Asian Journal of Applied Sciences (ISSN: 2321 – 089)
Volume 04 – Issue 03, June 2016

Riabouchinsky Flow of a Pressure-Dependent Viscosity Fluid in Porous Media

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ABSTRACT— *Two-dimensional flow of a fluid with pressure-dependent viscosity through a variable permeability porous structure is considered. Exact solutions are obtained for a Riabouchinsky type flow using a procedure that is based on a method implemented by the late Professor Dr. R.K. Naeem of Karachi University in his study of Navier-Stokes flow with pressure-dependent viscosity. Viscosity is considered proportional to fluid pressure due to the importance and uniqueness of validity of this type of relation in the study of Poiseuille flow, and the effects of changing the proportionality constant on the pressure distribution are discussed. Since a variable permeability introduces an additional variable in the flow equations and renders the governing equations under-determined, the current work devises a methodology to determine the permeability function through satisfaction of a condition derived from the specified streamfunction. Illustrative examples are used to demonstrate how the variable permeability is determined, and how the arising parameters are determined. Although the current work considers flow in an infinite domain and does not handle a particular engineering problem, it nevertheless initiates the study of flow of fluids with pressure-dependent viscosity through variable-permeability media and sets the stage for future work in stability analysis of this type of flow. It is expected that the current work will be of value in transition layer analysis and the determination of variable permeability functions suitable for such analysis.*

The corresponding author (M.H. Hamdan) respectfully dedicates this contribution to the memory of Professor Dr. Rana Khalid Naeem who prematurely departed this world in November, 2015. Naeem was a very dear friend, and a classmate during our MSc and PhD studies under the supervision of Professor Dr. Ronald M. Barron at the University of Windsor. His contributions to the advancement of exact solutions of Navier-Stokes equations and special flows have been as far reaching as his great contributions to the advancement of fluid dynamics.

Keywords— Porous media, pressure-dependent viscosity, variable permeability

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1. INTRODUCTION

It has long been recognized that viscosity of a flowing fluid changes with flow conditions. For example, changes in temperature result in changes in viscosity, and excessive increase in pressure could affect the viscosity (although it has been customary to assume constant shear viscosity when Newtonian liquids flow in the absence of temperature variations), (*cf.* [8] and the references therein). An assumption of constant viscosity clearly ignores effects of pressure on viscosity in fluid flow conditions when the fluid pressure is of the order of 1000 atm. ([3], [15], [16]. A further factor that has an influence on viscosity is the flow domain (such as flow in constrictions and flow through porous media) where it has been suggested by a large number of researchers that the effective viscosity of a fluid in a porous structure is different than the viscosity of the base fluid (*cf.* [2] and the references therein).

High pressures arise in industrial applications that involve chemical and process technologies, such as medical tablet production, crude and fuel oil pumping, food processing, fluid-film lubrication theory, and in microfluidics, [3], [6], [7], [9], [10], [11]. In these and many other applications, the form of momentum equations used must include a shear viscosity that is a function of pressure. It has been argued that in some applications the effects of high pressure on increasing viscosity is much more significant than the effect of pressure on increasing density, [8]. Accordingly, one can ignore compressibility effects but must take into account the dependence of viscosity on pressure, [3], [5]. This realization is even more important when high pressures are encountered in the flow of fluids through porous media due to the important applications of groundwater recovery and oil production. [9], [13], [14], [15]. In this regard, governing equations must be derived to accommodate the different types of fluid flow, flow conditions, and the anticipated porous structure, [4], [14]. While many models governing fluid flow through porous media have been developed over the past sixteen decades, equations governing flow through porous media with variable viscosity have been developed more recently, [1], [6], [7], [13], [15].

