

# Effect of Non-homogeneity on Thermally Induced Vibration of Parallelogram Plate of Parabolically Varying Thickness

Arun Kumar Gupta<sup>1</sup> and Kumud<sup>2</sup>

<sup>1</sup>Department of Mathematics, M.S. College,  
Saharanpur, U.P., India

<sup>2</sup>Department of Mathematics, M.S. College,  
Saharanpur, U.P., India

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**ABSTRACT-** *The paper present here is to study the effect of non-homogeneity on thermally induced vibration of parallelogram plate of parabolically varying thickness. Thermal Induced vibration of such plates has been taken as one dimensional temperature distribution in linear form only. For non-homogeneity of the plate material, density is assumed to vary linearly. Using the method of separation of variables; the governing differential equation is solved. An approximate but quite convenient frequency equation is derived by Rayleigh-Ritz technique with two term deflection function. The frequencies corresponding to the first two modes of vibration has been computed for a clamped parallelogram plate for different values of non-homogeneity constant, aspect ratio, thermal constant, taper constant and skew angle.*

**Keywords-** Thermal, vibration, non-homogeneous, parallelogram plate, parabolically varying thickness.

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## 1. INTRODUCTION

Parallelogram plates have quite a good number of applications in modern structures. In the modern time, people have started taking lot of interest in effect of temperature on solids. This type of plates with variable thickness are of great importance in a wide variety of engineering applications i.e. construction of wings, fins of rockets, missiles. It has a lot of role in space technology, high-speed atmospheric flights and in nuclear energy applications. In addition, the non-homogeneity of longitudinal to transverse modules of these new materials demands improvement in the existing analytical tools. As a result, the analysis of plate's vibrations has attracted many research works, and has been considerably improved to achieve realistic results. As space technology has advanced, the need of the study of vibration of plates of certain aspect ratios with some simple restraints on the boundaries has also increased. The information for the first few modes of vibrations is essential for a constitution engineer before finalizing a design.

A survey of literature on vibration problems of skew plates shows that the vibration of skew plates has received rather less attention than that given to the other type i.e. rectangular, circular and elliptic plates. Leissa [1] contains an excellent discussion of the subject of vibrating plates. Gupta and Singhal [2-3] solved the problem of thermal effect on free vibration of non-homogeneous orthotropic visco-elastic rectangular plate of parabolically varying thickness. Gupta et al. [4-5] work out to investigate thermal effect on vibration of non-homogeneous orthotropic rectangular plate having bi-directional varying thickness. Gupta and Sharma [6] solved the problem of thermally induced vibration of orthotropic trapezoidal plate of linearly varying thickness. Gupta et al. [7] discuss the problem of thermal effect on vibrations of parallelogram plate of linearly varying thickness. Vibration of visco-elastic orthotropic parallelogram plate with linearly thickness variation is discussed by Gupta et al. [8]. Dokainish and Kumar [9] have discussed the problem of vibration of orthotropic parallelogram plates with variable thickness. Gupta et al. [10] solved the problem of visco elastic parallelogram plate of parabolically varying thickness. Vibration of skew plates was studied by Nair and Durvasula [11]. Singh and Saxena [12] have discussed transverse vibration of skew plates with variable thickness.

In the case of variable thickness plates, the governing differential equation of motion is found to have variable coefficient, this fact increases the difficulty of the solution. According to the study of plates thermal field generates non-homogeneity in elastic bodies and the material properties are not constant but vary with the position in random manner. It is well known [13] that in the presence of constant thermal gradient, the elastic coefficient of homogeneous materials become function of the space variable.

The aim of the present study is to determine the effect of non-homogeneity on free vibration of linear thermal gradient clamped parallelogram plate of parabolically varying thickness. The Rayleigh-Ritz technique has been used to determine the frequency equation of the plate. The frequencies of the first two modes of vibration are obtained for a

clamped parallelogram plate for various values non-homogeneity constant ( $\alpha_1$ ), aspect ratio ( $a/b$ ), thermal constant ( $\alpha$ ), taper constant ( $\beta$ ) and skew angle ( $\theta$ ).

