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Thermally Induced Vibration of Non-homogeneous Trapezoidal Plate Whose Thickness Varies Linearly in One and Parabolically in Other Direction with Linearly Varying Density

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ABSTRACT— ***Based on classical plate theory, the natural frequencies of thermally induced vibration of non-homogeneous trapezoidal plate with varying thickness linearly in x-direction and parabolically in y-direction has been calculated by Rayleigh–Ritz method. Due to linear variation in density non-homogeneity arises in plate's material. The frequency equation has been obtained by assuming the two-term deflection function with clamped-simply supported- clamped-simply supported boundary condition. For a symmetric, non-homogeneous trapezoidal plate the effect of non-homogeneity constant, aspect ratios, thermal gradient and taper constants on the frequencies has been studied for first two modes of vibration. All the numerical results which have been obtained presented in tabular and graphical form.***

Keywords— non-homogeneous problem, thickness, density, trapezoidal plate, frequencies

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1. INTRODUCTION

In engineering, research and nuclear reactor technology the study of vibration of plates has attained tremendous significance. It has influence not only in constructive but also in destructive fields like aircraft engineering and earthquake respectively. Basically with the help of vibration analysis the perfect design of a system can be made and its performance can be increased. Vibration is required in various operations, so this phenomenon is not undesirable in every case. There has been a great fascination about vibration in various shapes of plates over the last century.

Vibrations of plates depend on non-homogeneity of materials which come up because of variation in density. In modern technology trapezoidal plates of variable thickness are commonly used as structural constituent in various engineering applications like aircraft, ships, space craft and gas turbines to take into account the availability of resources, materials and cost of production to provide better efficiency and strength.

Vibration analysis of different shapes of plates has attracted the interest of several researchers. Some of them are discussed here. Hassan and Makary [2] worked on the transverse vibrations of elliptical plate of linearly varying thickness with half of the boundary clamped and the rest simply supported. Chakraverty et al. [3] discussed the vibration of non-homogeneous orthotropic elliptic and circular plates with variable thickness. Lal et al. [4] used Chebyshev collocation method in the study of transverse vibrations of non-uniform rectangular orthotropic plates. Laura et al. [5] studied the transverse vibrations of a trapezoidal cantilever plate of variable thickness. McGee and Butalia [6] discussed the natural vibrations of shear deformable cantilevered skewed trapezoidal and triangular thick plates. Bhardwaj and Gupta [7] had worked on the axisymmetric vibrations of polar orthotropic circular plates of quadratically varying thickness resting on elastic foundation. Kavita et al. [8] studied the thermally induced vibration of non-homogeneous trapezoidal plate with parabolically thickness variation in both directions. Kavita et al. [9] discussed the temperature behaviour on thermally induced vibration of non-homogeneous trapezoidal plate with bi-linearly varying thickness. Liew and Lim [10] studied the transverse vibration of trapezoidal plates of variable thickness: symmetric trapezoids. Hosokawa et al. [11] discussed the free vibration analysis of cantilevered laminated trapezoidal plates. Larrondo et al. [12] studied the vibration of simply supported rectangular plates with varying thickness and same aspect ratio cutouts.

Gupta et al. [13] worked on the vibration of visco-elastic parallelogram plate with parabolic thickness variation. Leung et al. [14] presented the free vibration of laminated composite plates subjected to in-plane stresses using trapezoidal p-element. Gupta et al. [15] did the vibration analysis of visco-elastic rectangular plate with thickness varies linearly in one and parabolically in other direction. Gupta and Sharma [16] observed the thermal effect on vibration on non-homogeneous trapezoidal plate of linearly varying thickness. Gupta and Sharma [17] had studied the thermal gradient effect on frequencies of a trapezoidal plate of linearly varying thickness.

It is identified from the analysis of literature that up to now no author encountered the paper with linearly varying thickness in x-direction and parabolically in y-direction and linearly varying density in x-direction with thermal effect. Therefore, the vibrational behaviour of symmetric, isotropic and non-homogeneous trapezoidal plate has been studied by considering variation in thickness and density. Rayleigh-Ritz method is applied to find the natural frequencies for first and second mode of vibration for C-S-C-S boundary condition by assuming a two term deflection function, where C and S stand for clamped and simply supported, respectively. Hence, the objective of this research paper is to examine the frequencies for both modes of vibration for different values of taper constants (β_1, β_2), thermal gradient (α), non-homogeneity constant (β) and aspect ratio (c/b). All the numerical results are shown in tabular and graphical form.

2. MATHEMATICAL ANALYSIS AND EQUATIONS OF MOTION

With variable thickness and density a symmetric, isotropic and non-homogeneous trapezoidal plate has been considered and shown in figure1.