Solutions to the available models are at least as challenging as solutions to Navier-Stokes equations with variable viscosity, due in part to the additional drag term that involves permeability. When permeability is a variable function of position (and time, when clogging is taken into account), a further complication is added to the equations. Solutions, and modelling of flow, thus necessitate approximations and simplifying assumptions, [14]. Considerations of special cases of flow, linearization, and simplifying assumptions are not at all new in the study of Navier-Stokes flow in general, and pressure-dependent viscosity analysis in particular, as has been amply demonstrated in the work of Naeem [12] and the work of Naeem and co-workers (*cf.* [12] and the references therein).

Solutions to flow of a fluid with pressure-dependent viscosity have been obtained under various assumptions and using a plethora of techniques have been reviewed, reported and implemented in the work of Naeem [12] and the vast literature reported in his work. One such technique is based on the concept of Riabouchinsky flow in which the streamfunction of a two-dimensional flow is assumed to be a function of one space variable, or combinations of functions of single variables. This approach has received considerable success in the understanding of flow phenomena and the introduction of methodologies based on this approach. Naeem [12] successfully implemented this approach in his study of Navier-Stokes flow of a fluid with pressure-dependent viscosity. This approach is also conveniently valid, and suitable in the sequel where we study the same flow in a porous structure that is of either constant or variable permeability. We hasten to point out that solutions of motions of fluids with pressure-dependent viscosities through variable permeability porous media is at its infancy. The permeability function introduces an additional variable in the governing equations, thus rendering the system of governing equations under-determined. In order to circumvent this situation, this work introduces a condition, derived from the specified streamfunction, that must be satisfied by the permeability function. This approach is demonstrated by considering some illustrative examples that are intended to bolster viability of the devised methodology.

2. GOVERNING EQUATIONS

The equations governing variable-viscosity fluid flow through porous media, in the absence of heat transfer effects, are given by the following continuity and linear momentum equations, [1]:

Continuity Equation:

$$\nabla \bullet \vec{u} = 0 \qquad \ldots(1)$$

Momentum Equations:

$$\rho\vec{u} \bullet \nabla\vec{u} = -\nabla p + \eta\nabla \bullet \left(\nabla\vec{u} + (\nabla\vec{u})^T\right) + (\nabla\eta) \bullet \left(\nabla\vec{u} + (\nabla\vec{u})^T\right) - \frac{\eta}{k}\vec{u}\,. \qquad \ldots(2)$$

wherein $\vec{u}$ is the velocity field, ρ is the fluid density, η is the viscosity, p is the pressure and k is the permeability.

For two-dimensional flow, we take $\vec{u} = (u, v)$, and write equation (1) as:

$$u_x + v_y = 0\,. \qquad \ldots(3)$$

Equations (2) are written as:

$$\frac{1}{2}(q^2)_x - v(v_x - u_y) = -P_x + \frac{1}{\rho}[(2\eta u_x)_x + \{\eta(u_y + v_x)\}_y] - \frac{\eta}{\rho k}u \qquad ...(4)$$

$$\frac{1}{2}(q^2)_y + u(v_x - u_y) = -P_y + \frac{1}{\rho}[(2\eta v_y)_y + \{\eta(u_y + v_x)\}_x] - \frac{\eta}{\rho k}v \qquad ...(5)$$

where

$$P = \frac{p}{\rho} \qquad ...(6)$$

and the square of the speed is defined by

$$q^2 = u^2 + v^2. \qquad ...(7)$$

Assuming that viscosity depends on the pressure according to the relationship:

$$\eta = a\rho P \qquad ...(8)$$

where a is a constant, then using (8) in (5) and (6) yields, respectively:

$$(\frac{q^2}{2})_x - v(v_x - u_y) = (2au_x - 1)P_x - aP[v_x - u_y]_y + aP_y(u_y + v_x) - \frac{aP}{k}u \qquad ...(9)$$

$$(\frac{q^2}{2})_y + u(v_x - u_y) = -(2au_x + 1)P_y + aP[v_x - u_y]_x + aP_x(u_y + v_x) - \frac{aP}{k}v. \qquad ...(10)$$

