Performances of Some Combined Algorithms for Adaptive Beamforming in Smart Antenna Using Linear Array

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ABSTRACT—*A comparative study of beamforming techniques using combined algorithms, like, sample matrix inversion with least mean square (SMI-LMS), sample matrix inversion with recursive least square (SMI-RLS) and least mean square with recursive least square (LMS-RLS) algorithm is presented in this paper. Side lobe level, null depth and error plot of adaptive beam formation for different signal-to-noise (SNR) with different element spacing are simulated and compared. It is found that performance of LMS-RLS algorithm is better than SMI-LMS and SMI-RLS algorithm.*

Keywords—Smart antenna, beamforming, LMS algorithm, RLS algorithm, SMI algorithm

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INTRODUCTION

Overall performance in mobile communication can be improved using smart antenna which is an antenna array and signal received at each antenna element is adaptively combined using smart signal processing module. By producing only radiation beam along the direction of arrival (DOA) of signal, appreciable power saving can be achieved using smart antenna [1, 2]. The smart antenna technology can significantly improve wireless system performance by increasing signal quality, network capacity and coverage area. The digital beamforming method using smart antenna is shown in Fig. 1. Signals are processed adaptively in order to exploit the spatial domain of the mobile radio channel. Usually the signals received at the different antennas are multiplied with complex weights and then adaptively weights are summed up.

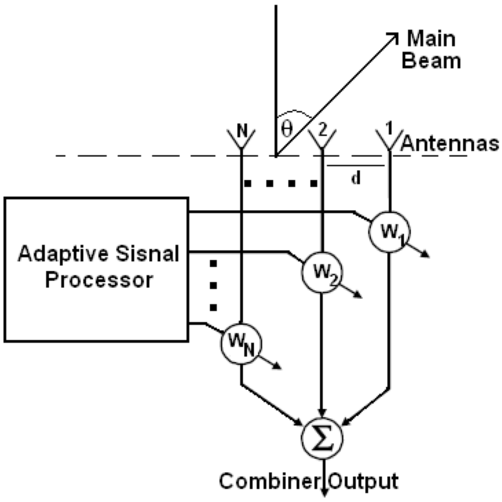

Fig. 1:Beamforming network of a smart antenna

Basically, there are two types of smart antennas, viz., switched beam smart antenna and adaptive smart antenna. In switched beam smart antenna, antenna system has several fixed beam patterns and according to detected condition most appropriate beam is used for communication. Whereas, in adaptive smart antenna, beam can be steered in any direction according to DOA estimation and at the same timenull can be generated in the direction of the interferer (Fig. 2). Smart antenna estimates direction of arrival of incoming signals and the direction of interfering signals.

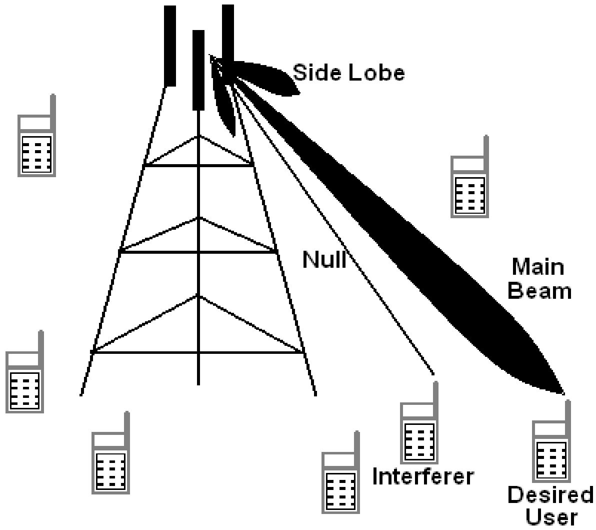

Fig. 2:Main beam toward desired user and null toward interferer

Then using beamforming algorithm, antenna beam is generated toward the desired direction and null is generated toward the direction of interferer. There are various types of algorithms for beamforming, having their advantages and disadvantages [3-7]. Different beamforming schemes and adaptive algorithms for weight adjustments on antennas are described in [3, 4]. A complex quaternion LMS algorithm is used [6] for beamforming of polarization-sensitive electromagnetic vector-sensor.A comparative study of beamforming techniques in smart antenna using LMS algorithm and its variants are reported in [7] where it is found that all values of step-size parameter have not same performance in adaptive beamforming. Most popular beamforming algorithms are least mean square (LMS), recursive least square (RLS) and sample matrix inversion (SMI) algorithms [8-13]. One of the drawbacks of the LMS adaptive scheme is that the algorithm must go through many iterations before satisfactory convergence is achieved. SMI algorithm uses a block adaptive approach which would give a better performance than a continuous approach. The SMI has a faster convergence rate because it exploits the direct inversion of the covariance matrix. Sample matrix is a time average estimate of the array correlation matrix using K-time samples. From literature survey, it is found that a number of publications are available on the adaptive beamforming algorithms, like, LMS, RLS, SMI etc., but report on the beamforming of adaptive antennas using combined algorithms is relatively less.

