# Some Fixed Point Theorems for Generalized Contractive Mappings in Complete 2 – Metric Spaces

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**ABSTRACT—** ***In this paper we introduce new concepts of Ciric type and JS type – contraction and established some fixed point theorems for such contraction in complete 2 – metric spaces.***

**Keywords—**Fixed Point, Ciric type contraction, JS type contraction, Complete 2-metric space.

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## 1. INTRODUCTION

The Banach contraction principle is the first important result on fixed points for contractive-type mappings, many authors have obtained interesting extensions and generalizations of the Banach contraction principle. The concept of Ciric contraction and JS-contraction have been introduced, respectively, by Ciric [6] and Hussain et al.[4]. In this paper we introduced Ciric type and JS type – contraction and established some fixed point theorems for such contraction in complete 2 – metric spaces.

## 2. PRELIMINARIES

***Definition 2.1.**([2, 3, 17])*

A 2-metric space is a set X with a real valued non -negative function is defined on $X \times X \times X$ such that

(i) for all $x, y \in X$, $(x \neq y)$, there exists a point $z \in X$ such that $\sigma(x, y, z) \neq 0$
(ii) $\sigma(x, y, z) = 0$ if at least two of points x, y, z coincide.
(iii) $\sigma(x, y, z) = \sigma(x, z, y) = \sigma(y, z, x) = \sigma(y, x, z)$
(iv) $\sigma(x, y, z) \leq \sigma(x, y, w) + \sigma(x, w, z) + \sigma(w, y, z)$

The function σ is called 2-metric for the space and (X, σ) is called a 2-metric space.

***Definition 2.2 :** ([6, 7])*

Let (X, σ) be a 2 – metric space. A mapping $T : X \rightarrow X$ is said to be Ciric type contraction.
If there exist non negative numbers a, b, c, d with $0 \leq a + 2d < 1$, $0 \leq b < 1$, $0 \leq c + d < 1$, Such that
$$\sigma(Tx, Ty, z) \leq a\,\sigma(x, y, z) + b\sigma(x, Tx, z) + c\sigma(y, Ty, z) + d[\sigma(x, Ty, z) + \sigma(y, Tx, z)] \quad \text{for all } x, y, z \in X, \qquad (1)$$

***Definition 2.3 :** ([8])*

Let (X, σ) be a 2 – metric space. A mapping $T : X \rightarrow X$ is said to be JS type contraction.
If there exist $\psi \in \Psi$ and non-negative numbers. a, b, c, d with $0 \leq a + 2d < 1$, $0 \leq b < 1$, $0 \leq c + d < 1$, Such that
$$\psi[\sigma(Tx, Ty, z)] \leq [\psi(\sigma(x, y, z))]^a [\psi(\sigma(x, Tx, z))]^b [\psi(\sigma(y, Ty, z))]^c [\psi(\sigma(x, Ty, z) + \sigma(y, Tx, z))]^d \text{ for all } x, y, z \in X, \quad (2)$$
Where Ψ is the set of all functions $\psi : [0, \infty) \rightarrow [1, \infty)$ satisfying the following conditions:

($\psi_1$) ψ is non-decreasing, and ψ(t) = 1 if and only if t = 0;

($\psi_2$) for each sequence $\{t_n\} \subset (0, \infty)$, $\lim_{n\to\infty} \psi(t_n) = 1$ *if and only if* $\lim_{n\to\infty} t_n = 0$;

($\psi_3$) there exist $r \in (0, 1)$ and $\ell \in (0, \infty]$ such that $\lim_{t\to 0^+} \frac{\psi(t)-1}{t^r} = \ell$;

($\psi_4$) $\psi(a + b + c) \leq \psi(a)\, \psi(b)\, \psi(c)$ for all a, b, c > 0.

***Definition 2.4 :** ([17])*

A sequence $\{x_n\}$ in X is said to be Cauchy sequence if for each $z \in X$,

$\lim_{m,n\to\infty} \sigma(x_n, x_m, z) = 0$

A sequence $\{x_n\}$ in X is convergent to an element $x \in X$ if for each $z \in X$,

$\lim_{n\to\infty} \sigma(x_n, x, z) = 0$

A complete 2 – metric space is one in which every Cauchy sequence in X converges to an element of X.

***Definition 2.5.** ([3])*

A mapping T : X → X of a 2-metric space (X, σ ) in to itself is said to be asymptotically regular at a point x ϵ X

if $\lim_{n\to\infty} \sigma(T^n x, T^{n+1}x, z) = 0$, (z ϵ X).