We note that equations (9) and (10) are valid for both variable and constant permeability. Now, equation (3) implies the existence of the streamfunction ψ such that:

$$u = \psi_y \qquad ...(11)$$

$$v = -\psi_x \qquad ...(12)$$

and the vorticity, ω, is defined as:

$$\omega = \nabla \times \vec{u} = v_x - u_y \qquad ...(13)$$

and expressed in terms of the streamfunction as

$$\omega = -\psi_{xx} - \psi_{yy} = -\nabla^2\psi. \qquad ...(14)$$

Using equations (11) and (12) in equations (9) and (10), we obtain, respectively

$$(\frac{\psi_y^2 + \psi_x^2}{2})_x - \psi_x\nabla^2\psi = (2a\psi_{yx} - 1)P_x + aP\nabla^2\psi_y + aP_y(\psi_{yy} - \psi_{xx}) - \frac{aP}{k}\psi_y \qquad ...(15)$$

$$(\frac{\psi_y^2 + \psi_x^2}{2})_y - \psi_y\nabla^2\psi = -(2a\psi_{xy} + 1)P_y - aP\nabla^2\psi_x + aP_x(\psi_{yy} - \psi_{xx}) + \frac{aP}{k}\psi_x. \qquad ...(16)$$

Equations (15) and (16) must be satisfied by the streamfunction $\psi(x,y)$ and the pressure $P(x,y)$, if the permeability is constant. However, if $k = k(x,y)$ is variable, then (15) and (16) must be satisfied by the three functions. This implies that at least one of the functions must be specified.

3. METHOD OF SOLUTION

Solutions to (15) and (16) are obtained by assuming a functional form of $\psi(x,y)$ and then solving for the pressure $P(x,y)$. Viscosity is then determined using equation (8).

Assuming that the streamfunction is a function of *y* only, namely

$$\psi(x,y) = f(y) \qquad \text{…(17)}$$

we obtain

$$\psi_x = \psi_{xx} = \psi_{xy} = 0 \qquad \text{...(18)}$$
$$\psi_y = f'(y) \qquad \text{…(19)}$$
$$\psi_{yy} = f''(y). \qquad \text{…(20)}$$

Velocity components and vorticity, equations (11), (12), and (13), take the following forms, respectively

$$u = f'(y) \qquad \text{…(21)}$$
$$v = 0 \qquad \text{…(22)}$$
$$\omega = -f''(y). \qquad \text{…(23)}$$

Using (17)-(20) in equations (15) and (16), we obtain, respectively

$$P_x = aPf'''(y) + aP_y f''(y) - \frac{aP}{k} f'(y) \qquad \text{...(24)}$$

$$P_y = aP_x f''(y). \qquad \text{...(25)}$$

Upon substituting (25) in (24), and simplifying, we obtain

$$\frac{P_x}{P} = \frac{af'''(y)}{[1-a^2 f''^2(y)]} - \frac{af'(y)}{k[1-a^2 f''^2(y)]}. \qquad \text{…(26)}$$

The terms on the *RHS* of (26) can be expressed as follows:

$$\frac{af'''(y)}{[1-a^2 f''(y)^2]} = \frac{af'''(y)}{2[1+af''(y)]} + \frac{af'''(y)}{2[1-af''(y)]} = \frac{1}{2}\frac{d}{dy}\ln\frac{[1+af''(y)]}{[1-af''(y)]} \qquad \text{…(27)}$$

$$\frac{af'(y)}{k[1-a^2 f''(y)^2]} = \frac{af'(y)}{2k[1+af''(y)]} + \frac{af'(y)}{2k[1-af''(y)]}. \qquad \text{…(28)}$$

Equation (26) is then expressed as:

$$\frac{\partial(\ln P)}{\partial x} = \frac{1}{2}\frac{d}{dy}\ln\frac{[1+af''(y)]}{[1-af''(y)]} - \frac{af'(y)}{2k[1+af''(y)]} - \frac{af'(y)}{2k[1-af''(y)]}. \quad \ldots(29)$$