In this paper, beams of smart antenna are generated by combined approaches using SMI-LMS, SMI-RLS and LMS-RLS algorithms. These combined algorithms are applied for beamforming for various values of step size (for LMS), forgetting factor (for RLS) and block length (for SMI) and best results are reported. In LMS-RLS, first weights are updated using LMS and then again updated using RLS. Similar procedures are followed for SMI-LMS and SMI-RLS algorithms for element spacing 0.3λ and 0.5λ. A comparative study of performances of the combined algorithms is presented.

ADAPTIVE BEAMFORMING ALGORITHMS

Least mean square (LMS) algorithm is a stochastic and a steepest descent method, where iterative procedure is used making successive corrections to the weight vector in the direction of the negative of the gradient vector which eventually leads to the minimum mean square error [9]. Here LMS Algorithm is used for a uniform linear array with N isotropic elements, which forms the integral part of the adaptive beamforming system.

Adaptive algorithm is used to minimize the error *e(n)* between desired signal *d(n)* and array output *y(n),* as

$$e(n) = d(n) - y(n) \quad (1)$$

Output of adaptive beamformer, at time 'n', is given by a linear combination of the data at the N antenna elements, can be expressed as [9]

$$y(n) = w^H x(n) \quad (2)$$

$$w = [w_1, w_2 \dots, w_N]^H \quad (3)$$

Where, H denotes Hermitian (complex conjugate) transpose. The weight vector w is a complex vectors. Signal received by multiple antenna elements is

$$x(n) = [x_1(n), x_2(n), \dots, x_N(n) \quad (4)$$

LMS algorithm updates the weight vectors according to the equation [5, 9, 10]

$$w(n+1) = w(n) + \mu x(n) e^{*}(n) \tag{5}$$

Where, μ is the step size parameter and e(n) is the error between array output and the desired signal.

Recursive least square (RLS) algorithm is a deterministic algorithm, where weight vectors are updated recursively according to the equation [13]

$$\bar{w}(k) = \bar{w}(k-1) + \bar{g}(k)[d^{*}(k) - \bar{x}^{H}(k)\bar{w}(k-1)] \tag{6}$$

Where, $\bar{g}(k) = \hat{R}_{xx}^{-1}(k)\bar{x}(k)$ is gain vector, $\hat{R}_{xx}$ is correlation matrix and given by

$$\hat{R}_{xx}(k) = \propto \hat{R}_{xx}(k-1) + \bar{x}(k)\bar{x}^{H}(k) \tag{7}$$

'α' is forgetting factor or exponential weighting factor and is a positive constant, $0 \le \alpha \le 1$.

Sample matrix inversion (SMI) algorithm is discontinuous adaptive algorithms. Sample matrix is a time average estimate of the array correlation matrix using K-time samples [11, 13]. For ergodic random process, the time-average estimation will be equal the actual correlation matrix. It's used in discontinuous transmission, however it requires the number of interferers and their positions remain constant during the duration of the block acquisition. The sample matrix is defined as the time average estimate of the array correlation, which uses N samples, and if the random process is ergodic in correlation, then time average estimate is equal to the real correlation matrix [11, 12]

$$R_{xx} \approx \frac{1}{N}\sum_{n=1}^{N} x(n)x^{H}(n) \tag{8}$$

$$r = \frac{1}{N}\sum_{n=1}^{N} d^{*}(n)x(n) \tag{9}$$

The matrix $x_N(n)$is defined as the n-th block of vectors *x* ranges over N -data snapshots

$$x_N(n) = \begin{bmatrix} x_1(1+nN)x_1(2+nN) & \cdots & x_1(N+nN) \\ \vdots & \ddots & \vdots \\ x_N(1+nN)x_N(2+nN) & \cdots & x_N(N+nN) \end{bmatrix} \tag{10}$$

where, 'n' represents the block number, and N is the block length. So, R_{xx} can be given by

$$R_{xx}(n) = \frac{1}{N} x_N(n) x_N^{H}(n) \tag{11}$$

If the desired signal is [11, 12]

$$d(n) = [\, d(1+nK)d(2+nK)d(3+nK) \ldots \ldots \ldots d(N+nK) \tag{12}$$

Then,

$$r = \frac{1}{N} d^{*}(n) x_N(n) (13)$$

The sample matrix inversion weights of the *n* th block can be computed as

$$w_{SMI}(n) = R_{xx}^{-1}(n) r(n) = [x_N(n)x_N^{H}(n)]^{-1} d^{*}(n) x_N(n) \tag{14}$$

