Kind of Weak Separation Axioms by $D_{\omega}, D_{\alpha-\omega}, D_{pre-\omega}, D_{b-\omega}$ and $D_{\beta-\omega}$ −Sets

Mustafa H. Hadi
(Babylon/ Hillah, Iraq)
University of Babylon, College of Education for Pure Science,
Mathematics Department

Email: mustafahh1984 [AT] uobabylon.edu.iq

ABSRACT---- ***In this paper we define new types of sets we call them $D_{\omega}, D_{\alpha-\omega}, D_{pre-\omega}, D_{pre-\omega}, D_{b-\omega}$, and $D_{\beta-\omega}$ −sets and use them to define some associative separation axioms. Some theorems about the relation between them and the weak separation axioms introduced in [5] are proved, with some other simple theorems.***

Keywords--- Separation axioms, weak open sets, T_i spaces.

---

1. INTRODUCTION

Throughout this paper , (X, T) stands for topological space. Let (X, T) be a topological space and A a subset of X. A point x in X is called condensation point of A if for each U in T with x in U, the set $U \cap A$ is uncountable [6]. In 1982 the ω −closed set was first introduced by H. Z. Hdeib in [6], and he defined it as: A is ω −closed if it contains all its condensation points and the ω −open set is the complement of the ω −closed set. Equivalently. A sub set W of a space (X, T), is ω −open if and only if for each $x \in W$, there exists $U \in T$ such that $x \in U$and $U \backslash W$ is countable. The collection of all ω −open sets of (X, T) denoted T_{ω} form topology on X and it is finer than T. Several characterizations of ω −closed sets were provided in [1,6,7].

In [3,8,9] some authors introduced α −open , pre −open, b −open , and β −open sets. On the other hand in [10] T. Noiri, A. Al-Omari, M. S. M. Noorani introduced the notions $\alpha-\omega$ −open, pre −ω −open, $\beta-\omega$ −open, and b −ω −open sets in topological spaces. In [2,5] used the ω − open sets to define types of weak separation axioms called $\omega - R_0$, $\omega - R_1$and $\omega^* - T_1$ spaces .They defined them as follows:

Definition 1.1. [10] A subset A of a space X is called:

1. $\alpha-\omega$ −open if $A \subseteq \mathrm{int}_{\omega}\left(\mathrm{cl}\left(\mathrm{int}_{\omega}(A)\right)\right)$ and the complement of the $\alpha-\omega$ −open set is called $\alpha-\omega$ −closed set.

2. pre −ω −open if $A \subseteq \mathrm{int}_{\omega}\left(\mathrm{cl}(A)\right)$ and the complement of the pre − ω −open set is called pre − ω −closed set.

3. b −ω −open if $A \subseteq \mathrm{int}_{\omega}\left(\mathrm{cl}(A)\right) \cup \mathrm{cl}\left(\mathrm{int}_{\omega}(A)\right)$ and the complement of the b − ω −open set is called b − ω −closed set.

4. $\beta-\omega$ −open if $A \subseteq \mathrm{cl}\left(\mathrm{int}_{\omega}\left(\mathrm{cl}(A)\right)\right)$ and the complement of the $\beta-\omega$ −open set is called $\beta-\omega$ −closed set.

In [10] T. Noiri, A. Al-Omari, M. S. M. Noorani introduced relationships among the weak open sets above by the lemma below:

Lemma 1.2. [10] In any topological space:

1. Any open set is ω −open.

2. Any ω −open set is $\alpha-\omega$ −open.

3. Any $\alpha-\omega$ −open set is pre – ω −open.

4. Any pre − ω −open set is b – ω −open.

5. Any b − ω −open set is $\beta-\omega$ −open.

The converse is not true [10].

For our results in this paper we need the following definitions:

Definition 1.3. [10] A subset A of a space X is called

1. An $\omega - t -$set, if $\mathrm{int}(A) = \mathrm{int}_{\omega}(\mathrm{cl}(A))$.

2. An $\omega - B -$set if $A = U \cap V$, where U is an open set and V is an $\omega - t -$set.

3. An $\omega - t_{\alpha} -$set, if $\mathrm{int}(A) = \mathrm{int}_{\omega}(\mathrm{cl}(\mathrm{int}_{\omega}(A)))$.

4. An $\omega - B_{\alpha} -$set if $A = U \cap V$, where U is an open set and V is an $\omega - t_{\alpha} -$set.

5. An $\omega -$set if $A = U \cap V$, where U is an open set and $\mathrm{int}(V) = \mathrm{int}_{\omega}(V)$.

Definition 1.4. [5] Let (X, T) be topological space. It said to be satisfy

1. The $\omega -$condition if every $\omega -$open set is $\omega - t -$set.

2. The $\omega - B_{\alpha} -$condition if every $\alpha - \omega -$open set is $\omega - B_{\alpha} -$set.

3. The $\omega - B -$condition if every $\mathrm{pre} - \omega -$open is $\omega - B -$set.

Lemma 1.5. [10] For any subset A of a space X, We have

1. A is open if and only if A is $\omega -$open and $\omega -$set.

2. A is open If and only if A is $\alpha - \omega -$open and $\omega - B_{\alpha} -$set.

3. A is open if and only if A is $\mathrm{pre} - \omega -$open and $\omega - B -$set.

Lemma 1.6. If (X, T) is a door space, then

1.Every $\mathrm{pre} - \omega -$open set is $\omega -$open. [10]

2.Every $\beta - \omega -$open set is is $b - \omega -$open.[5]

Lemma 1.7. [10] Let (X, T) be a topological space and let $A \subseteq X$. If A is $b - \omega -$open set such that $\mathrm{int}_{\omega}(A) = \emptyset$, then A is $\mathrm{pre} - \omega -$open.

The classes of the sets in Definition 1.1 are larger than that sets in [3,8,9]. In [5] we introduce some weak separation axioms by utilizing the notions of T. Noiri, A. Al-Omari, M. S. M. Noorani. Let us summarize them in the following definitions.

Definition 1.3.[5] Let X be a topological space. If for each $x \neq y \in X$, either there exists a set U, such that $x \in U, y \notin U$,or there exists a set U such that $x \notin U, y \in U$. Then X called

1. $\omega - T_0$ space, whenever U is $\omega -$open set in X.

2. $\alpha - \omega - T_0$ space, whenever U is $\alpha - \omega -$open set in X.

3. $\mathrm{pre}-\omega - T_0$ space, whenever U is $\mathrm{pre} -\omega -$open set in X.

4. $b - \omega - T_0$ space, whenever U is $b - \omega -$open set in X.

5. $\beta - \omega - T_0$ space, whenever U is $\beta - \omega -$open set in X.

Definition 1.4.[5] Let X be a topological space. For each $x \neq y \in X$, there exists a set U, such that $x \in U, y \notin U$, and there exists a set V such that $y \in V, x \notin V$, then X is called

1. $\omega - T_1$ space if U is open and V is $\omega -$open sets in X.

2. $\alpha - \omega - T_1$ space if U is open and V is $\alpha - \omega -$open sets in X.

3. $\omega^{\star} - T_1$ space [1] if U and V are $\omega -$open sets in X.

4. $\alpha - \omega^{\star} - T_1$ space if U is ω-open and V is $\alpha - \omega -$open sets in X.

5. $\alpha - \omega^{\star\star} - T_1$ space if U and V are $\alpha - \omega -$open sets in X.

6. $\mathrm{pre} -\omega - T_1$ space if U is open and V is $\mathrm{pre} - \omega -$open sets in X.

7. $\mathrm{pre} -\omega^{\star} - T_1$ space if U is ω-open and V is $\mathrm{pre} -\omega -$open sets in X.

8. $\alpha - \mathrm{pre} -\omega - T_1$ space if U is $\alpha - \omega -$ open and V is $\mathrm{pre} -\omega -$open sets in X.

9. pre $-\omega^{\star\star} - T_1$ space if U and V are pre $-\omega$ $-$open sets in X.

10. $b - \omega - T_1$ space if U is open and V is $b - \omega$ $-$open sets in X.

11. $b - \omega^{\star} - T_1$ space if U is ω $-$open and V is $b - \omega$ $-$open sets in X.

12. $\alpha - b - \omega - T_1$ space if U is $\alpha - \omega$ $-$open and V is $b - \omega$ $-$open sets in X.

13.pre $- b - \omega - T_1$ space if U is pre $-\omega$ $-$open and V is $b - \omega$ $-$open sets in X.

14. $b - \omega^{\star\star} - T_1$ space if U and V are $b - \omega$ $-$open sets in X.

15. $\beta - \omega - T_1$ space if U is open and V is $\beta - \omega$ $-$open sets in X.

16. $\beta - \omega^{\star} - T_1$ space if U is ω $-$open and V is $\beta - \omega$ $-$open sets in X.

17. $\alpha - \beta{-}\omega - T_1$ space if U is $\alpha - \omega$ $-$open and V is $\beta - \omega$ $-$open sets in X.

18.▯▯▯ − ▯ − ▯ − ▯▯ space if U is pre $-\omega$ $-$open and V is $\beta - \omega$ $-$open sets in X.