## 3. MAIN RESULTS

We first introduce a new 2-metric ρ in a given 2- metric space (X, σ) induced by the metric σ,
Define , [11] , $\rho(x, y, z) = \eta(\sigma(x, y, z)) = \ln(\psi[\sigma(x, y, z)])$

For $\psi \in \Psi$ and $t \in [0, \infty)$, set $\eta(t) = \ln(\psi(t))$. Then $\eta : [0, \infty)\to[0, \infty)$ has the following properties:

($\eta_1$) η is non-decreasing, and η (t) = 0 if and only if t = 0;

($\eta_2$) for each sequence $\{t_n\} \subset (0, \infty)$, $\lim_{n\to\infty} \eta(t_n) = 0$ *if and only if* $\lim_{n\to\infty} t_n = 0$;

($\eta_3$) $\eta(a + b + c) \leq \eta(a)\, \eta(b)\, \eta(c)$ for all a, b, c > 0.

***Lemma 3.1:*** [11] Let (X, σ) be a 2-metric space then (X, ρ) is a 2-metric space,
where $\rho(x, y, z) = \eta(\sigma(x, y, z)) = \ln(\psi[\sigma(x, y, z)])$.

***Proof:*** For each x, y, z ∈ X. we have
σ(x, y, z) = 0 if at least two of points x, y, z coincide
this imply η(σ(x, y, z)) = 0 if at least two of points x, y, z coincide, by ($\eta_1$)
Hence

(i) ρ(x, y, z) = η(σ(x, y, z)) = 0; if at least two of points x, y, z coincide,

(ii) ρ(x, y, z) = η(σ(x, y, z)) = η(σ(x, z, y) = ρ(x, z, y); similarly we get
ρ(x, y, z) = ρ(x, z, y) = ρ(y, x, z) = ρ(y, z, x) = …..

(iii) ρ(x, y, z) = η(σ(x, y, z))

≤ η(σ(x, y, w) + σ(x, w, z)+ (σ(w, y, z))
≤ η(σ(x, y, w)) + η(σ(x, w, z))+ η(σ(w, y, z))
≤ ρ(x, y, w) + ρ(x, w, z)+ ρ(w, y, z)

Hence (X, ρ) is a 2-metric space.

***Lemma 3.2:*** Let (X, σ) be a 2-metric space with ψ ∈ Ψ then (X, ρ) is complete if and only if (X, σ) is complete.

where $\rho(x, y, z) = \eta(\sigma(x, y, z)) = \ln(\psi[\sigma(x, y, z)])$.

***Proof:***

Suppose that (X, σ) is complete and $\{x_n\}$ is a Cauchy sequence of (X, σ).

i.e $\lim_{m,n\to\infty} \rho(x_n, x_m, z) = 0.$ then we have $\lim_{m,n\to\infty} \eta(\sigma(x_n, x_m, z)) = 0.$

and hence $\lim_{m,n\to\infty} \sigma(x_n, x_m, z) = 0$ by ($\eta_2$).

More over by completeness of (X, σ) there exists x in X such that $\lim_{n \to \infty} \sigma(x_n, x, z) = 0$ and so

$\lim_{n \to \infty} \rho(x_n, x, z) = \lim_{n \to \infty} \eta(\sigma(x_n, x, z)) = 0$ by $(\eta_2)$.

Hence (X, ρ) is complete.
Similarly we can prove if (X, ρ) is complete then (X, σ) is complete.
Hence the proof.

***Lemma 3.3:*** Let (X, σ) be a 2-metric space and T : X → X be a JS type contraction with ψ ∈ Ψ then T is a Ciric type contraction in (X, ρ).

***Proof:*** For all x, y, z in X

$$\begin{aligned}\rho(Tx, Ty, z) &= \eta(\sigma(Tx, Ty, z)) = \ln(\psi[\sigma(Tx, Ty, z)])\\ &\le \ln\{[\psi(\sigma(x, y, z))]^a [\psi(\sigma(x, Tx, z))]^b [\psi(\sigma(y, Ty, z))]^c [\psi(\sigma(x, Ty, z) + \sigma(y, Tx, z))]^d\}\\ &\le a \ln[\psi(\sigma(x, y, z))] + b \ln[\psi(\sigma(x, Tx, z))] + c \ln[\psi(\sigma(y, Ty, z))] + d [\ln(\psi(\sigma(x, Ty, z)) + \ln(\psi\sigma(y, Tx, z))]\\ &\le a\, \rho(x, y, z) + b\, \rho(x, Tx, z) + c\, \rho(y, Ty, z) + d[\rho(x, Ty, z) + \rho(y, Tx, z)] \text{ for all } x, y, z \in X,\end{aligned}$$

that is, (1) is satisfied with respect to the metric ρ, and hence T is a Ciric type contraction in (X, ρ).