ADAPTIVE BEAM FORMATION USING COMBINED ALGORITHMS

Fig. 3 shows a uniform linear antenna array with inter-element spacing of 'd'. Antennas are excited by sinusoidal current with progressive phase shift of 'δ'.

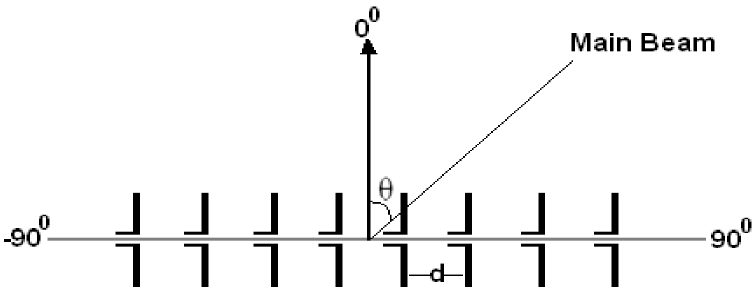

Fig. 3:Uniform linear antenna array

The array factor for N element linear array is given by [11]

$$AF = \sum_{n=0}^{N-1} A_n e^{(jn(\frac{2\pi d}{\lambda}cos\theta + \delta))} \quad (15)$$

Where to generate the main beam at wavelength λ toward the desired beam direction $\theta_0$from the broadside direction, the progressive phase shift is

$$\delta = \frac{-2\pi d}{\lambda} cos\theta_0 \quad (16)$$

Normalized array factor is

$$AF_n = \frac{AF}{AF_{max}} \quad (17)$$

In equation (17), AF_{max} is the maximum value of array factor AF, given by equation (15). Combined algorithms are programmed for 8 antenna elements and equation (15) is used as cost function in the computation. Number of elements in the uniform linear array is N = 8, desired direction for main beam = 30^0, desired direction for null = 60^0. Beams are generated using combined algorithms for different noisy environments with element spacing 0.5λ and 0.3λ. Here, different noisy environments are considered based on signal-to-noise ratio (SNR) of the signal. In all the results, shown below, the parameters for beamforming algorithms are K=1000, μ=0.02, α=0.8. Normalized array factors versus angle of arrival (AOA) are plotted in Fig. 4 and Fig. 5 for element spacing of 0.5λ with SNR=20dB and SNR=10dB.