19. $\beta - \omega^{\star\star} - T_1$ space if U and V are $\beta - \omega$ $-$open sets in X.

20. $b - \beta{-}\omega - T_1$ space if U is $b - \omega$ $-$open and V is $\beta - \omega$ $-$open sets in X

Definition 1.5. [5] Let X be a topological space. And for each $x \neq y \in X$, there exist two disjoint sets U and V with $x \in U$ and $y \in V$, then X is called:

1. $\omega - T_2$ space if U is open and V is ω $-$open sets in X.

2. $\alpha - \omega - T_2$ space if U is open and V is $\alpha - \omega$ $-$open sets in X.

3. $\omega^{\star} - T_2$ space if U and V are ω $-$open sets in X.

4. $\alpha - \omega^{\star} - T_2$ space if U is ω $-$open and V is $\alpha - \omega$ $-$open sets in X.

5. $\alpha - \omega^{\star\star} - T_2$ space if U and V are $\alpha - \omega$ $-$open sets in X.

6. pre $-\omega - T_2$ space if U is open and V is pre $-\omega$ $-$open sets in X.

7. pre $-\omega^{\star} - T_2$ space if U is ω $-$open and V is pre $-\omega$ $-$open sets in X.

8. $\alpha -$ pre $-\omega - T_2$ space if U is α $-$open and V is pre $-\omega$ $-$open sets in X.

9. pre $-\omega^{\star\star} - T_2$ space if U and V are pre $-\omega$ $-$open sets in X.

10. $b - \omega - T_2$ space if U is open and V is $b - \omega$ $-$open sets in X.

11. $b - \omega^{\star} - T_2$ space if U is ω $-$open and V is $b - \omega$ $-$open sets in X.

12. $\alpha - b - \omega - T_2$ space if U is $\alpha - \omega$ $-$open and V is $b - \omega$ $-$open sets in X.

13. pre $- b - \omega - T_2$ space if U is pre $-\omega$ $-$open and V is $b - \omega$ $-$open sets in X.

14. $b - \omega^{\star\star} - T_2$ space if U and V are $b - \omega$ $-$open sets in X.

15. $\beta - \omega - T_2$ space if U is open and V is $\beta - \omega$ $-$open sets in X.

16. $\beta - \omega^{\star} - T_2$ space if U is ω $-$open and V is $\beta - \omega$ $-$open sets in X.

17. $\alpha - \beta{-}\omega - T_2$ space if U is $\alpha - \omega$ $-$open and V is $\beta - \omega$ $-$open sets in X.

18. pre$-\beta - \omega - T_2$ space if U is pre $-\omega$ $-$open and V is $\beta - \omega$ $-$open sets in X.

19. $\beta - \omega^{\star\star} - T_2$ space if U and V are $\beta - \omega$ $-$open sets in X.

20. $b - \beta{-}\omega - T_2$ space if U is $b - \omega$ $-$open and V is $\beta - \omega$ $-$open sets in X.

2. $D_{\omega}, D_{\alpha-\omega}, D_{pre-\omega}, D_{b-\omega}$ AND $D_{\beta-\omega}$ $-$SETS

In this article we shall define new types of sets and use them to define new spaces with associative separation axioms.

Definition 2.1. A subset A of a topological space (X, T) is called D −set [4] (resp.D_{ω} −set , $D_{\alpha-\omega}$ −set, $D_{pre-\omega}$ −set, $D_{b-\omega}$ −set, $D_{\beta-\omega}$ −set). If there are two open (resp. ω −open, α − ω −open, pre − ω −open, β − ω −open, and b −ω −open) sets U and V with $U \neq X$ and $A = U \setminus V$.

Remark 2.2. It is true that every ω −open, (resp. α − ω −open, pre − ω −open, b −ω −open, and β − ω −open) set $U \neq X$ is D_{ω} −set (resp. $D_{\alpha-\omega}$ −set, $D_{pre-\omega}$ −set, $D_{b-\omega}$ −set, and $D_{\beta-\omega}$ −set) if $A = U$ and $V = \emptyset$.

Using Definition 2.1 and Lemma1.2, Lemma 1.6, and Lemma1.5 we can easily prove the following Propositions:

Proposition 2.3. In any topological space X.

1. Any D −set is D_{ω} −set.

2. Any D_{ω} −set is $D_{\alpha-\omega}$ −set.

3. Any $D_{\alpha-\omega}$ −set is $D_{pre-\omega}$ −set.

4. Any $D_{pre-\omega}$ −set is $D_{b-\omega}$ −set.

5. Any $D_{b-\omega}$ −set is $D_{\beta-\omega}$ −set.

Proposition 2.4. In any topological door space :

1.Any $D_{pre-\omega}$ −set is D_{ω} −set.