Integrating (29) with respect to *x*, we obtain:

$$\ln P = \left\{\frac{1}{2}\frac{d}{dy}\ln\frac{[1+af''(y)]}{[1-af''(y)]} - \frac{af'(y)}{2k[1+af''(y)]} - \frac{af'(y)}{2k[1-af''(y)]}\right\}x + \ln F(y) \quad \ldots(30)$$

or

$$P = F(y)\exp[\frac{x}{2}G(y)] \quad \ldots(31)$$

where $F(y)$ is a function to be determined, and

$$G(y) = \frac{d}{dy}\ln\frac{[1+af''(y)]}{[1-af''(y)]} - \frac{af'(y)}{k[1+af''(y)]} - \frac{af'(y)}{k[1-af''(y)]}. \quad \ldots(32)$$

From equation (31) we obtain

$$P_x = \frac{F(y)G(y)}{2}\exp[\frac{x}{2}G(y)] \quad \ldots(33)$$

$$P_y = [F'(y) + \frac{x}{2}F(y)G'(y)]\exp[\frac{x}{2}G(y)]. \quad \ldots(34)$$

Using (33) and (34) in equation (25), we obtain

$$[F'(y) + \frac{x}{2}F(y)G'(y)] = \frac{a}{2}f''(y)F(y)G(y) \quad \ldots(35)$$

and, upon equating coefficients of similar powers of *x*, we obtain

$$F(y)G'(y) = 0 \quad \ldots(36)$$

and

$$\frac{F'(y)}{F(y)} = \frac{a}{2}f''(y)G(y). \quad \ldots(37)$$

Equations (36) and (37) yield:

$$F(y) = \exp\left[\frac{a}{2}\int f''(y)G(y)dy\right] \quad \ldots(38)$$

and

$$G(y) = C_2 \quad \ldots(39)$$

where C_2 is a constant.

Using (39), and integrating the RHS of equation (38) once, equation (38) is replaced by

$$F(y) = C_4\exp[\frac{aC_2}{2}f'(y)] \quad \ldots(40)$$

where C_4 is a constant. In addition, using (39), equation (32) becomes

$$\frac{d}{dy}\ln\frac{[1+af''(y)]}{[1-af''(y)]}-\frac{af'(y)}{k[1+af''(y)]}-\frac{af'(y)}{k[1-af''(y)]}=C_2 \qquad \text{…(41)}$$

and the pressure equation (31) is replaced by:

$$P=F(y)\exp[\frac{C_2x}{2}]. \qquad \text{…(42)}$$

Using (40) in (42) we obtain

$$P=C_4\exp[\frac{C_2}{2}(x+af'(y))]. \qquad \text{…(43)}$$

The pressure distribution and flow quantities thus hinge on the function $f(y)$ that must be chosen such that (41) is satisfied. Equation (41) can be expressed in the following form:

$$f'''+\frac{aC_2}{2}f''^2-\frac{f'(y)}{k}=\frac{C_2}{2a}. \qquad \text{…(44)}$$

In the analysis to follow, we will select and substitute a suitable function $f(y)$ in equation (44), one that satisfies (44) and helps determine the constant C_2. With the knowledge of C_2, the pressure function in (43) can then be determined and other flow quantities and viscosity can be determined from equations (8), (17), and (21)-(23). The next section will provide some examples.

4. RESULTS AND ANALYSIS

4.1. Example 1

Let $f(y)=Ay+B$, where A and B are known constants, and assume that permeability k is constant. Equation (44) yields $C_2=\frac{-2aA}{k}$ and equation (43) gives the following pressure distribution:

$$P=C_4\exp[\frac{-aA}{k}(x+aA)]. \qquad \text{…(45)}$$

Equation (8) renders the viscosity function as:

$$\eta=a\rho C_4\exp[\frac{-aA}{k}(x+aA)]. \qquad \text{…(46)}$$

The constant C_4 in the pressure distribution can be determined with the imposition of a condition on the pressure. For instance, if $P(0,0)=P_0$ then (45) yields $C_4=P_0\exp[\frac{a^2A^2}{k}]$, and the pressure distribution and viscosity become, respectively,

