2)^T$.

Calculation of Cable Tension: Eq.(9) is the condition adjustment model. The correction number Δ can be calculated by each order frequency value and the least square principle. Each order frequency value is

$$\hat{f}_i = f_i + \Delta_i \quad (10)$$

Where, $\hat{f}_i$: the correction value of each order frequency; f_i : the measured value of each order frequency. Solving Eq.(2) for cable tension T, substituting for $\hat{f}$ from Eq.(10), then

$$\hat{T} = (\frac{n^2}{k^2}\cdot \hat{f}_k^2 - \frac{k^2}{n^2}\cdot \hat{f}_n^2)\cdot \frac{4\rho AL^2}{n^2-k^2} \qquad (11)$$

The Precision Estimation of Tension: In order to ensure the correct and accuracy for cable tension, according to propagation of error and Eq.(11).The measurement accuracy of cable tension can be obtained as Eq.(12):

$$m_T = \pm\sqrt{c_1^2 m_{f_k}^2 + c_2^2 m_{f_n}^2 + c_3^3 m_L^2}\cdot \frac{4\rho AL^2}{n^2-k^2} \qquad (12)$$

Where, $c_1 = \frac{2n^2\cdot \hat{f}_k}{k^2}$, $c_2 = \frac{2k^2\cdot \hat{f}_n}{n^2}$, $c_3 = \frac{2}{L}(\frac{n^2}{k^2}\cdot \hat{f}_k^2 - \frac{k^2}{n^2}\cdot \hat{f}_n^2)$; m_{f_k} and m_{f_n} are the mean square error of measurement in the *K* and *N* order frequency; m_L is the mean square error of cable length. Setting $m_{f_k} = m_{f_n} = \pm 0.01H_Z$; $m_L = \pm 1mm$ [17].

In the process of hoisting steel tube arch, the average of the calculated tension in each order frequency regards as cable tension. The mean square error of arithmetic average is

$$m_{\bar{T}} = \pm\frac{m_T}{\sqrt{n}} \qquad (13)$$

Where, $m_{\bar{T}}$ is the mean square error of arithmetic average. The *n* is the measurement time of tension.

4. RESULT AND DISCUSSION

In the process of hoisting steel tube arch, it is an important part of construction that the tension of stay cable can be measured. In addition, it is one of the general concern problems for the designer. The forced state of each cable hoisting before and after, influences directly on the quality of engineering construction, the alignment of completed bridge main girder and main arch, the internal force distribution of completed bridge main girder and main arch[18].

Tab.1 shows the measured value of four order frequency and the correction value of each order frequency by least square method; Tab.2 shows the calculated value of cable tension in each frequency.

Tab.1 The calculation results of Natural frequency

Cable	Measured frequency /Hz		Correction value /Hz	adjusted value /Hz
A	f_1	11.00	-0.27	10.73
	f_2	21.50	-0.07	21.43
	f_3	32.00	0.26	32.26
	f_4	43.25	-0.10	43.15
B	f_1	8.50	0.00	8.50
	f_2	17.00	-0.09	16.91
	f_3	25.25	0.10	25.35
	f_4	33.75	-0.03	33.72
C	f_1	18.50	-0.12	18.38
	f_2	37.00	-0.21	36.79
	f_3	55.00	0.32	55.32
	f_4	74.00	-0.11	73.89

Tab.2 The calculation results of cable tension

Cable		T_1/kg	$T_{1,2}$/kg	$T_{1,3}$/kg	$T_{1,4}$/kg	$T_{2,3}$/kg	$T_{2,4}$/kg	$T_{3,4}$/kg	$m_{\bar{T}}$/kg	Relative Error /%
A	pre	7132.95	7239.65	7186.01	7148.87	6896.56	6785.73	7148.87		
	post	6786.91	6796.35	6786.32	6785.15	6781.57	6789.23	6783.95	17.28	0.26
B	pre	4269.14	4259.14	4269.75	4263.26	4325.75	4279.77	4149.49		
	post	4270.14	4272.88	4270.36	4269.26	4272.46	4269.39	4269.44	11.91	0.278
C	pre	20175.63	20175.63	20221.03	20175.63	20465.08	20175.63	19348.57		
	post	19914.49	19912.72	19905.65	19901.52	19898.57	19897.91	19896.04	51.38	0.258

Where “pre” refers to the frequency without correction; “post” refers to the frequency with correction; The relative error is ratio of the mean square error of arithmetic average and the average of cable tension, its values reflects the precision of cable tension. And compared with Fang Z.’s[19] research, the calculation results are less error and more reliable. From Tab.1 and Tab.2

（1）It has been revised by least square method that each order frequency is obtained by vibration testing method. The K order frequency is about k times of the first order frequency. The difference value between any adjacent two order natural vibration frequency equal to the first order natural vibration frequency. Each order natural vibration frequency is equal interval.

（2）When the cable natural frequency has been corrected, the calculation of cable tension in each order frequency is approximate and error is small.

5. CONCLUSION

According to test cable tension by vibration testing method, this paper established condition adjustment model, to calculate the cable force with modified frequency by least square method. Data processing is tight, its results are also reliable and high precision. It provides a safety guarantee for bridge hoisting. In addition, after each order frequency has been corrected, the calculation of cable tension is almost same; Meanwhile, in the actual engineering calculation of cable force, without considering the influence of bending rigidity, the amendatory first order frequency can be calculated directly into practical calculation, namely $T_1 = 4\rho AL^2 \hat{f}_1$.

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