2.Any $D_{\beta-\omega}$ −set is $D_{b-\omega}$ −set.

Proposition 2.5. In any topological space satisfies ω −condition. Any D_{ω} −set is D −set**.**

Proposition 2.6. In any topological space satisfies $\omega - B_{\alpha}$ −condition. Any $D_{\alpha-\omega}$ −set is D −set.

Proposition 2.7. In any topological space satisfies ω − B −condition. Any $D_{pre-\omega}$ −set is D −set.

Proposition 2.8. In any topological space. Any $D_{b-\omega}$ −set with empty ω −interior is $D_{pre-\omega}$ −set .

Proof:

Let X be a topological space, and let A be a $D_{b-\omega}$ −set with empty ω −interior in X, then there are two b − ω −open which are by Lemma 1.7 also pre − ω −open sets U and V with $U \neq X$, and $A = U \setminus V$

Similarly we can prove the other cases.

From the lemmas above we can get the following figure:

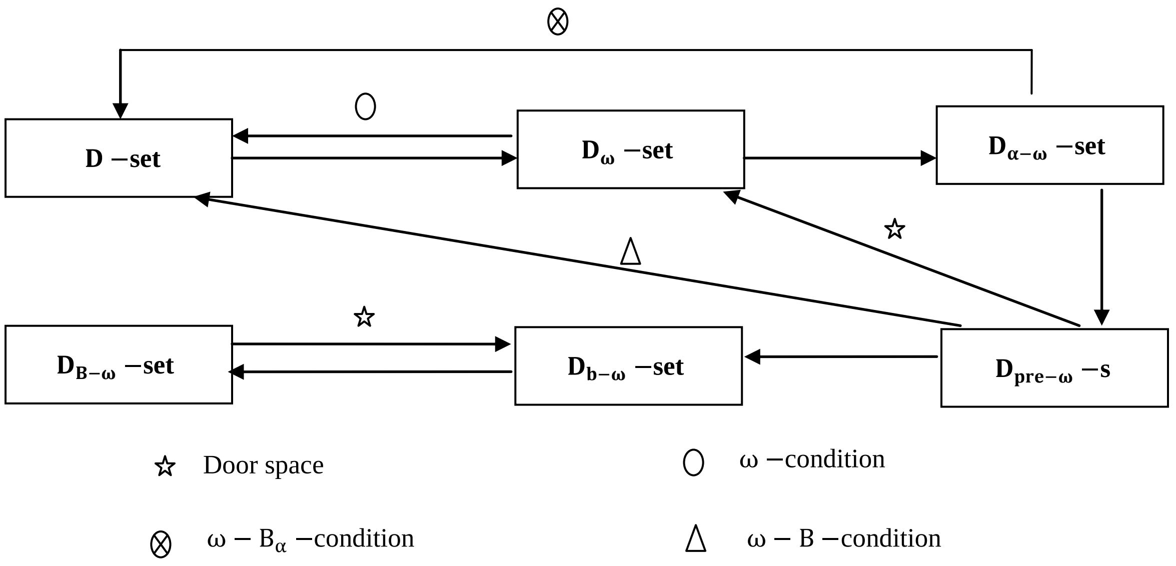

Figure 1: Relation among the weak D −sets

3. $D_{\omega}, D_{\alpha-\omega}, D_{pre-\omega}, D_{b-\omega}$ AND $D_{\beta-\omega}$ −SETS AND ASSOCIATIVE SEPARATION AXIOMS

Utilizing the weak D_ω sets we can define our separation axioms as follows:

Definition 3.1. Let X be a topological space. If $x \neq y \in X$, either there exists a set U, such that $x \in U$, $y \notin U$,or there exists a set U such that $x \notin U, y \in U$. Then X called

1. $\boldsymbol{\omega - D_0}$ ***space***, whenever U is D_ω −set in X.

2. $\boldsymbol{\alpha - \omega - D_0}$ ***space,*** whenever U is $D_{\alpha-\omega}$ −set in X.

3. $\boldsymbol{pre{-}\omega - D_0}$ ***space***, whenever U is $D_{pre-\omega}$ −set in X.

4. $\boldsymbol{b - \omega - D_0}$ ***space***, whenever U is $D_{b-\omega}$ −set in X.

5. $\beta - \omega - D_0$ space, whenever U is $D_{\beta-\omega}$ −set in X.