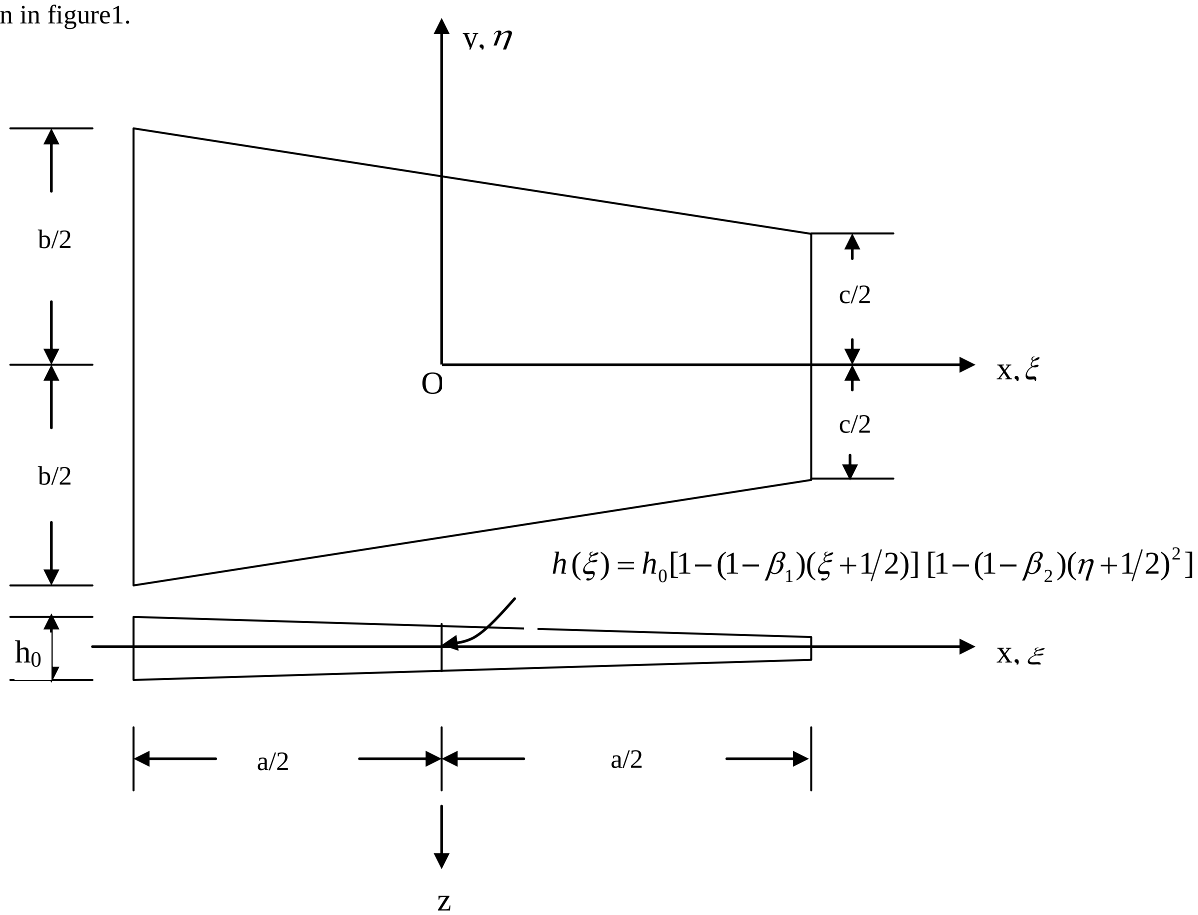

Figure 1. Geometry of the trapezoidal plate

For a non-homogeneous trapezoidal plate temperature is assumed to vary linearly along the length of the plate i.e. x-axis as

$$\tau=\tau_0\left(\frac{1}{2}-\xi\right) \tag{1}$$

where τ denotes the excess above the reference temperature at a distance $\xi=\dfrac{x}{a}$ and τ_0 denotes the temperature excess above the reference temperature at the end $\xi=-\dfrac{1}{2}$.

The temperature dependence of the modulus of elasticity for most of the elastic materials is described [1] as

$$E=E_0\left(1-\gamma\tau\right) \tag{2}$$

where E_0 is the value of Young's modulus at reference temperature $\tau = 0$ and γ is the slope of variation of E with τ.

By using equation (1) into equation (2), modulus of variation becomes

$$E = E_0\left(1-\alpha\left(\frac{1}{2}-\xi\right)\right) \tag{3}$$

where $\alpha = \gamma\,\tau_0 (0 \le \alpha \le 1)$ known as thermal gradient.

Plates of variable thickness are generally come across in engineering applications and have greater efficiency for vibration as compared to plates of uniform thickness. In the present study thickness of symmetric and non-homogeneous trapezoidal plate is assumed to vary linearly in x-direction and parabolically in y-direction. Therefore, thickness can be expressed as

$$h(\xi) = h_0\left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)^2\right] \tag{4}$$

where $h_0 = h$ at $\xi = \eta = -\frac{1}{2}$ and β_1, β_2 are taper constants.

The non-homogeneity occurs in bodies because of imperfection of materials and it is assumed to arise due to linear variation in density along x-axis. Thus, it can be taken as

$$\rho = \rho_0\left[1-(1-\beta)\left(\xi+\frac{1}{2}\right)\right] \tag{5}$$

where $\rho_0 = \rho$ is the mass density at $\xi = -\frac{1}{2}$ and β is non-homogeneity constant.

The governing differential equations of kinetic energy T and strain energy V for a non –homogeneous trapezoidal plate as expressed by [10] are

$$T = \frac{ab}{2}\omega^2 \int_A h(\xi)\rho w^2 dA \tag{6}$$

$$V = \frac{ab}{2}\int_A D(\xi)\left\{\left(\frac{1}{a^2}\frac{\partial^2 w}{\partial \xi^2}+\frac{1}{b^2}\frac{\partial^2 w}{\partial \eta^2}\right)^2 - 2(1-\nu)\left(\frac{1}{a^2b^2}\frac{\partial^2 w}{\partial \xi^2}\frac{\partial^2 w}{\partial \eta^2}-\left(\frac{1}{ab}\frac{\partial^2 w}{\partial \xi \partial \eta}\right)^2\right)\right\}dA \tag{7}$$

where ω is the angular frequency of vibration, ν is the Poisson ratio, A is the area of the plate and $D(\xi)$ is the flexural rigidity of the plate which can be defined as

$$D(\xi) = D_0\left[\left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)^2\right]\right]^3 \tag{8}$$

where $\xi = \frac{x}{a}, \eta = \frac{y}{b}$ are non-dimensional variables. Here,

$$D_0 = \frac{Eh_0^{\,3}}{12(1-\nu^2)} \tag{9}$$

Substituting equation (3) in equation (9), we obtain

$$D_0 = \frac{E_0 h_0^{\,3}\left(1-\alpha\left(\frac{1}{2}-\xi\right)\right)}{12(1-\nu^2)} \tag{10}$$

By using equation (10) into equation (8) the value of flexural rigidity becomes

$$D(\xi) = \frac{E_0 h_0^{\,3}}{12(1-\nu^2)}\left[\left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)^2\right]\right]^3\left(1-\alpha\left(\frac{1}{2}-\xi\right)\right). \tag{11}$$

On substituting equations (4), (5) in eq. (6) and eq. (11) in eq. (7), one can obtain the kinetic energy and strain energy as

$$T = \frac{ab}{2}\rho_0 h_0 \omega^2 \int_A \left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)^2\right]\times\left[1-(1-\beta)\left(\xi+\frac{1}{2}\right)\right]w^2 dA, \tag{12}$$

and

$$V=\frac{ab}{2}\frac{E_0h_0^3}{12\left(1-\nu^2\right)}\int_A\left[\left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)^2\right]\right]^3\left(1-\alpha\left(\frac{1}{2}-\xi\right)\right)\times$$
$$\times\left\{\left(\frac{1}{a^2}\frac{\partial^2 w}{\partial\xi^2}+\frac{1}{b^2}\frac{\partial^2 w}{\partial\eta^2}\right)^2-2\left(1-\nu\right)\left(\frac{1}{a^2b^2}\frac{\partial^2 w}{\partial\xi^2}\frac{\partial^2 w}{\partial\eta^2}-\left(\frac{1}{ab}\frac{\partial^2 w}{\partial\xi\partial\eta}\right)^2\right)\right\}dA. \tag{13}$$

3. METHOD OF SOLUTION

The natural frequency of the existing problem is determined through Rayleigh-Ritz method which is based on the principle of conservation of energy i.e. the maximum strain energy must be equal to the kinetic energy. Thus, the consequent equation can be written as

$$\delta\left(V-T\right)=0\ . \tag{14}$$

Two term deflection function for a non-homogeneous trapezoidal plate which satisfies the clamped simply-supported clamped simply- supported boundary condition can be defined as

$$w=A_1\left\{\left(\xi+\frac{1}{2}\right)\left(\xi-\frac{1}{2}\right)\right\}^2\left\{\eta-\left(\frac{b-c}{2}\right)\xi+\left(\frac{b+c}{4}\right)\right\}\left\{\eta+\left(\frac{b-c}{2}\right)\xi-\left(\frac{b+c}{4}\right)\right\}+$$
$$+A_2\left\{\left(\xi+\frac{1}{2}\right)\left(\xi-\frac{1}{2}\right)\right\}^3\left\{\eta-\left(\frac{b-c}{2}\right)\xi+\left(\frac{b+c}{4}\right)\right\}^2\left\{\eta+\left(\frac{b-c}{2}\right)\xi-\left(\frac{b+c}{4}\right)\right\}^2, \tag{15}$$

where A_1 and A_2 are two unknowns to be determined.