$$P=P_0\exp[-\frac{aA}{k}x]. \qquad \text{…(47)}$$

$$\eta = a\rho P_0 \exp[-\frac{aA}{k}x]. \qquad \ldots(48)$$

The flow quantities of streamfunction, vorticity and velocity components are determined from equations (17), (21), (22) and (23) as:

$$\psi = Ay + B \qquad \ldots(49)$$

$$u = A \qquad \ldots(50)$$

$$v = 0 \qquad \ldots(51)$$

$$\omega = 0. \qquad \ldots(52)$$

In **Figures 1** and **2**, we provide a sketch of $\frac{P}{P_0}$ vs. *x*, as given by equation (47). **Figure 1** illustrates the effect of parameter *a* that appears in equation (8). We therefore take $0 \le x \le 1$ and $0 \le y \le 1$ and choose the following values of permeability *k* and constant *A*, and vary *a*. The main intention here is to illustrate the effects of increase or decrease of the constant of proportionality, *a*, however, in a specific industrial situation some ranges of the constants might be pre-determined. For the sake of illustration, the following three cases are considered:

Case 1: *k=1, A=1, a=1*
Case 2: *k=1, A=1, a=5*
Case 3: *k=1, A=1, a=10.*

Figure 1 shows the pressure distribution curves for various values of *a*, while keeping other parameters fixed. Those same values of *a* represent "multipliers" of the pressure to produce the viscosity values, as can also be seen in equation (47).

Figure 2 illustrates the effect of increasing permeability while holding other parameters fixed, and shows the relative increase in pressure as permeability increases.

Case 1: *k=0.1, A=1, a=1*
Case 2: *k=1, A=5, a=5*
Case 3: *k=10, A=1, a=1*

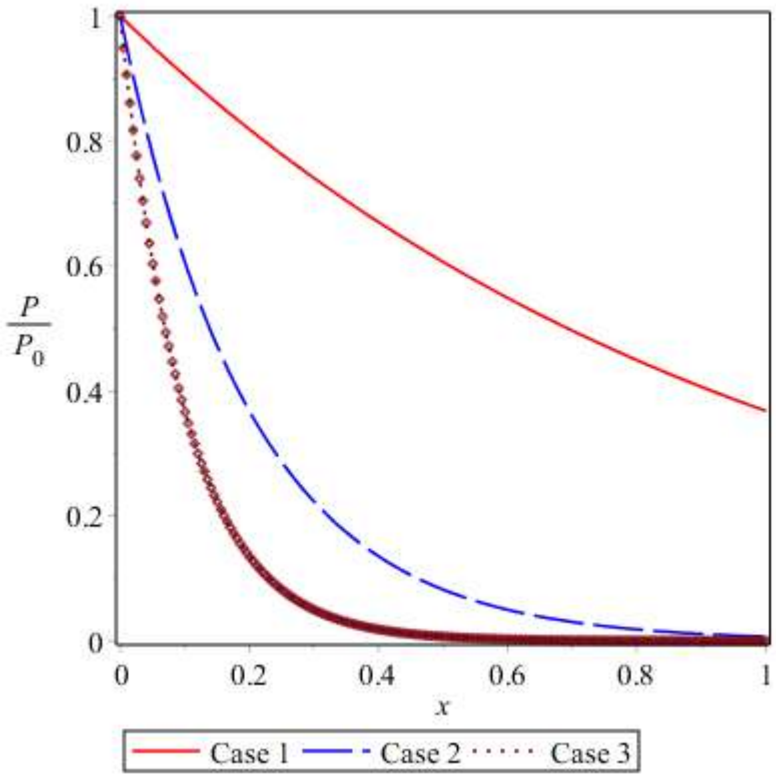

Figure 1. Effect of parameter *a* on $\frac{P}{P_0}$, fixed permeability and parameter *A*