Definition 3.2. We can define the spaces $\omega - D_i$, $\alpha - \omega - D_i$, $pre - \omega - D_i$, $b - \omega - D_i$, $\beta - \omega - D_i$, for $i = 0,1,2$. And $\omega^\star - D_i$, $\alpha - \omega^\star - D_i$, $\alpha - \omega^{\star\star} - D_i$, $pre - \omega^\star - D_i$, $\alpha - pre - \omega - D_i$, $\mathrm{pre} - \omega^{\star\star} - D_i$, $b - \omega^\star - D_i$, $\mathrm{pre} - b - \omega - D_i$, $\alpha - b - \omega - D_i$, $b - \omega^{\star\star} - D_i$, $\beta - \omega^\star - D_i$, $\alpha - \beta{-}\omega - D_i$, $\mathrm{pre} - \beta - \omega - D_i$, $\beta - \omega^{\star\star} - D_i$, and $b - \beta{-}\omega - D_i$, for $i = 1,2$, by replacing the sets: open, ω −open, $\alpha - \omega$ −open, pre − ω-open, b −ω-open, β − ω-open, by the D −set, D_ω −set , $D_{\alpha-\omega}$ −set, $D_{pre-\omega}$ −set, $D_{b-\omega}$ −set, and $D_{\beta-\omega}$ −set respectively, in Definition 1.3, Definition 1.4, and Definition 1.5.

Remark 3.3. For the relations among weak D_i, $i = 0,1,2$ we can make a figures coincide with these for weak T_is spaces in [5].

Theorem 3.4. Let (X, T) be a topological space:

1. If (X, T) is $\omega - T_i$, (resp. $\alpha - \omega - T_i$, $pre - \omega - T_i$, $b - \omega - T_i$, $\beta - \omega - T_i$, for $i = 0,1,2$, and $\omega^\star - T_i$, $\alpha - \omega^\star - T_i$, $\alpha - \omega^{\star\star} - T_i$, $pre - \omega^\star - T_i$, $\alpha - pre - \omega - T_i$, $b - \omega - T_i$, $pre - \omega^{\star\star} - T_i$, $b - \omega - T_i$, $b - \omega^\star - T_i$, $pre - b - \omega - T_i$, $\alpha - b - \omega - T_i$, $pre - b - \omega - T_i$, $b - \omega^{\star\star} - T_i$, $\beta - \omega^\star - T_i$, $\alpha - \beta{-}\omega - T_i$, $pre - \beta - \omega - T_i$, $\beta - \omega^{\star\star} - T_i$, and $b - \beta{-}\omega - T_i$ for $i = 1,2$), then it is $\omega - D_i$, (resp. $\alpha - \omega - D_i$, $pre - \omega - D_i$, $b - \omega - D_i$, $\alpha - b - \omega - D_i$, $\beta - \omega - D_i$, for $i = 0,1,2$, and $pre - b - \omega - D_i$, $b - \omega^{\star\star} - D_i$, $\omega^\star - D_i$, $\alpha - \omega^\star - D_i$, $\alpha - \omega^{\star\star} - D_i$, $pre - \omega^\star - D_i$, $\alpha - pre - \omega - D_i$, $pre - \omega^{\star\star} - D_i$, $b - \omega - D_i$, $b - \omega^\star - D_i$, $pre - b - \omega - D_i$, $\beta - \omega^\star - D_i$, $\alpha - \beta{-}\omega - D_i$, $pre - \beta - \omega - D_i$, $\beta - \omega^{\star\star} - D_i$, and $b - \beta{-}\omega - D_i$ for $i = 1,2$).

2. If (X, T) is $\omega - D_i$, (resp. $\alpha - \omega - D_i$, $\omega^\star - D_i$, $\alpha - \omega^\star - D_i$, $\alpha - \omega^{\star\star} - D_i$, $pre - \omega - D_{1i}$, $pre - \omega^\star - D_i$, $\alpha - pre - \omega - D_i$, $b - \omega - D_i$, $pre - \omega^{\star\star} - D_i$, $b - \omega - D_i$, $b - \omega^\star - D_i$, $pre - b - \omega - D_i$, $\alpha - b - \omega - D_i$, $pre - b - \omega - D_i$, $b - \omega^{\star\star} - D_i$, $\beta - \omega - D_i$, $\beta - \omega^\star - D_i$, $\alpha - \beta{-}\omega - D_i$, $pre - \beta - \omega - D_i$, . $\beta - \omega^{\star\star} - D_i$, $b - \beta{-}\omega - D_i$), then it is $\omega - D_{i-1}$, (resp. $\alpha - \omega - D_{i-1}$, $\omega^\star - D_{i-1}$, $\alpha - \omega^\star - D_{i-1}$, $\alpha - \omega^{\star\star} - D_{i-1}$, $pre - \omega - D_{i-1}$, $pre - \omega^\star - D_{i-1}$, $\alpha - pre - \omega - D_{i-1}$, $b - \omega - D_{i-1}$, $pre - \omega^{\star\star} - D_{i-1}$, $b - \omega - D_{i-1}$, $b - \omega^\star - D_{i-1}$, $pre - b - \omega - D_{i-1}$, $\alpha - b - \omega - D_{i-1}$, $pre - b - \omega - D_{i-1}$, $b - \omega^{\star\star} - D_{i-1}$, $\beta - \omega - D_{i-1}$, $\beta - \omega^\star - D_{i-1}$, $\alpha - \beta{-}\omega - D_{i-1}$, $pre - \beta - \omega - D_{i-1}$, $\beta - \omega^{\star\star} - D_{i-1}$, and $b - \beta{-}\omega - D_{i-1}$), for $i = 1,2$.