In this way, the boundary conditions are defined by four straight lines as follows:

$$\eta=\frac{c}{4b}-\frac{\xi}{2}+\frac{1}{4}+\frac{c\xi}{2b},\ \eta=-\frac{c}{4b}+\frac{\xi}{2}-\frac{1}{4}-\frac{c\xi}{2b},\ \ \xi=-\frac{1}{2},\ \ \xi=\frac{1}{2}. \tag{16}$$

On using the boundary condition (16) into equations (12) and (13), we get

$$T=\frac{ab}{2}\rho_0h_0\omega^2\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{c}{4b}+\frac{\xi}{2}-\frac{1}{4}-\frac{c\xi}{2b}}^{\frac{c}{4b}-\frac{\xi}{2}+\frac{1}{4}+\frac{c\xi}{2b}}\left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)^2\right]\times$$
$$\times\left[1-\left(1-\beta\right)\left(\xi+\frac{1}{2}\right)\right]w^2d\eta\,d\xi\,, \tag{17}$$

and

$$V=\frac{ab}{2}\frac{E_0h_0^3}{12\left(1-\nu^2\right)}\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{c}{4b}+\frac{\xi}{2}-\frac{1}{4}-\frac{c\xi}{2b}}^{\frac{c}{4b}-\frac{\xi}{2}+\frac{1}{4}+\frac{c\xi}{2b}}\left[\left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)^2\right]\right]^3\left(1-\alpha\left(\frac{1}{2}-\xi\right)\right)\times$$
$$\times\left\{\left(\frac{1}{a^2}\frac{\partial^2 w}{\partial\xi^2}+\frac{1}{b^2}\frac{\partial^2 w}{\partial\eta^2}\right)^2-2\left(1-\nu\right)\left(\frac{1}{a^2b^2}\frac{\partial^2 w}{\partial\xi^2}\frac{\partial^2 w}{\partial\eta^2}-\left(\frac{1}{ab}\frac{\partial^2 w}{\partial\xi\partial\eta}\right)^2\right)\right\}d\eta\,d\xi\ . \tag{18}$$

Substituting the values of T and V from equations (17) and (18) into equation (14), one gets

$$\delta\left(V_1-\lambda^2T_1\right)=0 \tag{19}$$

where

$$T_1=\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{c}{4b}+\frac{\xi}{2}-\frac{1}{4}-\frac{c\xi}{2b}}^{\frac{c}{4b}-\frac{\xi}{2}+\frac{1}{4}+\frac{c\xi}{2b}}\left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)^2\right]\times$$
$$\times\left[1-\left(1-\beta\right)\left(\xi+\frac{1}{2}\right)\right]w^2d\eta d\xi\qquad, \tag{20}$$

$$V_1=\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{c}{4b}+\frac{\xi}{2}-\frac{1}{4}-\frac{c\xi}{2b}}^{\frac{c}{4b}-\frac{\xi}{2}+\frac{1}{4}+\frac{c\xi}{2b}}\left[\left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)^2\right]\right]^3\left(1-\alpha\left(\frac{1}{2}-\xi\right)\right)\times$$
$$\times\left\{\left(\frac{1}{a^2}\frac{\partial^2 w}{\partial\xi^2}+\frac{1}{b^2}\frac{\partial^2 w}{\partial\eta^2}\right)^2-2\left(1-\nu\right)\left(\frac{1}{a^2b^2}\frac{\partial^2 w}{\partial\xi^2}\frac{\partial^2 w}{\partial\eta^2}-\left(\frac{1}{ab}\frac{\partial^2 w}{\partial\xi\partial\eta}\right)^2\right)\right\}d\eta d\xi\ , \tag{21}$$

and $\lambda^2 = \dfrac{12\omega^2\rho_0 a^4\left(1-\nu^2\right)}{E_0 h_0^{\,2}}$ is a frequency parameter.

Two unknowns A_1 and A_2 which are involved in Eq. (19) arise due to the use of the deflection function w is given by Eq. (15). These two unknowns can be determined from Eq. (19) in the subsequent method:

$$\frac{\partial}{\partial A_m}\left(V_1-\lambda^2 T_1\right)=0, \quad m=1,\ 2\ . \tag{22}$$

After solving equation (22), we get following form

$$b_{m1}A_1+b_{m2}A_2=0\ , \qquad m=1,\ 2\ . \tag{23}$$

where b_{m1}, b_{m2} (m=1, 2) involves parametric constants and the frequency parameter. The determinant of co-efficient of equation (23) must be zero for a non-zero solution. Thus, the obtained frequency equation for a symmetric and non-homogeneous trapezoidal plate can be defined as

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix}=0 \tag{24}$$

From the solution of equation (24) one can get a quadratic equation in λ^2 which presents the two values of λ^2 known as first and second modes of vibration respectively.

4. RESULTS AND DISCUSSION

The purpose of the present work is to study the vibrational characteristics of symmetric, non-homogeneous and isotropic trapezoidal plate with variable thickness linearly in one and parabolically in other direction. Under the C-S-C-S boundary condition all the numerical values of frequency parameter λ of trapezoidal plate has been calculated. In order to obtain the first two modes of vibration computation has been made for various values of taper constants β_1, β_2, thermal gradient α, aspect ratios a/b, c/b and non-homogeneity constant β. In order to attain all the values of frequency parameter Poisson's ratio ν is considered as 0.33. All the acquired results have been presented in tabular and graphical form. First mode and Second mode of vibrations are presented in Figure (a) and Figure (b) respectively.

In **table 1** numerical values of frequency parameter λ has been calculated for taper constant β_1 varies from 0.0 to 1.0, taper constant $\beta_2=0.6$, thermal gradient $\alpha=0.0, 0.4$, non-homogeneity constant $\beta=0.4, 1.0$ and aspect ratios a/b=1.0, c/b=0.5. So, one can observe from the **table 1** that the value of frequency parameter λ increases on increasing the values of taper constant β_1. Moreover, it is also noticed from the tabulated values that on increasing the value of non-homogeneity constant β the frequency parameter decreases for both the modes of vibration. The variation of frequency parameter λ with taper constant β_1 is shown in **figures 2(a)** and **2(b)** which represents that as taper constant increases the frequency parameter also increases for both modes of vibration.