## 2. TRANSVERSE EQUATION OF MOTION

The parallelogram plate is assumed to be non-uniform, thin and isotropic. The skew co-ordinate system used is shown in figure 1. The skew co-ordinate are related as

$$\xi = x - y \tan \theta \quad \text{and} \quad \eta = y \sec \theta \quad \dots \dots \dots (1)$$

The boundaries of the plate in skew coordinates are  
 $\xi = 0, \xi = a, \eta = a \eta = b \dots \dots \dots (2)$

For free vibration of the plate, the displacement function is periodic in time so it can expressed as  
 $W(\xi, \eta, t) = W(\xi, \eta) \sin \omega t \dots \dots \dots (3)$

where  $W(\xi, \eta)$  is the maximum displacement at time  $t$  and  $\omega$  is the angular frequency.

The maximum kinetic energy,  $T$ , and the strain energy  $V$  in the plate when it is executing transverse vibration mode shape  $W(\xi, \eta)$  are [7]:

$$T = \frac{1}{2} \omega^2 \cos \theta \iint h \rho W^2 d\xi d\eta \dots \dots \dots (4)$$

and

$$V = \frac{D}{2 \cos^3 \theta} \iint [W_{,\xi\xi}^2 - 4 \sin \theta W_{,\xi\xi} W_{,\xi\eta} + 2(\sin^2 \theta + \nu \cos^2 \theta) W_{,\xi\xi} W_{,\eta\eta} + 2(1 + \sin^2 \theta - \nu \cos^2 \theta) W_{,\xi\eta}^2 - 4 \sin \theta W_{,\eta\eta} W_{,\xi\eta} + W_{,\eta\eta}^2] d\xi d\eta \dots \dots \dots (5)$$

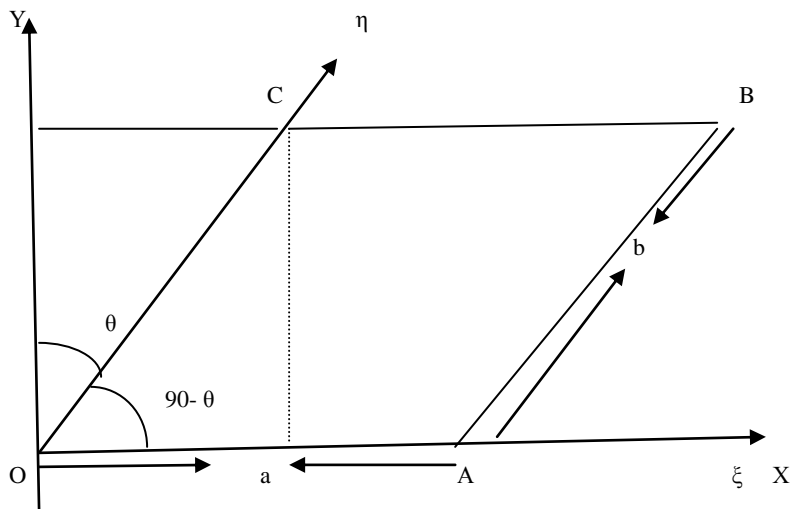


Figure 1: Geometry of parallelogram plate

A comma followed by a suffix denotes partial derivative with respect to that variable. Here  $D$  the flexural rigidity and given by

$$D = \frac{Eh^3}{24(1-\nu^2)} \dots \dots \dots (6)$$

Here  $\nu$  is the Poisson Ratio.

### 3. MATHEMATICAL ANALYSIS

Olsson [14] assumed that the parallelogram plate is subjected to a study one dimensional temperature distribution along the length, i.e. in  $\xi$  - direction.

$$\tau = \tau_0 \left(1 - \frac{\xi}{a}\right) \quad (7)$$

where  $\tau$  denotes the temperature excess above the reference temperature at any point at a distance  $\frac{\xi}{a}$  and  $\tau_0$  at  $\xi = a$ .