Proof:

1. Follows immediately by the Remark 3.3.

2. Directly from Definition 2.1. Definition 3.1, and Definition 3.2.

By the following theorems we recognize the importance of the weak D_i-spaces, for $i = 0,1,2$.

Theorem 3.5. Let (X, T) be a topological space. Then X is $\omega - D_1$,(resp. $\alpha - \omega - D_1$, $\omega^\star - D_1$, $\alpha - \omega^\star - D_1$, $\alpha - \omega^{\star\star} - D_1$, $pre - \omega - D_1$, $pre - \omega^\star - D_1$, $\alpha - pre - \omega - D_1$, $b - \omega - D_1$, $pre - \omega^{\star\star} - D_1$, $b - \omega - D_1$, $b - \omega^\star - D_1$, $pre - b - \omega - D_1$, $\alpha - b - \omega - D_1$, $pre - b - \omega - D_1$, $b - \omega^{\star\star} - D_1$, $\beta - \omega - D_1$, $\beta - \omega^\star - D_1$, $\alpha - \beta{-}\omega - D_1$, $pre - \beta - \omega - D_1$, $\beta - \omega^{\star\star} - D_1$, $b - \beta{-}\omega - D_1$) if and only if it is $\omega - D_2$,(resp. $\alpha - \omega - D_2$, $\omega^\star - D_2$, $\alpha - \omega^\star - D_2$, $\alpha - \omega^{\star\star} - D_2$, $pre - \omega - D_2$, $pre - \omega^\star - D_2$, $\alpha - pre - \omega - D_2$, $b - \omega - D_2$, $pre - \omega^{\star\star} - D_2$, $b - \omega - D_2$, $b - \omega^\star - D_2$, $pre - b - \omega - D_2$, $\alpha - b - \omega - D_2$, $pre - b - \omega - D_2$, $b - \omega^{\star\star} - D_2$, $\beta - \omega - D_2$, $\beta - \omega^\star - D_2$, $\alpha - \beta{-}\omega - D_2$, $pre - \beta - \omega - D_2$, $\beta - \omega^{\star\star} - D_2$, $b - \beta{-}\omega - D_2$).

Proof:

The proof of the forward direction is a step by step similar to that of Theorem 4.8 in [4] .The inverse direction follows immediately from (2) of theorem 3.4 above.

Theorem 3.6. Let (X, T), be a topological space. Then X is $\alpha - \omega - T_0$ (resp. $\omega - T_0$, $pre{-}\omega - T_0$, $b - \omega - T_0$, $\beta - \omega - T_0$) if and only if it is $\alpha - \omega - D_0$ (resp. $\omega - D_0$, $pre{-}\omega - D_0$, $b - \omega - D_0$, $\beta - \omega - D_0$) .

Proof:

The forward direction follows immediately from (1). of Theorem 3.4.

For the opposite side let X be $\alpha-\omega-D_0$, so for $x \neq y \in X$, there is a $D_{\alpha-\omega}$ −set U such that $x \in U$, but $y \notin U$. Then by the definition of the $D_{\alpha-\omega}$ −set , $U = W \backslash V$, where V and $W \neq X$ are $\alpha-\omega-$ open sets. Now if $x \in W$, but $y \notin W$, and W is an $\alpha-\omega-$ open set in X. So X is $\alpha-\omega-T_0$. Then whenever $x \in U = W \backslash V$ and $y \in (W \cap V)$. Then $y \in V$, and $x \notin V$. Thus X is $\alpha-\omega-T_0$ space.

For the following definition we need the definition of the ω −neighbourhood from [5]:

Definition 3.7.[5] Let (X, T) be a topological space. A subset U of X is ω −neighbourhood of a point $x \in X$, if and only if there exists an ω −open set V such that $x \in V \subseteq U$.