In **table 2** numerical values of frequency parameter λ has been calculated for taper constant β_2 which varies from 0.0 to 1.0, taper constant $\beta_1=0.6$, thermal gradient $\alpha=0.0, 0.4$, non-homogeneity constant $\beta=0.4, 1.0$ and aspect ratios a/b=1.0, c/b=0.5. As a result it is quite clear from the **table 2** that the value of frequency parameter λ increases on increasing the values of taper constant β_2. It is also found that frequency parameter decreases when the value of non-homogeneity constant β increases for both modes of vibration. The variation of frequency parameter λ with taper constant β_2 is shown in **figures 3(a)** and **3(b)** which represents that as taper constant increases the frequency parameter also increases for both modes of vibration.

In **table 3** numerical values of frequency parameter λ has been calculated for thermal gradient α varies from 0.0 to 1.0, taper constants $\beta_1=0.0, 0.2\ \&\ \beta_2=0.0, 0.6$, non-homogeneity constant $\beta=0.4, 1.0$ and aspect ratios a/b=1.0, c/b=0.5. From the tabulated values one can observe that the value of frequency parameter λ decreases on increasing the values of thermal gradient α. In addition, frequency decreases when values of non-homogeneity constant β increases for both modes of vibration. The behaviour of frequency parameter for both the modes of vibration with thermal gradient is shown in **figures 4(a)** and **4(b)**. So, one can observe from these figures that frequency parameter decreases on increasing the values of thermal gradient.

In **table 4** and **table 5** numerical values of frequency parameter λ has been calculated for aspect ratio c/b varies from 0.25 to 1.0, non-homogeneity constant $\beta = 0.4,$ aspect ratio a/b=0.75 & 1.0 and different combinations of thermal gradient α and taper constants β_1 & β_2 .Therefore, these combinations have been taken as follows:

- $\beta_1 = \beta_2 = 0.0, \alpha = 0.0$
- $\beta_1 = \beta_2 = 0.0, \alpha = 0.4$
- $\beta_1 = \beta_2 = 0.6, \alpha = 0.0$
- $\beta_1 = \beta_2 = 0.6, \alpha = 0.4$

Table 4 and **Table 5** demonstrate that frequency parameter λ decreases on increasing the values of aspect ratio c/b. It is also observed from these tables that values of frequency parameter increases on increasing the values of taper constants. In addition, comparison of **table 4** and **table 5** give detail about aspect ratio that as aspect ratio a/b increases from 0.75 to 1.0 the values of frequency parameter λ also increases for both modes of vibration. For both modes of vibration, the behaviour of frequency parameter with aspect ratio c/b is shown in **figures 5(a)** and **5(b)**. So, one can observe from these figures that frequency parameter decreases on increasing the values of aspect ratio c/b.

In **table 6** numerical values of frequency parameter λ has been calculated for non-homogeneity constant β varies from 0.0 to 1.0, taper constants $\beta_1 = 0.0, 0.2$ & $\beta_2 = 0.0, 0.6,$ thermal gradient $\alpha = 0.0, 0.4$ and aspect ratios a/b=1.0, c/b=0.5. Table 6 clearly shows that the frequency parameter λ decreases for both modes of vibration when the values of non-homogeneity constant β increases. For both modes of vibration the behaviour of frequency parameter λ with non-homogeneity constant β is shown in **figures 6(a)** and **6(b)**. So, one can view from these figures that frequency parameter decreases on increasing the values of non-homogeneity constant.

Table 1: Values of frequency parameter (λ) for different values of taper constant (β_1), thermal gradient $(\alpha = 0.0, 0.4)$, non-homogeneity constant $(\beta = 0.4, 1.0)$, taper constant $(\beta_2 = 0.6)$ and aspect ratios (a/b=1.0, c/b=0.5).

β_1	$\beta = 0.4, \beta_2 = 0.6$				$\beta = 1.0, \beta_2 = 0.6$			
	$\alpha = 0.0$		$\alpha = 0.4$		$\alpha = 0.0$		$\alpha = 0.4$	
	First mode	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**
0.0	32.7052	151.485	31.5861	143.459	28.6357	131.474	27.6582	124.498
0.2	33.2307	160.556	31.9209	150.719	28.9760	138.178	27.8368	129.699
0.4	34.2231	172.462	32.6563	160.583	29.7431	147.542	28.3843	137.366
0.6	35.6453	186.494	33.7572	172.401	30.8966	158.837	29.2625	146.821
0.8	37.4483	202.121	35.1834	185.688	32.3889	171.548	30.4318	157.592
1.0	39.5780	218.950	36.8920	200.088	34.1693	185.312	31.8514	169.342

Table 2: Values of frequency parameter (λ) for different values of taper constant (β_2), thermal gradient $(\alpha = 0.0, 0.4)$, non-homogeneity constant $(\beta = 0.4, 1.0)$, taper constant $(\beta_1 = 0.6)$ and aspect ratios (a/b=1.0, c/b=0.5).

β_2	$\beta = 0.4, \beta_1 = 0.6$				$\beta = 1.0, \beta_1 = 0.6$			
	$\alpha = 0.0$		$\alpha = 0.4$		$\alpha = 0.0$		$\alpha = 0.4$	
	First mode	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**
0.0	31.1280	172.361	29.4797	159.276	26.9824	146.553	25.5559	135.415
0.2	32.3424	173.730	30.6291	160.581	28.0335	147.818	26.5508	136.619
0.4	33.8499	178.412	32.0567	164.928	29.3396	151.887	27.7877	140.395
0.6	35.6453	186.494	33.7572	172.401	30.8966	158.837	29.2625	146.821
0.8	37.7081	197.798	35.7114	182.839	32.6869	168.521	30.9586	155.763
1.0	40.0082	211.979	37.8909	195.927	34.6841	180.649	32.8513	166.955

Table 3: Values of frequency parameter (λ) for different values of thermal gradient (α), taper constants ($\beta_1 = 0.0, 0.2, \beta_2 = 0.0, 0.6$), non-homogeneity constant ($\beta = 0.4, 1.0$) and aspect ratios (a/b=1.0, c/b=0.5).