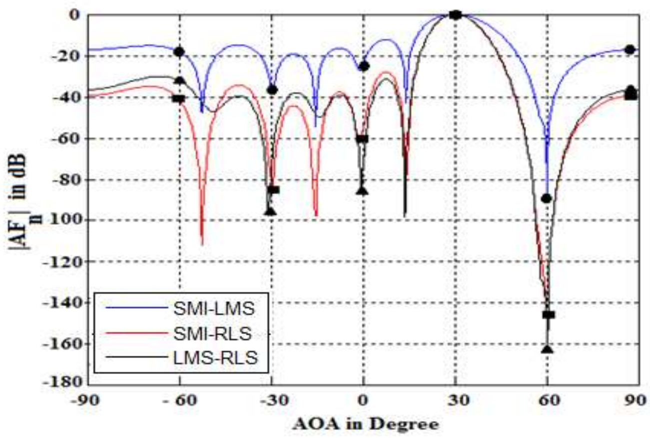

Fig. 4:Normalized array factor for SNR=20dB and d=0.5λ

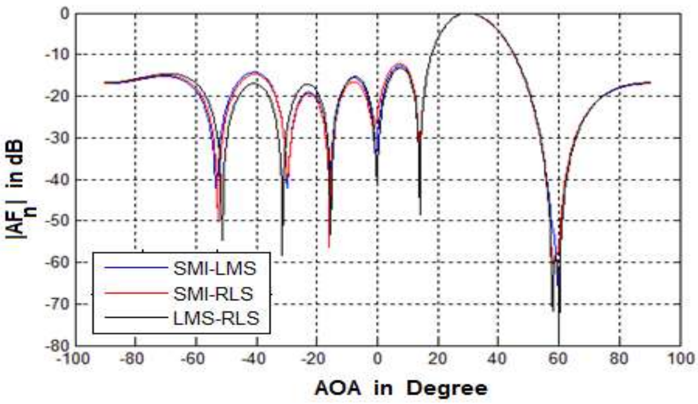

Fig. 5:Normalized array factor for SNR = 10dB, d=0.5λ

Fig. 6 and Fig. 7 show simulation results for beamforming using combined algorithms with element spacing 0.3λ and SNR values of 20dB and 10dB respectively.

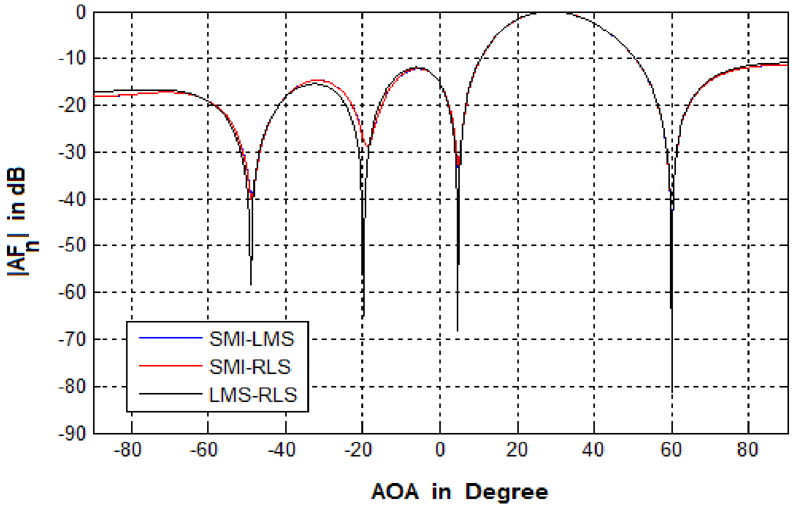

Fig. 6:Normalized array factor for SNR = 20 dB, d=0.3λ

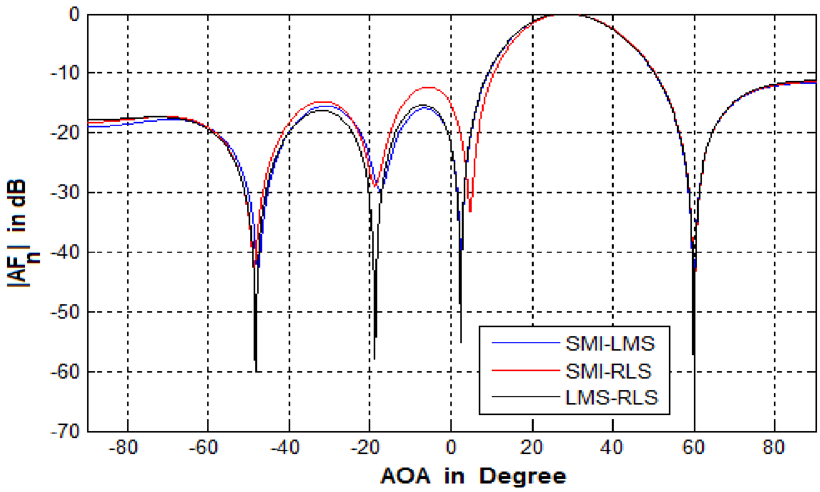

Fig. 7:Normalized array factor for SNR = 10 dB, d=0.3λ

Beam patterns, achieved using SMI-LMS algorithm for different values block length, are plotted in Fig. 8 for SMI-LMS algorithm, where step size is 0.02.