Definition 3.8. A point $x \in X$ which has only X as ω −neighbourhood is called an ω −net point.

Proposition 3.9. Let (X, T) be a topological space If X is $\omega-D_1$ space, then it has no ω −net point.

Proof:

Since X is $\omega-D_1$ so each point x of X contained in a D_ω −set $W = U \backslash V$, $U \neq X$, and U and V are ω −open sets. So it contained in the ω −open set $U \neq X$, which implies x is no ω −net point.

Theorem 3.10. Let X be a door topological space, has no ω −net point . Then it is $\omega-D_1$ space.

Proof:

Since (X, T) be a door topological space, so for each point x in X , {x} is either ω −open or ω −closed. This implies for each $x \neq y \in X$, at least one of them say x has ω −neighbourhood $U \neq X$ containing x but not y, U is D_ω −set. If X has no ω −net point, then y is not ω −net point , so there is an ω −neighbourhood $V \neq X$ of y. Thus $V \backslash U$ is D_ω −set containing y but not x. Hence X is $\omega-D_1$ space .

To introduce Theorem 3.12 we need the following Definition from [5]:

Definition 3.11. [5] Let (X, σ) and (Y, τ) be two topological spaces. A map $f: (X, \sigma) \to (Y, \tau)$ is called ω-continuous (resp. α − ω-continuous, pre −ω −continuous, b −ω −continuous and β − ω −continuous) at $x \in X$, if and only if for each ω −open (resp. α − ω −open, pre −ω −open, b −ω −open and β − ω −open) set V containing f(x), there exists an ω −open (resp. α − ω −open, pre −ω −open, b −ω −open and β − ω −open) set U containing x, such that $f(U) \subset V$. If f is ω − continuous (resp. α − ω − continuous, pre −ω − continuous, b −ω − continuous and β − ω − continuous) at each $x \in X$, we call it ω − continuous (resp. α − ω − continuous, pre −ω − continuous, b −ω −continuous and β − ω −continuous).

Theorem 3.12. If $f: (X, \tau) \to (Y, \sigma)$ is ω − continuous (resp. α − ω -continuous , pre −ω -continuous, β − ω − continuous, b −ω − continuous) onto function and A is D_ω − set (resp. $D_{\alpha-\omega}$ − set, $D_{pre-\omega}$ − set, $D_{b-\omega}$ − set, $D_{\beta-\omega}$ −set) in Y, then $f^{-1}(A)$ is also D_ω −set (resp. $D_{\alpha-\omega}$ −set, $D_{pre-\omega}$ −set, $D_{b-\omega}$ −set, $D_{\beta-\omega}$ −set) in X.

Proof:

Let A be D_ω − set in Y, so there are two ω − open sets $U \neq Y$, V in Y such that $A = U \backslash V$. Then by the ω −continuous function definition, we have $f^{-1}(U)$ and $f^{-1}(V)$ are ω −open sets in X, such that $f^{-1}(U) \neq X$. And $f^{-1}(A) = f^{-1}(U \backslash V) = f^{-1}(U) \backslash f^{-1}(V)$ is D_ω −set in X .

The other cases are the same .

Theorem 3.13. For any two topological spaces (X, τ) and (Y, σ).

1. If (Y, σ) be an $\omega^\star-D_1$ and $f: (X, \tau) \to (Y, \sigma)$ is an ω −continuous bijection, then (X, τ) is $\omega^\star-D_1$.

2. If (Y, σ) be an , $\alpha-\omega^{\star\star}-D_1$ and $f: (X, \tau) \to (Y, \sigma)$ is an α − ω −continuous bijection, then (X, τ) is, $\alpha-\omega^{\star\star}-D_1$.

3. If (Y, σ) be a , $pre-\omega^{\star\star}-D_1$ and $f: (X, \tau) \to (Y, \sigma)$ is a pre − ω-continuous bijection, then (X, τ) is $pre-\omega^{\star\star}-D_1$.

4. If (Y, σ) be a , $b-\omega^{\star\star}-D_1$ and $f: (X, \tau) \to (Y, \sigma)$ is a b − ω −continuous bijection, then (X, τ) is $b-\omega^{\star\star}-D_1$.

5. If (Y, σ) be a , $\beta-\omega^{\star\star}-D_1$ and $f: (X, \tau) \to (Y, \sigma)$ is a β − ω −continuous bijection, then (X, τ) is $\beta-\omega^{\star\star}-D_1$.