The temperature dependence of the modulus of elasticity is given by

$$E(\tau) = E_0 (1 - \gamma \tau) \dots\dots\dots (8)$$

where  $E_0$  is the value of Young modulus at  $\tau = 0$

Using (7) in (8), one obtains

$$E(\xi) = E_0 \left(1 - \alpha \left(1 - \frac{\xi}{a}\right)\right) \dots\dots\dots (9)$$

where  $\alpha = \gamma \tau_0 (0 < \alpha < 1)$ , a parameter known as thermal constant.

The thickness variation of the plate is assumed to be parabolic in  $\xi$  direction only, therefore one has

$$h = h_0 \left(1 + \beta \left(\frac{\xi}{a}\right)^2\right) \dots\dots\dots (10)$$

where  $\beta$  are taper constants &  $h = h_0$  at  $\xi = 0$

$$\text{Density of the plate } \rho = \rho_0 \left(1 - \alpha_1 \frac{\xi}{a}\right) \dots\dots\dots (11)$$

$\alpha_1$  is the non -homogeneity constant

Using the equation (10) and (11) in equation (4) and (5) one gets

$$T = \frac{1}{2} h_0 \rho_0 \omega^2 \cos \theta \int_{\eta=0}^b \int_{\xi=0}^a \left(1 - \alpha_1 \frac{\xi}{a}\right) \left(1 + \beta \left(\frac{\xi}{a}\right)^2\right) W^2 d\xi d\eta \dots\dots\dots (12)$$

$$V = \frac{E_0 h_0^3}{24(1-\nu^2)\cos^4\theta} \int_{\eta=0}^b \int_{\xi=0}^a \left(1 - \alpha \left(1 - \frac{\xi}{a}\right)\right) \left(1 + \beta \left(\frac{\xi}{a}\right)^2\right)^3 \left[ W_{,\xi\xi}^2 - 4 \left(\frac{a}{b}\right) \sin\theta W_{,\xi\xi} W_{,\xi\eta} + 2 \left(\frac{a}{b}\right)^2 (\sin^2\theta + \nu \cos^2\theta) W_{,\xi\xi} W_{,\xi\eta} + 2 \left(\frac{a}{b}\right)^2 (1 + \sin^2\theta - \nu \cos^2\theta) W_{,\xi\eta}^2 - 4 \left(\frac{a}{b}\right)^3 \sin\theta W_{,\xi\eta} W_{,\eta\eta} + \left(\frac{a}{b}\right)^4 W_{,\eta\eta}^2 \right] d\xi d\eta \dots\dots (13)$$

### 4. SOLUTION AND FREQUENCY EQUATION

Rayleigh-Ritz technique requires that the maximum strain energy must be equal to the maximum kinetic energy .It is therefore, necessary for the problem under consideration that

$$\delta (V-T) = 0 \dots\dots\dots (14)$$

for arbitrary variation of  $W$  satisfying relevant geometric boundary conditions.

For a parallelogram plate clamped along all four edges the boundary conditions are

$$W = W_{,\xi} = 0 \text{ at } \xi = 0, a \text{ and } W = W_{,\eta} = 0 \text{ at } \eta = 0, b \text{ and}$$

corresponding two term deflection is taken as

$$W(\xi, \eta) = \left(\frac{\xi^2}{a^2}\right) \left(\frac{\eta^2}{b^2}\right) \left(1 - \frac{\xi}{a}\right)^2 \left(1 - \frac{\eta}{b}\right)^2 \times [A_1 + A_2 \left(\frac{\xi}{a}\right) \left(\frac{\eta}{b}\right) \left(1 - \frac{\xi}{a}\right) \left(1 - \frac{\eta}{b}\right)] \dots\dots\dots (15)$$