α	$\beta = 0.4$				$\beta = 1.0$			
	$\beta_1 = \beta_2 = 0.0$		$\beta_1 = 0.2,\ \beta_2 = 0.6$		$\beta_1 = \beta_2 = 0.0$		$\beta_1 = 0.2,\ \beta_2 = 0.6$	
	First mode	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**
0.0	28.5361	139.839	33.2307	160.556	24.9842	121.221	28.9760	138.178
0.2	28.0475	136.110	32.5842	155.715	24.5573	117.984	28.4137	134.005
0.4	27.5463	132.276	31.9209	150.719	24.1195	114.656	27.8368	129.699
0.6	27.0311	128.328	31.2391	145.553	23.6696	111.229	27.2440	125.245
0.8	26.5001	124.257	30.5363	140.198	23.2059	107.694	26.6330	120.628
1.0	25.9510	120.049	29.8096	134.632	22.7265	104.040	26.0015	115.829

Table 4: Values of frequency parameter (λ) for different values of aspect ratio for different combinations of thermal gradient (α), taper constants (β_1&β_2), non-homogeneity constant (β=0.4) and aspect ratio (a/b=0.75).

c/b	$\beta = 0.4$							
	$\beta_1 = \beta_2 = 0.0,\ \alpha = 0.0$		$\beta_1 = \beta_2 = 0.0,\ \alpha = 0.4$		$\beta_1 = \beta_2 = 0.6,\ \alpha = 0.0$		$\beta_1 = \beta_2 = 0.6,\ \alpha = 0.4$	
	First mode	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**
0.25	33.5813	135.268	32.6579	129.777	40.1377	168.972	38.6114	158.948
0.50	26.0026	106.976	25.1894	102.638	31.4673	133.122	30.0214	124.595
0.75	20.7018	84.8782	19.9550	81.4414	25.8711	106.816	24.3660	99.2952
1.0	17.2317	68.5711	16.5164	65.7228	22.7596	89.1680	21.0619	82.0190

Table 5: Values of frequency parameter (λ) for different values of aspect ratio for different combinations of thermal gradient (α), taper constants (β_1&β_2), and non-homogeneity constant (β=0.4) and aspect ratio (a/b=1.0).

c/b	$\beta = 0.4$							
	$\beta_1 = \beta_2 = 0.0,\ \alpha = 0.0$		$\beta_1 = \beta_2 = 0.0,\ \alpha = 0.4$		$\beta_1 = \beta_2 = 0.6,\ \alpha = 0.0$		$\beta_1 = \beta_2 = 0.6,\ \alpha = 0.4$	
	First mode	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**
0.25	36.5554	175.053	35.4182	165.973	45.1242	232.040	43.0943	215.908
0.50	28.5361	139.839	27.5463	132.276	35.6453	186.494	33.7572	172.401
0.75	22.8059	110.358	21.9189	104.229	29.2467	148.966	27.3770	136.817
1.0	18.9175	87.3524	18.0935	82.4016	25.3851	120.233	23.4029	109.605

Table 6: Values of frequency parameter (λ)for different values of non-homogeneity constant (β), with different combinations of thermal gradient α and taper constant (β_1&β_2) and aspect ratios (a/b=1.0, c/b=0.5).

β	$\beta_1 = \beta_2 = 0.0$				$\beta_1 = 0.2, \beta_2 = 0.6$			
	$\alpha = 0.0$		$\alpha = 0.4$		$\alpha = 0.0$		$\alpha = 0.4$	
	First mode	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**
0.0	31.9645	158.443	30.8529	149.887	37.3900	183.507	35.9120	172.285
0.2	30.1049	148.266	29.0595	140.253	35.1271	170.871	33.7408	160.411
0.4	28.5361	139.839	27.5463	132.276	33.2307	160.556	31.9209	150.719
0.6	27.1894	132.710	26.2472	125.528	31.6115	151.924	30.3668	142.610
0.8	26.0170	126.574	25.1160	119.721	30.2079	144.559	29.0194	135.692
1.0	24.9842	121.221	24.1195	114.656	28.9760	138.178	27.8368	129.699

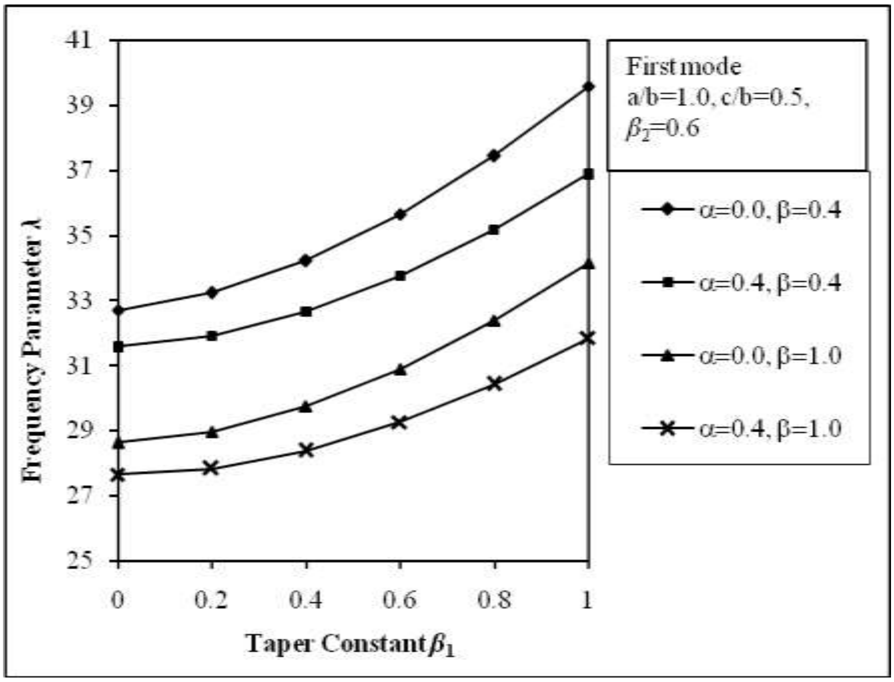

Figure 2(a): Variation of frequency parameter λ for different values of taper constant β_1 for the first mode.

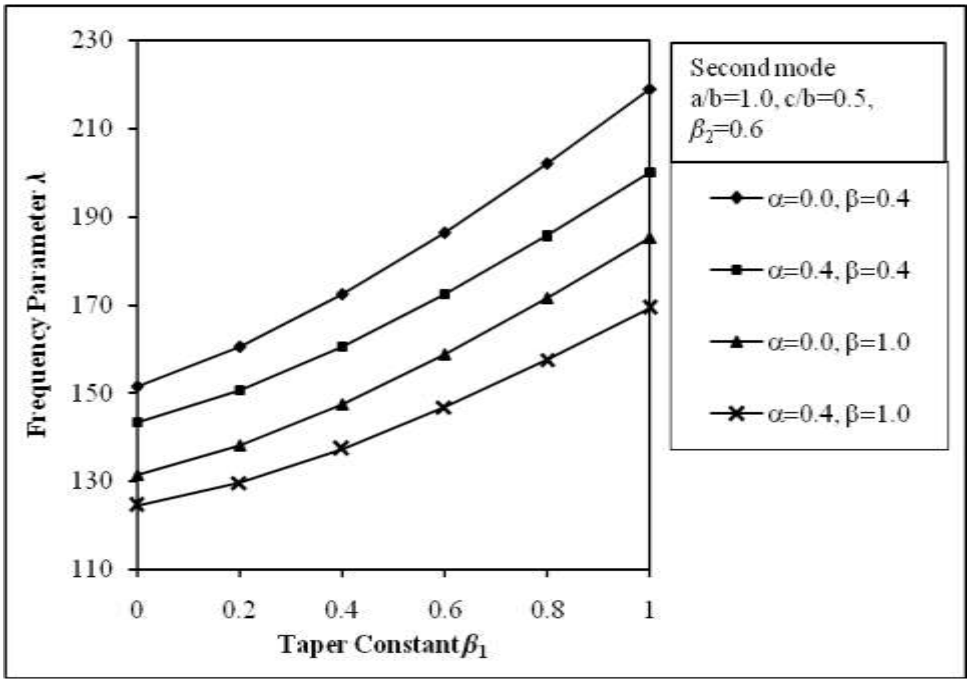

Figure 2(b): Variation of frequency parameter λ for different values of taper constant β_1 for the second mode.