Proof of (1):

Let Y be an $\omega^{\star} - D_1$ space. Let $x \neq y \in X$, since f is bijective and Y is $\omega^{\star} - D_1$ space , so there exist two D_ω −sets U and V such that U containing f(x) but not f(y) and V containing f(y) but not f(x), then by Theorem 3.12. $f^{-1}(U)$ and $f^{-1}(V)$ are D_ω −sets such that $f^{-1}(U)$ containing x but not y and $f^{-1}(V)$ containing y but not x. So (X, τ) is $\omega^{\star} - D_1$.

By the same way we can prove the other cases .

Theorem 3.14. A topological space (X, T) is $\omega^{\star} - D_1$ (resp. $\alpha - \omega^{\star\star} - D_1$, pre $-\omega^{\star\star} - D_1$, , b $-\omega^{\star\star} - D_1$, $\beta - \omega^{\star\star} - D_1$) if and only if for each pair of distinct points $x, y \in X$, there exists an ω −continuous (resp. $\alpha - \omega$ −continuous, pre − ω-continuous, b − ω-continuous, β − ω-continuous) onto function $f: (X, \tau) \to (Y, \sigma)$ such that f(x) and f(y) are distinct, where (Y, σ) is $\omega^{\star} - D_1$ (resp. $\alpha - \omega^{\star\star} - D_1$, pre $-\omega^{\star\star} - D_1$, b $-\omega^{\star\star} - D_1$, $\beta - \omega^{\star\star} - D_1$) space.

Proof:

Let (X, τ) be an $\omega^{\star} - D_1$, let $x, y \in X$, then we can find an onto function $f: (X, \tau) \to (Y, \sigma)$, where (Y, σ) is an $\omega^{\star} - D_1$ is defined by $f(x) = x$, such that $f(x)$ and $f(y)$ distinct. For the opposite direction. Let $x \neq y \in X$, and $f: (X, \tau) \to (Y, \sigma)$ be an onto ω −continuous function such that $f(x)$ and $f(y)$ distinct, and (Y, σ) is $\omega^{\star} - D_1$ space. We must prove (X, τ) is $\omega^{\star} - D_1$ space. Since (Y, σ) is an $\omega^{\star} - D_1$ space and $f(x)$ and $f(y)$ are distinct points in it , then by Theorem 3.5 there are two distinct disjoint D_ω −sets U and V in Y such that U containing $f(x)$ and V containing $f(y)$. Then since f is ω −continuous function so $f^{-1}(U)$ and $f^{-1}(V)$ are two disjoint D_ω −sets in X such that $f^{-1}(U)$ containing x and $f^{-1}(V)$ containing y. So (X, T) is $\omega^{\star} - D_2$, and by Theorem 3.5. again , we get (X, T) is $\omega^{\star} - D_1$ space.

4. REFERENCES

[1]. A. Al-Omari and M. S. M. Noorani, " *Regular generalized ω-closed sets*", Internat. J. Math. Math. Sci., vo. 2007. Article ID 16292, doi: 10.1155/2007/16292, 11 pages, 2007.

[2]. *Luay. A. Al Swidi , Mustafa. H. Hadi ," Characterizations of Continuity and Compactness with Respect to Weak Forms of ω-Open Sets ", European Journal of Scientific Research,Vol.57, No.4, pp.577-582, 2011.*

[3]. D. Andrijevic, " *On b-open sets*" , Mat. Vesnik 48 ,pp. 59-64, 1996.

[4]. *S. Athisaya and M. Lellis, " Another form of separation axioms", Methods of Functional Analysis and Topolog,. vol.13,no.4,pp. 380-385 ,2007.*

[5]. *M. H. Hadi, " Weak forms of ω-open sets and decomposition of separation axioms" , M. Sc. Thesis, Babylon University (2011).*

[6]. H. Z. Hdeib, " *ω-closed mappings*", Rev. Colomb. Maht,vol.16,no. 3-4, pp. 65-78, 1982.

[7]. H. Z. Hdeib, " *ω-continuous functions*", Dirasat, vol. 16, no.2, pp. 136-142 1989.

[8]. A. S. Mashhour, M. E. Abd El- Monsef and S. N. El- Deeb, " *On pre-continuous and weak pre-continuous functions* ", Proc. Math. Phys. Soc. Egypt, no. 51, pp. 47-53, 1982.

[9]. O. Njastad, " *On some classes of nearly open sets* ", Pacific. J. Math. 15, pp. 961-970, 1965.

[10]. T. Noiri, A. Al-Omari and M. S. M. Noorani, " *Weak forms of ω-open sets and decomposition of continuity*", E.J.P.A.M. vol.2 , no. 1, pp. 73-84, 2009.