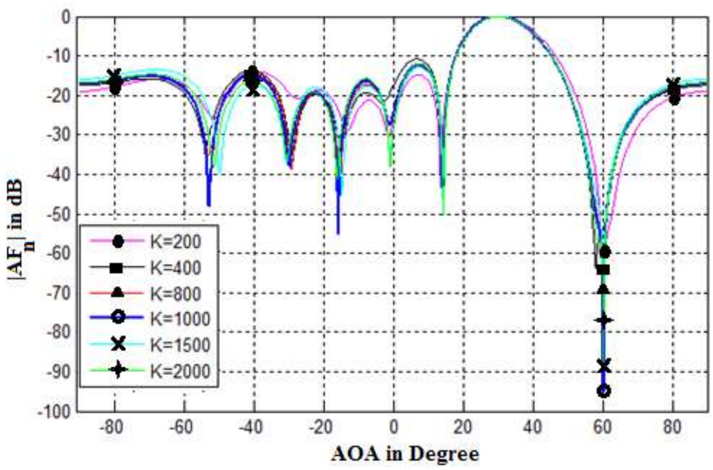

Fig. 8:Normalized array factor using SMI-LMS algorithm for d=0.5λ

In Fig. 8, deepest null is achieved for K=1000 with step size=0.02. But side lobe levels vary with block length 'K'. SLL obtained for K=200, 400, 800, 1000, 1500 and 2000 are -14.27dB, -10.79dB, -12.44dB, -12.24dB, -12.86dB and -12.07dB respectively.Beam patterns, obtained for different values of block length with forgetting factor of 0.8, using SMI-RLS algorithm, are plotted in Fig. 9. In this case, null depth increases with decrease of block length.

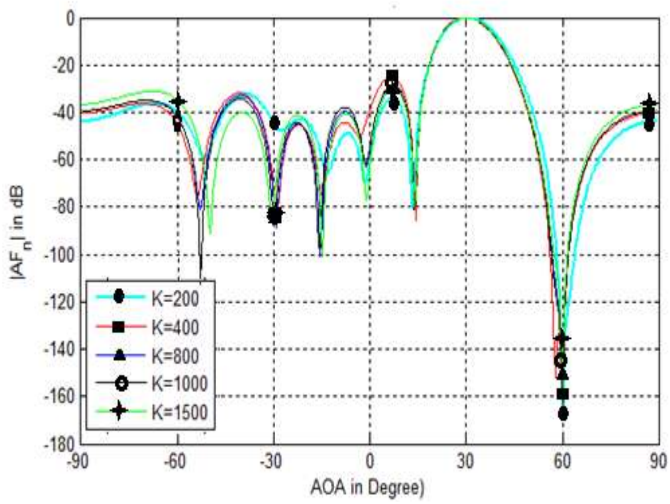

Fig. 9:Normalized array factor using SMI-RLS algorithm for α=0.8 for d=0.5λ

In Fig. 9, side lobe levels obtained for K=200, 400, 800, 1000 and 1500 are -32.44dB, -24.8dB, -28.61dB, -28.16dB and -29.58dB respectively.

The parameters, like, maximum side lobe level (SLL_{max}), first-null beamwidth (FNBW), direction of user and direction of interferer of SMI-LMS, SMI-RLS and LMS-RLS algorithms in adaptive beamforming are tabulated in Table 1.

Table 1: Performances of combined adaptive beamforming algorithms

Parameters	**Algorithms**	**SNR=20dB**		**SNR=10dB**	
		d=0.5λ	**d=0.3λ**	**d=0.5λ**	**d=0.3λ**
SLL_{max}	SMI-LMS	-12.3dB	-12dB	-13.2 dB	-15.5dB
	SMI-RLS	-28.3dB	-12dB	-12.1 dB	-12.3dB
	LMS-RLS	-31dB	-12dB	-13.2 dB	-15.3dB
FNBW	SMI-LMS	54^0	55.5^0	42^0	57^0
	SMI-RLS	54^0	55.5^0	42^0	55^0
	LMS-RLS	54^0	55.5^0	42^0	57.5^0
Direction of Main Beam	SMI-LMS	30^0	30^0	30^0	30^0
	SMI-RLS	30^0	30^0	30^0	30^0
	LMS-RLS	30^0	30^0	30^0	30^0
Direction of Null	SMI-LMS	60^0	60^0	60^0	60^0
	SMI-RLS	60^0	60^0	60^0	60^0
	LMS-RLS	60^0	60^0	60^0	60^0

From Table 1, it is evident that beamwidth is narrower for element spacing of 0.5λ compared to spacing of 0.3λ and hence directivity is higher.

Mean square errors, for the combined algorithms, are plotted in Fig. 10, Fig. 11 and Fig. 12 for SNR=20 dB and element spacing 0.5λ.