Now equation (14) becomes after using equations (12) and (13)

$$\delta (V_1 - \lambda^2 T_1) = 0 \dots \dots \dots (16)$$

where

$$V_1 = \int_{\eta=0}^b \int_{\xi=0}^a \left( 1 - \alpha \left( 1 - \frac{\xi}{a} \right) \right) \left( 1 + \beta \left( \frac{\xi}{a} \right)^2 \right)^3 \left[ W_{\xi\xi}^2 - 4 \left( \frac{a}{b} \right) \sin\theta W_{\xi\xi} W_{\xi\eta} + 2 \left( \frac{a}{b} \right)^2 (\sin^2\theta + \nu \cos^2\theta) W_{\xi\xi} W_{\xi\eta} + 2 \left( \frac{a}{b} \right)^2 (1 + \sin^2\theta - \nu \cos^2\theta) W_{\xi\eta}^2 - 4 \left( \frac{a}{b} \right)^3 \sin\theta W_{\xi\eta} W_{\eta\eta} + \left( \frac{a}{b} \right)^4 W_{\eta\eta}^2 \right] d\xi d\eta \dots \dots \dots (17)$$

and

$$T_1 = \cos^4\theta \int_{\eta=0}^b \int_{\xi=0}^a \left( 1 - \alpha_1 \frac{\xi}{a} \right) \left( 1 + \beta \left( \frac{\xi}{a} \right)^2 \right) W^2 d\xi d\eta \dots \dots \dots (18)$$

Here

$$\lambda^2 = \frac{12a^4 \omega^2 \rho_0 (1-\nu^2)}{E_0 h_0^2} \dots \dots \dots (19)$$

is a frequency parameter.

Equation (16) involves the unknown  $A_1$  and  $A_2$  arising due to the substitution of  $W(\xi, \eta)$  from equation (15). These unknowns are to be determined from equation (16) for which

$$\frac{\partial}{\partial A_n} (V_1 - \lambda^2 T_1) = 0, \quad n=1, 2 \dots \dots \dots (20)$$

The above equation simplifies to

$$b_{n1} A_1 + b_{n2} A_2 = 0, \quad n=1, 2 \dots \dots \dots (21)$$

where  $b_{n1}, b_{n2}$  ( $n=1, 2$ ) involve parametric constants and the frequency parameter.

For a non-trivial solution the determinant of the coefficient of equation (21) must be zero.

Therefore one gets the frequency equation as

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} = 0 \dots \dots \dots (22)$$

From equation (22), one can obtain quadratic equation in  $\lambda^2$  from which two values of  $\lambda^2$  can be found.

## 5. RESULTS AND DISCUSSION

The frequency equation (22) is a quadratic equation in  $\lambda^2$  from which two values of  $\lambda^2$  can be found. The frequency parameter  $\lambda$  corresponding to the first two modes of vibration of a clamped parallelogram plate has been computed for various values non-homogeneity constant ( $\alpha_1$ ), aspect ratio ( $a/b$ ), thermal constant ( $\alpha$ ), taper constant ( $\beta$ ) and skew angle ( $\theta$ ). The Poisson's ratio ( $\nu$ ) is taken as 0.3. These results are summarized in tables (1-12).

Tables 1-2 contains the value of frequency parameter of a clamped parallelogram plate for different values of thermal constant ( $\alpha$ ) for fixed value of aspect ratio ( $a/b$ )=1.0 for the first two modes of vibration for two values of taper constant ( $\beta$ ) and non-homogeneity constant ( $\alpha_1$ ). It is seen from the tables that as thermal constant increases frequency parameter decrease. And the thermal constant is more at angle  $60^\circ$  compare to  $0^\circ$ .

Tables 3-6 gives the value of frequency parameter of a clamped parallelogram plate for different values of aspect ratio ( $a/b$ ) and the combinations of  $\alpha$ ,  $\beta$  and  $\alpha_1$  i.e.  $\alpha=0.0$ ,  $\beta=0.0$  and  $\alpha_1=0.4$ ;  $\alpha=0.4$ ,  $\beta=0.0$  and  $\alpha_1=0.4$ ;  $\alpha=0.0$ ,  $\beta=0.4$  and  $\alpha_1=0.4$  and  $\alpha=0.4$ ,  $\beta=0.4$  and  $\alpha_1=0.4$ . It is seen from the tables that as aspect ratio increases frequency parameter increases in all the cases for both modes of vibration. Also the effect of aspect ratio thermal constant is more at angle  $60^\circ$  compare to  $0^\circ$ .