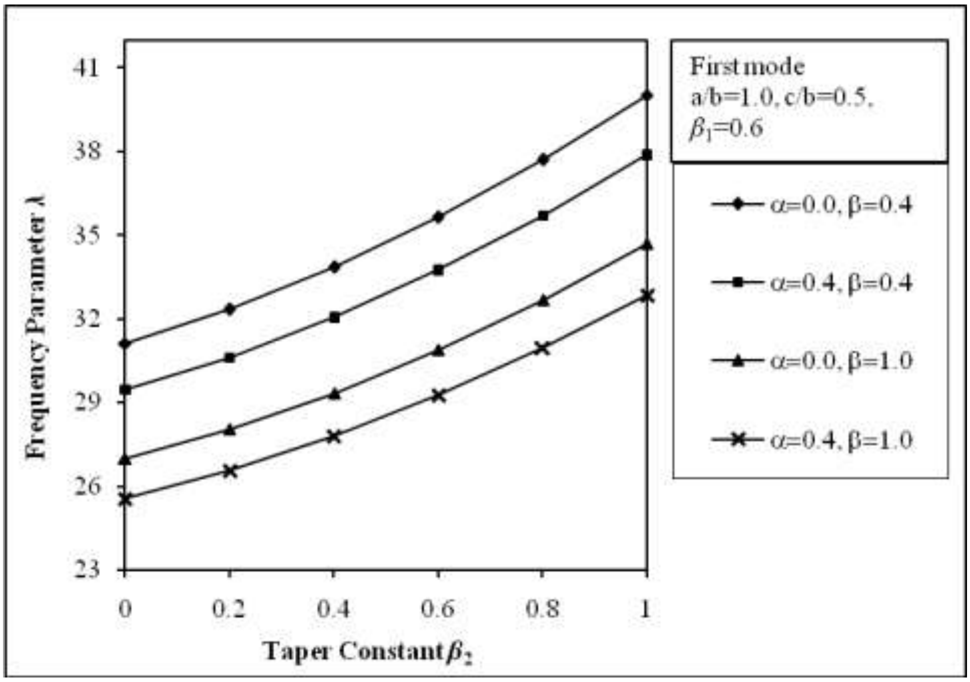

Figure 3(a): Variation of frequency parameter λ for different values of taper constant β_2 for the first mode.

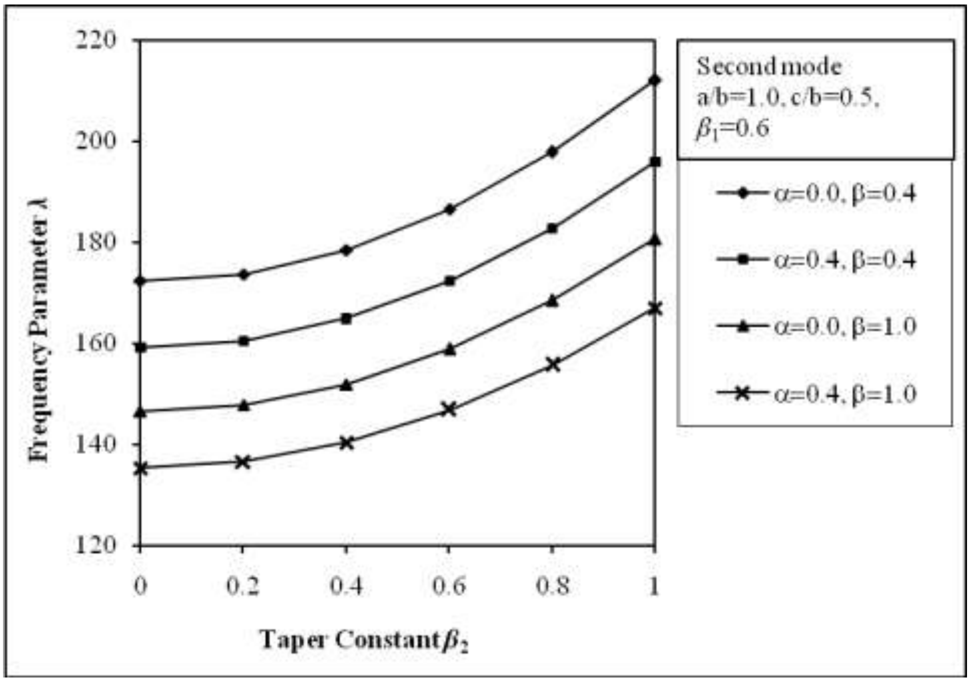

Figure 3(b): Variation of frequency parameter λ for different values of taper constant β_2 for the second mode.

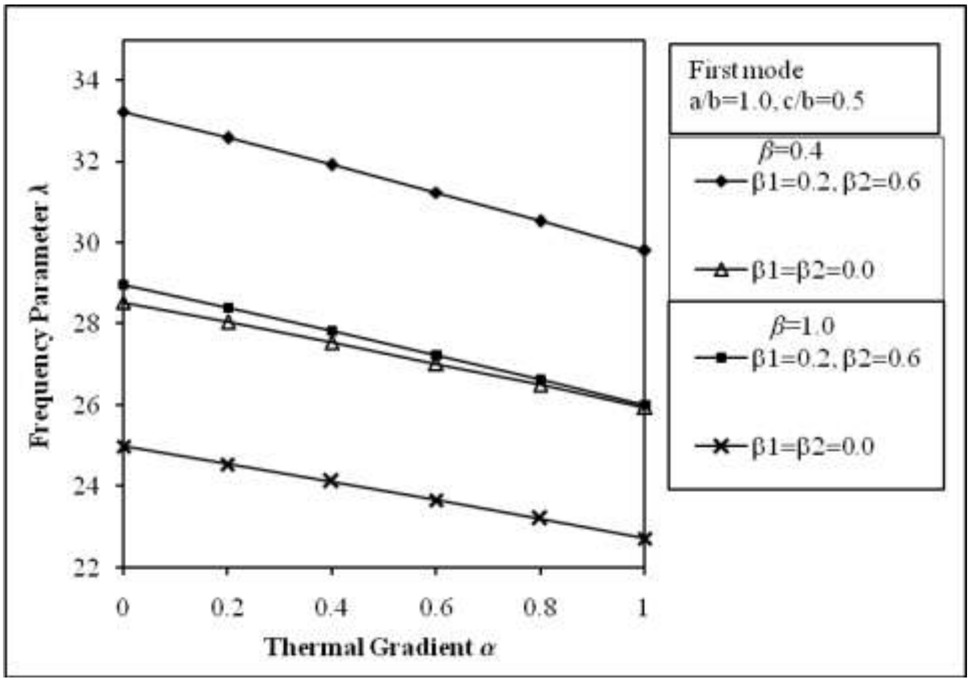

Figure 4(a): Variation of frequency parameter λ for different values of thermal gradient α for the first mode.