### *Theorem 3.4*

Let (X, σ) be a complete 2-metric space and T : X → X be a mapping such that the following condition is satisfied.

$$\sigma(Tx, Ty, z) \le a\, \sigma(x, y, z) + b\sigma(x, Tx, z) + c\sigma(y, Ty, z) + d[\sigma(x, Ty, z) + \sigma(y, Tx, z)] \text{ for all } x, y, z \in X,$$
$$0 \le a + 2d < 1,\ 0 \le b < 1,\ 0 \le c + d < 1 \quad (3)$$

If T is asymptotically regular at some point of X, then T has a unique fixed point in X.

***Proof:*** We shall assume that T is asymptotically regular at a point x ϵ X and consider the sequence $\{T^n x\}$. Then

$$\sigma(T^m x, T^n x, z) \le a\, \sigma(T^{m-1}x, T^{n-1}x, z) + b\sigma(T^{m-1}x, T^m x, z) + c\, \sigma(T^{n-1}x, T^n x, z) + d[\sigma(T^{m-1}x, T^n x, z) + \sigma(T^{n-1}x, T^m x, z)]$$

$$\begin{aligned}&\le a [\sigma(T^{m-1}x, T^{n-1}x, T^n x) + \sigma(T^{m-1}x, T^n x, z) + \sigma(T^n x, T^{n-1}x, z)] + b\sigma(T^{m-1}x, T^m x, z) + c\, \sigma(T^{n-1}x, T^n x, z)\\ &\quad + d [\sigma(T^{m-1}x, T^n x, T^m x) + \sigma(T^{m-1}x, T^m x, z) + \sigma(T^m x, T^n x, z) + \sigma(T^{n-1}x, T^m x, T^n x) + \sigma(T^{n-1}x, T^n x, z)\\ &\quad + \sigma(T^n x, T^m x, z)]\\ &\le a [\sigma(T^n x, T^{n-1}x, T^{m-1}x) + \sigma(T^{m-1}x, T^n x, T^m x) + \sigma(T^{m-1}x, T^m x, z) + \sigma(T^m x, T^n x, z) + \sigma(T^n x, T^{n-1}x, z)]\\ &\quad + b\, \sigma(T^{m-1}x, T^m x, z) + c\, \sigma(T^{n-1}x, T^n x, z) + d [\sigma(T^{m-1}x, T^n x, T^m x) + \sigma(T^{m-1}x, T^m x, z)\\ &\quad + \sigma(T^m x, T^n x, z) + \sigma(T^{n-1}x, T^m x, T^n x) + \sigma(T^{n-1}x, T^n x, z) + \sigma(T^m x, T^n x, z)]\end{aligned}$$

$$(1 - a - 2d)\, \sigma(T^m x, T^n x, z) \le (2a + b + 2d)\, \sigma(T^{m-1}x, T^m x, z) + (2a + c + 2d)\, \sigma(T^{n-1}x, T^n x, z)$$

$$\sigma(T^m x, T^n x, z) \le \frac{(2a+b+2d)}{(1-a-2d)} \sigma(T^{m-1}x, T^m x, z) + \frac{(2a+c+2d)}{(1-a-2d)} \sigma(T^{n-1}x, T^n x, z)$$

Since T is asymptotically regular at x.
$\sigma(T^m x, T^n x, z) \to 0$ as m, n → ∞;
Hence $\{T^n x\}$ is a Cauchy sequence.

Since (X, σ) is complete, there exist a point u in X such that $\lim_{n \to \infty} T^n x = u$.