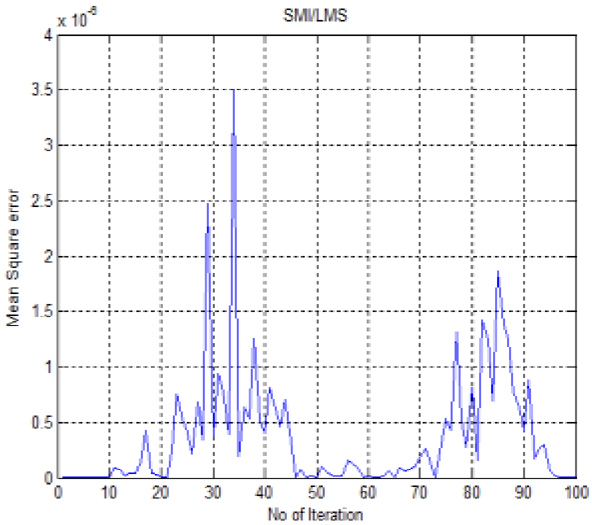

Fig. 10:Mean square error for 8 elements linear array using SMI-LMS algorithm

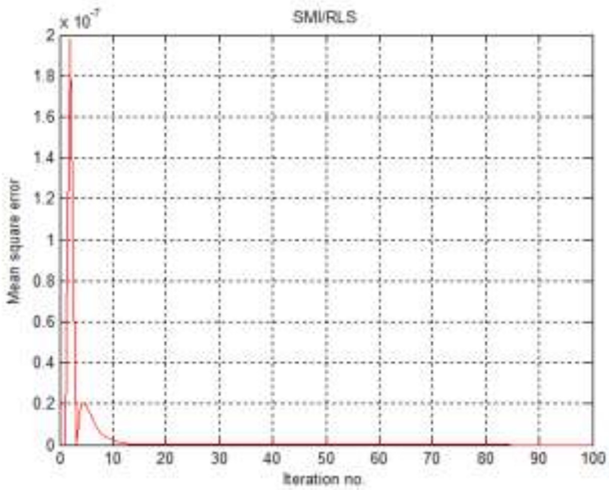

Fig. 11:Mean square error for 8 elements linear array using SMI-RLS algorithm

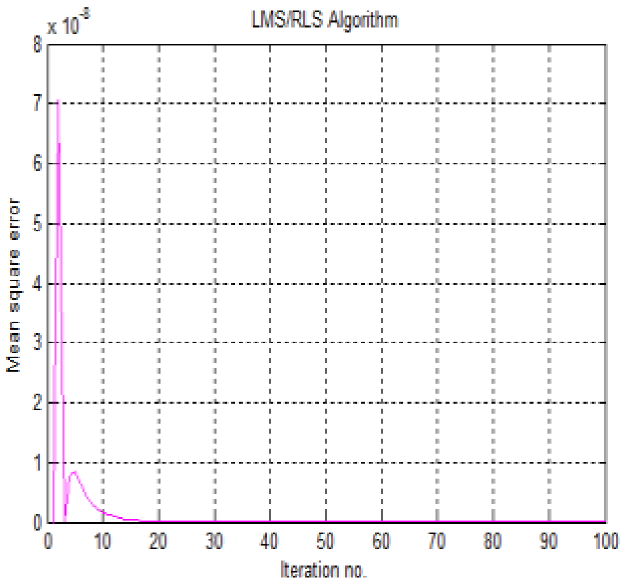

Fig. 12:Mean square error for 8 elements linear array using LMS-RLS algorithm

SMI-LMS algorithm is not able to generate null at desired direction for higher value of step size (0.1 and above). FNBW depends on SNR values for SMI-LMS, SMI-RLS and LMS-RLS algorithms. Also FNBW depends on inter-element spacing. FNBW is lower for higher values of inter-element spacing for all the combined algorithms. Minimum SLLs are obtained for d=0.5λ using SMI-RLS and LMS-RLS algorithms.

CONCLUSION

Performances of combined beamforming algorithms are compared in this paper for different noisy environments. Special attention is given on side lobe reduction using these combined adaptive algorithms. Program for each combined algorithm is run for 100 iterations. Narrower beamwidth is achieved for element spacing of 0.5λ. Reduced beamwidth is achieved for element spacing of 0.5λ. Also results for all the three algorithms are compared for different SNR values and it is found that the lowest side lobe levels can be achieved by LMS-RLS algorithms for spacing of 0.5λ. Also LMS-RLS algorithm produces deepest null as compared to SMI-LMS and SMI-RLS algorithms in noisy environment. However, minimum mean square error is achieved using LMS-RLS algorithm. Convergence of LMS-RLS and SMI-RLS are better than SMI-LMS. Future scope of this work may be to study the performances of planar and circular smart antennas using these combined algorithms.

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