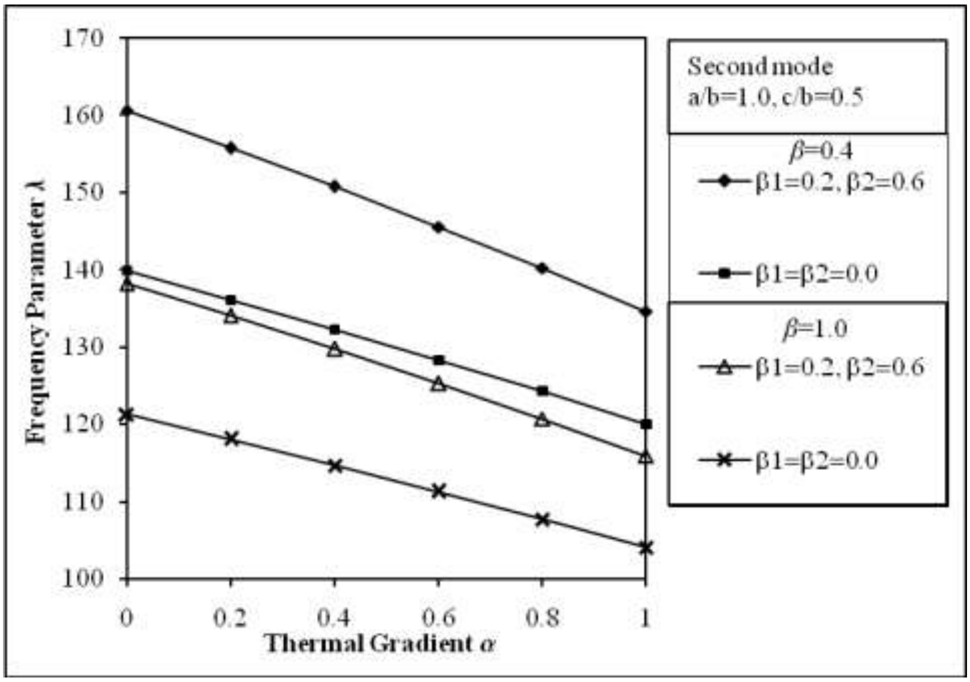

Figure 4(b): Variation of frequency parameter λ for different values of thermal gradient α for the second mode.

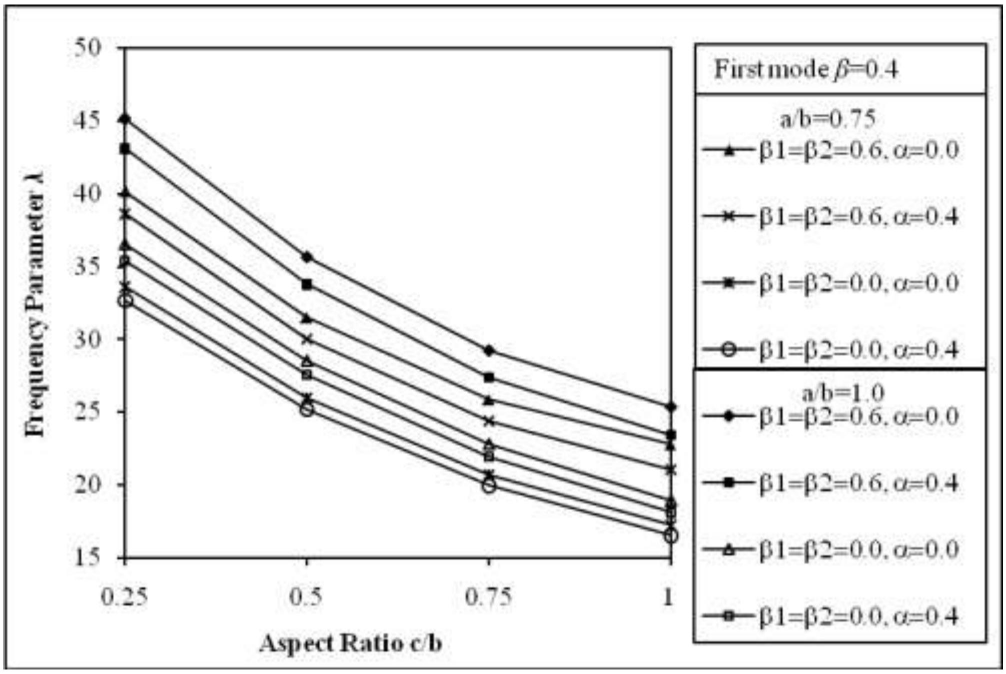

Figure 5(a). Variation of frequency parameter λ for different values of aspect ratio c/b for the first mode.