Suppose that u is not a fixed point of T
Then by (3), we obtain

$$\begin{aligned}\sigma(u,Tu,z) &\le \sigma(u, Tu, T^n x) + \sigma(u, T^n x, z) + \sigma(T^n x, Tu, z)\\ &\le \sigma(u, Tu, T^n x) + \sigma(u, T^n x, z) + a\, \sigma(T^{n-1}x, u, z) + b\, \sigma(T^{n-1}x, T^n x, z) + c\, \sigma(u,Tu, z)\\ &\quad + d [\sigma(T^{n-1}x, Tu, z) + \sigma(u, T^n x, z)]\\ &\le (1 + d)\, \sigma(u, T^n x, z) + a\, \sigma(T^{n-1}x, u, z) + b\, \sigma(T^{n-1}x, T^n x, z) + c\, \sigma(u, Tu, z) + d [\sigma(T^{n-1}x, Tu, z) + \sigma(u, T^n x, z)]\end{aligned}$$

Taking the limit as n → ∞, we obtain σ(u, Tu, z) ≤ (c + d) σ(u, Tu, z)
Which contradicts (c + d) < 1 unless u = Tu,
Suppose T has second fixed point v in X. Then by (3),
We obtain σ(u, v, z) ≤ (a +2d) σ(u, v, z)
Since (a + 2d) < 1 it follows that u = v.
Hence the fixed point is unique.

### *Theorem 3.5 [14]*

Let (X, σ) be a complete 2-metric space and $T : X \to X$ be a JS type contraction then T has a unique fixed point.

***Proof:***

Let x in X be an arbitrary point in X, for some n ∈ ℕ, we have $T^n x = T^{n+1}x$ then $T^n x$ will be a fixed point of T. So, without loss of generality, we can suppose that $\sigma(T^n x, T^{n+1}x, u) > 0$ for all n ∈ ℕ

Then by (2), we obtain

$$\begin{aligned}\psi[\sigma(T^n x, T^{n+1}x, z)] &\le [\psi(\sigma(T^{n-1}x, T^n x, z))]^a [\psi(\sigma(T^{n-1}x, T^n x, z))]^b [\psi(\sigma(T^n x, T^{n+1}x, z))]^c [\psi(\sigma(T^{n-1}x, T^{n+1}x, z) + \sigma(T^n x, T^n x, z))]^d \\ &\le [\psi(\sigma(T^{n-1}x, T^n x, z))]^a [\psi(\sigma(T^{n-1}x, T^n x, z))]^b [\psi(\sigma(T^n x, T^{n+1}x, z))]^c [\psi(\sigma(T^{n-1}x, T^{n+1}x, T^n x) + \sigma(T^{n-1}x, T^n x, z) \\ &\qquad + \sigma(T^n x, T^{n+1}x, z))]^d \\ &\le [\psi(\sigma(T^{n-1}x, T^n x, z))]^{a+b+2d} [\psi(\sigma(T^n x, T^{n+1}x, z))]^{c+d}\end{aligned}$$

$$\psi(\sigma(T^n x, T^{n+1}x, z)) \le [\psi(\sigma(T^{n-1}x, T^n x, z))]^{\frac{a+b+2d}{1-c-d}}, \ \forall n \in \mathbb{N}$$

$$\psi(\sigma(T^n x, T^{n+1}x, z)) \le [\psi(\sigma(T^{n-1}x, T^n x, z))]^{\frac{a+b+2d}{1-c-d}} \ldots\ldots\ldots \le .[\psi(\sigma(x, Tx, z))]^{\left(\frac{a+b+2d}{1-c-d}\right)^n} \tag{4}$$

Letting $n \to \infty$ in (4), we obtain

$\psi[\sigma(T^n x, T^{n+1}x, z)] \to 1$ as $n \to \infty$

which implies from ($\psi_2$) that $\lim\limits_{n\to\infty} \sigma(T^n x, T^{n+1}x, z) = 0$ (5)

From condition ($\psi_3$), ∃r ∈ (0, 1) and ℓ ∈ (0, ∞] such that $\lim\limits_{n\to\infty} \frac{\psi(\sigma(T^n x, T^{n+1}x, z) - 1}{(\sigma(T^n x, T^{n+1}x, z))^r} = \ell;$

Suppose that ℓ < ∞. In this case, let B = ℓ/2 > 0.

From the definition of the limit, there exists $n_0 \in \mathbb{N}$, such that

$\left| \frac{\psi(\sigma(T^n x, T^{n+1}x, z) - 1}{(\sigma(T^n x, T^{n+1}x, z))^r} - \ell \right| \le B$, for all $n \ge n_0$.

⇨ $\frac{\psi(\sigma(T^n x, T^{n+1}x, z) - 1}{(\sigma(T^n x, T^{n+1}x, z))^r} \ge \ell - B = B,$

then $n[\sigma(T^n x, T^{n+1}x, z)]^r \le nA\,[\psi(\sigma(T^n x, T^{n+1}x, z)) - 1]$, where A = 1/B.

Suppose now that ℓ = ∞, let B > 0 be an arbitrary positive number.