Tables 7-8 have the value of frequency parameter of a clamped parallelogram plate for different values of taper constant ( $\beta$ ) for fixed value of  $a/b=1.0$  and the combinations of  $\alpha$  and  $\alpha_1$  i.e.  $\alpha=0.4$ ,  $\alpha_1=0.0$  and  $\alpha=0.4$ ,  $\alpha_1=0.4$ . It is seen from the tables that as taper constant increases frequency parameter increase in all the cases for both modes of vibration. Also the effect of aspect ratio thermal constant is more at angle  $60^\circ$  compare to  $0^\circ$ .

Tables 9-10 contains the value of frequency parameter of a clamped parallelogram plate for different values of skew angle ( $\theta$ ) and the combinations of  $\alpha$ ,  $\beta$  and  $\alpha_1$  i.e.  $\alpha=0.0$ ,  $\beta=0.0$  and  $\alpha_1=0.0$ ;  $\alpha=0.0$ ,  $\beta=0.4$  and  $\alpha_1=0.0$ ;  $\alpha=0.4$ ,  $\beta=0.0$  and  $\alpha_1=0.0$  and  $\alpha=0.4$ ,  $\beta=0.4$  and  $\alpha_1=0.0$ . It is seen from the tables that as skew angle increases frequency parameter increases in all the cases for both modes of vibration.

Tables 11-12 have the value of frequency parameter of a clamped parallelogram plate for different values of non-homogeneity constant ( $\alpha_1$ ) for fixed value of aspect ratio ( $a/b$ ) =1.0 for the first two modes of vibration for two values of taper constant ( $\beta$ ) and thermal constant ( $\alpha$ ). It is seen from the tables that as non-homogeneity constant increases frequency parameter increases in all the cases for both modes of vibration. Also the effect of aspect ratio thermal constant is more at angle  $60^\circ$  compare to  $0^\circ$ .

**Table1:**

Frequency parameter ( $\lambda$ ) of a clamped parallelogram plate for different values of thermal constant ( $\alpha$ ), aspect ratio  $a/b=1.0$ ,  $\beta=0.4$  and  $\theta=0^\circ$

$\alpha$	$\alpha_1=0.0,$		$\alpha_1=0.4$	
	First Mode	Second Mode	First Mode	Second Mode
0.0	41.5999	162.2094	46.6037	182.1224
0.2	39.8020	155.2537	44.5892	174.3141
0.4	37.9158	147.9723	42.4758	166.1402
0.6	35.9262	140.3146	40.2465	157.5440
0.8	33.8133	132.2158	37.8789	148.4528

**Table2:**

Frequency parameter ( $\lambda$ ) of a clamped parallelogram plate for different values of thermal constant ( $\alpha$ ), aspect ratio  $a/b=1.0$ ,  $\beta=0.4$  and  $\theta=60^\circ$

$\alpha$	$\alpha_1=0.0,\beta=0.0,$		$\alpha_1=0.4,\beta=0.4$	
	First Mode	Second Mode	First Mode	Second Mode
0.0	191.8193	717.0958	214.9092	805.0626
0.2	183.5421	685.9152	205.6340	770.0631
0.4	174.8590	653.2516	195.9039	733.3992
0.6	165.7009	618.8722	185.6414	694.8099
0.8	155.9758	582.4750	174.7434	653.9562