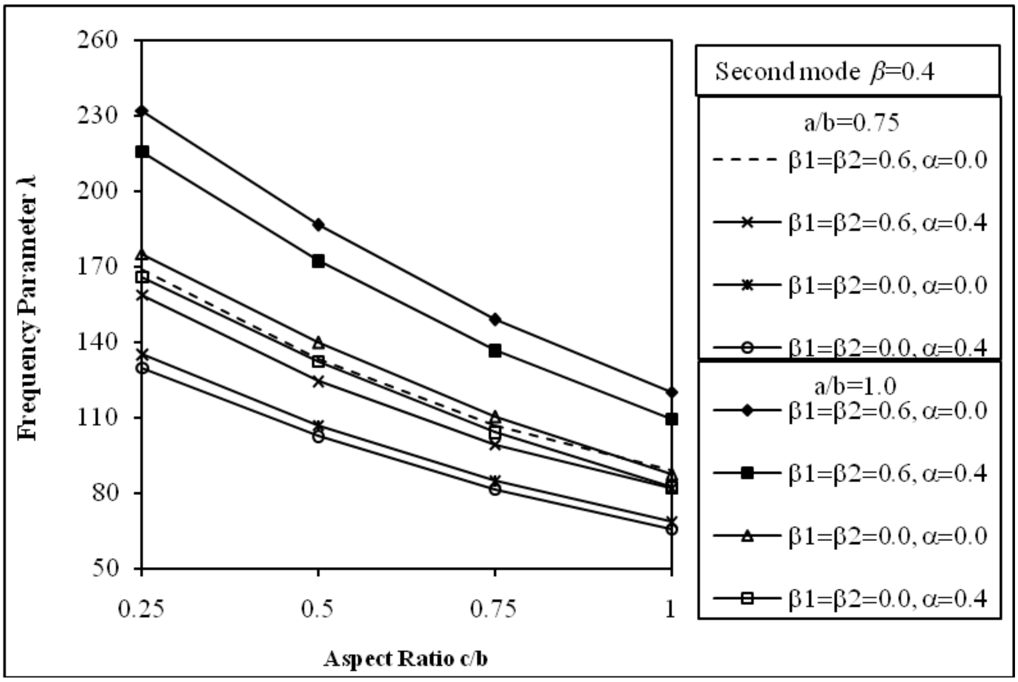

Figure 5(b). Variation of frequency parameter λ for different values of aspect ratio c/b for the second mode.

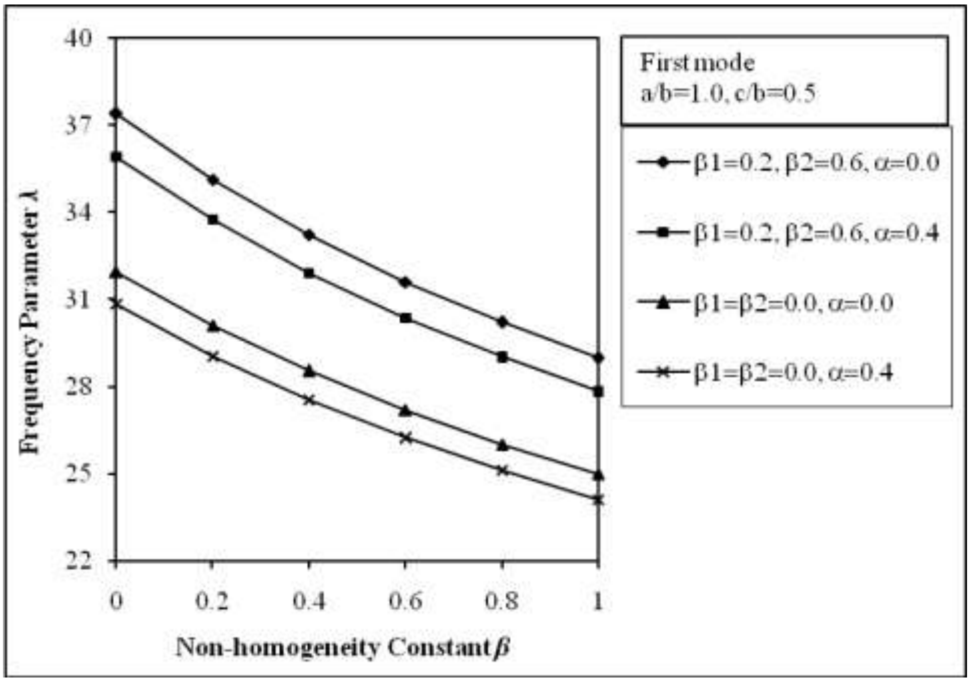

Figure 6(a). Variation of frequency parameter λ for different values of non-homogeneity constant β for the first mode.

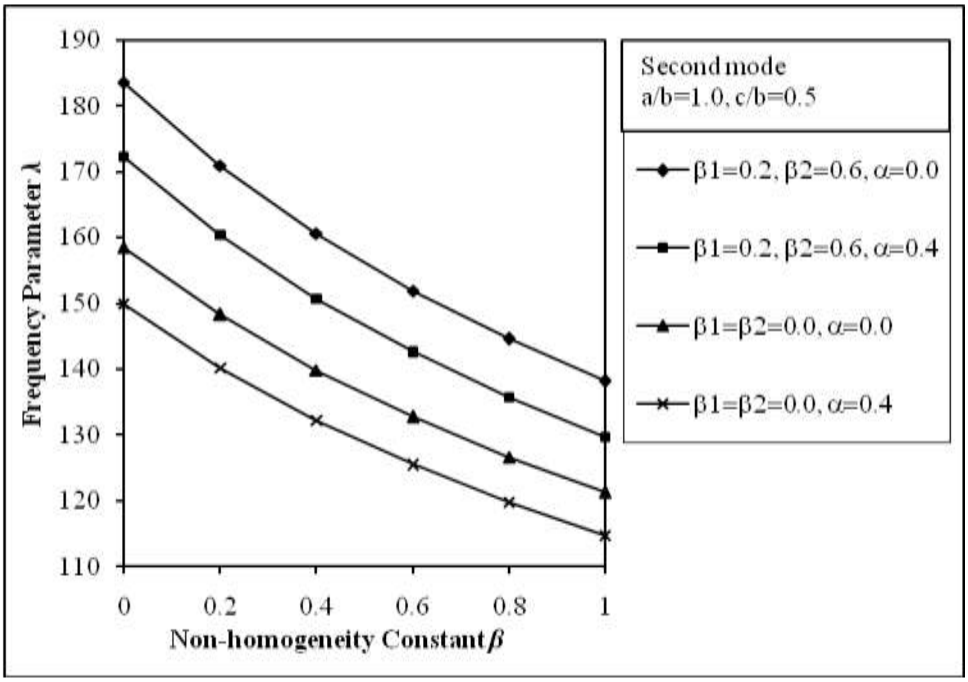

Figure 6(b): Variation of frequency parameter λ for different values of non-homogeneity constant β for the second mode.

5. CONCLUSION

On the basis of classical plate theory the vibration behaviour of non-homogeneous trapezoidal plate of variable thickness and density has been examined by taking a two term deflection function. Using Rayleigh-Ritz method numerical results of natural frequencies for first two modes of vibration corresponding to C-S-C-S boundary condition has been calculated. To study the effect of thermal gradient on the frequencies of non-homogeneous trapezoidal plate with other plate's parameters such as taper constants, non-homogeneity constant, aspect ratio is the significant purpose of the present work. From the graphical representation some of the important results have been obtained. Thus, it can be concluded that the frequency increases with the increase of taper constants and frequency decreases with the increase of thermal gradient, aspect ratio and non-homogeneity constant.

Our main motive of research is prosperity of human beings and the development of technology by providing a theoretical mathematical model for scientists and design engineers. The research work is done in such a way that the technologists can become conscious of their prospective in various fields and increase their effectiveness and lastingness practically according to the need of the hour. Therefore, mechanical engineers are suggested to provide much better structure and machines with economy with the help of study of present research paper.

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