From the definition of the limit, there exists $n_0 \in \mathbb{N}$, such that

$\frac{\psi(\sigma(T^n x, T^{n+1}x, z) - 1}{(\sigma(T^n x, T^{n+1}x, z))^r} \ge B,$

⇨ $n[\sigma(T^n x, T^{n+1}x, z)]^r \le nA\,[\psi(\sigma(T^n x, T^{n+1}x, z)) - 1]$, where A = 1/B , for all $n \ge n_0$.

Thus in all cases ∃A >0 and $n_0 \in \mathbb{N}$ such that

$n[\sigma(T^n x, T^{n+1}x, z)]^r \le nA\,[\psi(\sigma(T^n x, T^{n+1}x, z)) - 1]$, for all $n \ge n_0$.

Using (4), we obtain

$$n[\sigma(T^n x, T^{n+1}x, z)]^r \le nA\left([\psi(\sigma(x, Tx, z))]^{\left(\frac{a+b+2d}{1-c-d}\right)^n} - 1\right) \tag{6}$$

Letting $n \to \infty$, in (6), we obtain $\lim\limits_{n\to\infty} n[\sigma(T^n x, T^{n+1}x, z)]^r = 0$ . Thus, there exists $n_1 \in \mathbb{N}$ such that

$\sigma(T^n x, T^{n+1}x, z) \le \left(\frac{1}{n^{(1/r)}}\right)$ for all $n \ge n_1$.

Now for $m > n > n_1$, we have

$$\sigma(T^n x, T^m x, z) \le \sum_{i=n}^{m-1} \sigma(T^i x, T^{i+1} x, z) \le \sum_{i=n}^{m-1} \frac{1}{(i)^{\frac{1}{r}}},$$

Since $0 < r < 1$ then $\sum_{i=n}^{m-1} \frac{1}{(i)^{\frac{1}{r}}}$ converges and hence $\sigma(T^n x, T^m x, z) \to 0$ as $m, n \to \infty$.

Thus $\{T^n x\}$ is a Cauchy sequence.

Since $(X, \sigma)$ is complete, there exist a point $u \in X$ such that $\lim_{n \to \infty} T^n x = u$.

Suppose that u is not a fixed point of T

Then by (2), we obtain

$\psi(\sigma(u,Tu,z)) \le \psi[\sigma(u, T^n x, z) + \sigma(T^n x, Tu, z) + \sigma(u, Tu, T^n x)]$
$\le \psi(\sigma(u, T^n x, z))\, \psi(\sigma(T^n x, Tu, z))\, \psi(\sigma(u, Tu, T^n x))$
$\le \psi(\sigma(u, T^n x, z))\, \psi(\sigma(u, Tu, T^n x))\, [\psi(\sigma(T^{n-1}x, u, z))]^a\, [\psi(\sigma(T^{n-1}x, T^n x, z))]^b\, [\psi(\sigma(u, Tu, z))]^c$
$[\psi(\sigma(T^{n-1}x, Tu, z) + \sigma(u, T^n x, z)]^d$

Taking limit as $n \to \infty$

$\psi(\sigma(u,Tu,z)) \le [\psi(\sigma(u,Tu,z))]^{c+d}$.

which is contradiction, since $0 \le c + d < 1$. $\Rightarrow \sigma(u, Tu, z) = 0$.

i.e $Tu = u$.

Suppose T has second fixed point v in X then by (2), we obtain

$\psi(\sigma(u, v, z))$
$= \psi(\sigma(Tu, Tv, z))$
$\le [\psi(\sigma(u, v, z))]^a\, [\psi(\sigma(u, Tu, z))]^b\, [\psi(\sigma(v, Tv, z))]^c [\psi(\sigma(u, Tv, z) + \sigma(v, Tu, z))]^d$
$\le [\psi(\sigma(u, v, z))]^{a+2d}$.

which is contradiction, since $0 \le a + 2d < 1$. $\Rightarrow \sigma(u, v, z) = 0$.

It follows that $u = v$.

Hence the fixed point is unique.

***Theorem 3.6***

Let $(X, \sigma)$ be a complete 2-metric space and $T : X \to X$ be a JS type contraction. If T is asymptotically regular at some point of X, then T has a unique fixed point in X.

***Proof:***

Since $(X, \sigma)$ is Complete 2-metric space. $(X, \rho)$ is also a Complete 2-metric space by Lemma3.1 and Lemma 3.2:
Also T is a Ciric type contraction in $(X, \rho)$ by Lemma 3.3,
Therefore T has a unique fixed point in X by Theorem 3.4:, The proof is complete.

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