**Table3:**  
Frequency parameter ( $\lambda$ ) of a clamped parallelogram plate for different values of aspect ratio a/b and  $\alpha=0.0$ ,  $\beta=0.0$  and  $\alpha_1=.4$

a/b	$\theta=0^0$		$\theta=30^0$		$\theta=45^0$		$\theta=60^0$	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.5	118.1309	27.5374	149.6805	37.8190	228.7417	58.3268	465.9946	119.7898
1.0	157.5134	40.2490	217.8630	56.5494	338.2298	88.8843	698.9448	185.5117
1.5	268.8557	68.0318	368.8645	94.6244	568.5717	147.5178	1167.4780	305.7980
2.0	440.5238	110.1499	598.7221	151.2761	914.9669	233.3075	1863.9786	479.1592
2.5	667.4354	165.5802	901.7182	225.4423	1370.2828	345.023346	2776.8105	703.7210

**Table4:**  
Frequency parameter ( $\lambda$ ) of a clamped parallelogram plate for different values of aspect ratio a/b and  $\alpha=0.4$ ,  $\beta=0.0$  and  $\alpha_1=.4$

a/b	$\theta=0^0$		$\theta=30^0$		$\theta=45^0$		$\theta=60^0$	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.5	98.5041	24.63027	133.8783	33.8263	204.5928	52.169155	416.7982	107.1432
1.0	140.8843	35.9998	194.8625	50.5793	302.5220	79.5005	625.1553	165.9267
1.5	240.4719	60.8491	329.9224	84.6346	508.5461	131.9440	1044.2241	273.5140
2.0	394.0165	98.5210	535.5134	135.3054	818.3714	208.6766	1667.1931	428.5730
2.5	596.9725	148.0994	806.5213	201.6417	1225.6185	308.5982	2483.6552	629.4271

**Table5:**  
Frequency parameter ( $\lambda$ ) of a clamped parallelogram plate for different values of aspect ratio a/b and  $\alpha=0.0$ ,  $\beta=0.4$  and  $\alpha_1=.4$

a/b	$\theta=0^0$		$\theta=30^0$		$\theta=45^0$		$\theta=60^0$	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.5	131.8028	32.7142	178.7795	44.8913	272.7055	69.1840	554.5972	141.9966
1.0	182.1224	46.6037	251.5362	65.4885	390.0147	102.9523	805.0626	214.9092
1.5	303.3499	77.1339	416.2877	107.4110	641.8043	167.6261	1318.0942	347.8056
2.0	492.6417	123.8428	669.8959	170.2716	1024.2152	262.8713	2087.4470	540.3819
2.5	743.9635	185.5392	1005.5500	252.8380	1528.7061	387.2668	3099.0686	790.4878

**Table6:**

Frequency parameter ( $\lambda$ ) of a clamped parallelogram plate for different values of aspect ratio  $a/b$  and  $\alpha=0.4$ ,  $\beta=0.4$  and  $\alpha_1=0.4$

a/b	$\theta=0^0$		$\theta=30^0$		$\theta=45^0$		$\theta=60^0$	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.5	121.6174	30.0490	164.8589	41.2228	251.3213	63.5154	510.8221	130.3354
1.0	166.1402	42.4758	229.3428	59.6907	355.4406	93.8428	733.3992	195.9039
1.5	274.3260	69.8083	376.4834	97.2499	580.4710	151.8238	1192.1939	315.1197
2.0	444.0382	111.7432	603.9114	153.6979	923.4810	237.3717	1882.4251	488.1271
2.5	669.7417	167.2046	905.3726	227.9261	1376.6134	349.2141	2791.1298	713.0144

**Table7:**

Frequency parameter ( $\lambda$ ) of a clamped parallelogram plate for different values of taper constant ( $\beta$ ), aspect ratio  $a/b=1.0$ ,  $\alpha=0.4$  and  $\theta=0^0$

$\beta$	$\alpha_1=0.0$		$\alpha_1=0.4$	
	First Mode	Second Mode	First Mode	Second Mode
0.0	32.1992	126.0104	35.9998	140.8843
0.2	34.9359	136.4353	39.1011	152.8784
0.4	37.9158	147.9723	42.4758	166.1402
0.6	41.0841	160.4460	46.0627	180.4778
0.8	44.3991	173.7025	49.8152	195.7190

**Table8:**

Frequency parameter ( $\lambda$ ) of a clamped parallelogram plate for different values of taper constant ( $\beta$ ), aspect ratio  $a/b=1.0$ ,  $\alpha=0.4$  and  $\theta=60^0$

$\beta$	$\alpha_1=0.0$		$\alpha_1=0.4$	
	First Mode	Second Mode	First Mode	Second Mode
0.0	148.4094	559.1570	165.9267	625.1553
0.2	161.0754	603.9237	180.2871	676.6789
0.4	174.8590	653.2516	195.9039	733.3992
0.6	189.5041	706.4366	212.4912	794.5458
0.8	204.8147	762.8608	229.8302	859.4370

**Table9:**

Frequency parameter ( $\lambda$ ) of a clamped parallelogram plate for different values of skew angle  $\theta$  and aspect ratio  $a/b=1.0$

$\theta$	$\alpha =0.0, \beta=0.0, \alpha_1=0.0$		$\alpha =0.0, \beta=0.4, \alpha_1=0.0$		$\alpha =0.4, \beta=0.0, \alpha_1=0.0$		$\alpha =0.4, \beta=0.4, \alpha_1=0.0$	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
$0^0$	140.8840	35.9997	162.2094	41.5999	126.0104	32.1992	157.5134	40.2490
$30^0$	194.8625	50.5793	224.0401	58.4553	174.2903	45.2395	217.8630	56.5494
$45^0$	302.5225	79.5005	347.3909	91.8933	270.5843	71.1074	338.2298	88.8843
$60^0$	625.1565	165.9267	717.0958	191.8193	559.1570	148.4094	689.9448	185.5117

**Table10:**

Frequency parameter ( $\lambda$ ) of a clamped parallelogram plate for different values of skew angle  $\theta$  and aspect ratio  $a/b=1.0$

$\theta$	$\alpha =0.4, \beta=0.0, \alpha_1=.4$		$\alpha =0.0, \beta=0.4, \alpha_1=.4$		$\alpha =0.4, \beta=0.4, \alpha_1=0.0$		$\alpha =0.4, \beta=0.4, \alpha_1=.4$	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
$0^0$	140.8843	35.9998	182.1224	46.6037	147.9723	37.9158	166.1402	42.4758
$30^0$	194.8625	50.5793	251.5362	65.4885	204.2693	53.2810	229.3428	59.6907
$45^0$	302.5220	79.5005	390.0147	102.9523	316.5899	83.7637	355.4406	93.8428
$60^0$	625.1553	165.9267	805.0626	214.9092	653.2516	174.8590	733.3992	195.9039

**Table11:**

Frequency parameter ( $\lambda$ ) of a clamped parallelogram plate for different values of non-homogeneity constant ( $\alpha_1$ ), aspect ratio  $a/b=1.0$ ,  $\beta=0.4$  and  $\theta=0^0$

$\alpha_1$	$\alpha =0.0$		$\alpha =0.4$	
	First Mode	Second Mode	First Mode	Second Mode
0.0	41.5999	162.2094	37.9158	147.9723
0.2	43.8893	171.3037	40.0022	156.2695
0.4	46.6037	182.1224	42.4758	166.1402
0.6	49.8933	195.2907	45.4735	178.1550
0.8	53.9948	211.8044	49.2109	193.2227



**Table 12:**

Frequency parameter ( $\lambda$ ) of a clamped parallelogram plate for different values of non-homogeneity constant ( $\alpha_1$ ), aspect ratio  $a/b=1.0$ ,  $\beta=0.4$  and  $\theta=60^\circ$

$\alpha_1$	$\alpha=0.0$		$\alpha=0.4$	
	First Mode	Second Mode	First Mode	Second Mode
0.0	191.8193	717.0958	174.8590	653.2516
0.2	202.3830	757.2716	184.4872	689.8558
0.4	214.9092	805.0626	195.9039	733.3992
0.6	230.0919	863.2217	209.7413	786.3911
0.8	249.0259	936.1452	226.9968	852.8385

## 6. REFERENCES

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