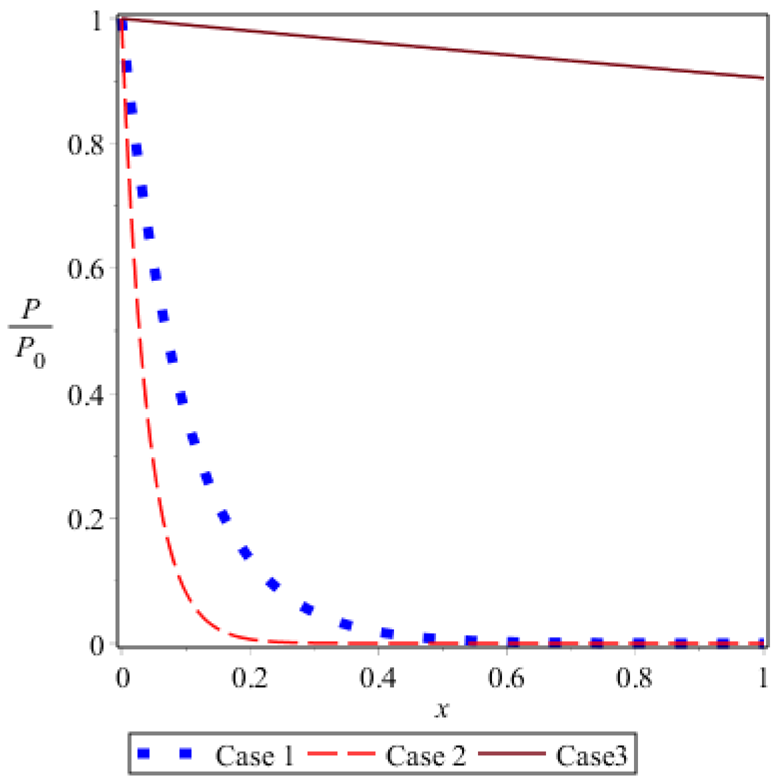

Figure 2. Effect of parameter combinations on $\frac{P}{P_0}$

4.2. Example 2

Let $f(y) = Ay^2 + By + C$, where *A*, *B*, *C* are known coefficients, and $k = 2Ay + B$ a variable permeability. Equation (44) yields $C_2 = \frac{2a}{4a^2A^2 - 1}$, hence equation (43) gives the following pressure distribution:

$$P = C_4 \exp[\frac{a}{4a^2A^2 - 1}\{x + a(2Ay + B)\}] \qquad \text{…(53)}$$

and equation (8) renders the viscosity as:

$$\eta = a\rho C_4 \exp[\frac{a}{4a^2A^2 - 1}\{x + a(2Ay + B)\}]. \qquad \text{…(54)}$$

If we impose the condition $P(0,0) = P_0$ then equation (53) gives $C_4 = P_0 \exp[\frac{a^2B}{1 - 4a^2A^2}]$. Pressure and viscosity distributions thus become:

$$P = P_0 \exp[\frac{ax + 2a^2Ay}{4a^2A^2 - 1}] \qquad \text{…(55)}$$

$$\eta = a\rho P_0 \exp[\frac{ax + 2a^2Ay}{4a^2A^2 - 1}]. \qquad \text{…(56)}$$

The flow quantities of streamfunction, vorticity and velocity components are determined from equations (17), (21), (22) and (23) as:

$$\psi = Ay^2 + By + C \qquad \text{…(57)}$$
$$u = 2Ay + B \qquad \text{…(58)}$$
$$v = 0 \qquad \text{…(59)}$$
$$\omega = -2A. \qquad \text{...(60)}$$

Three-dimensional pressure distribution described by (55) is illustrated in **Figures 3** and **4**, in order to demonstrate the effects of varying parameter *a*. Increasing the value of *a* is accompanied by increasing the pressure with decreasing *x*. We point out in this case that the permeability value is tied in with the value of parameter *A* and the constant pressure P_0.

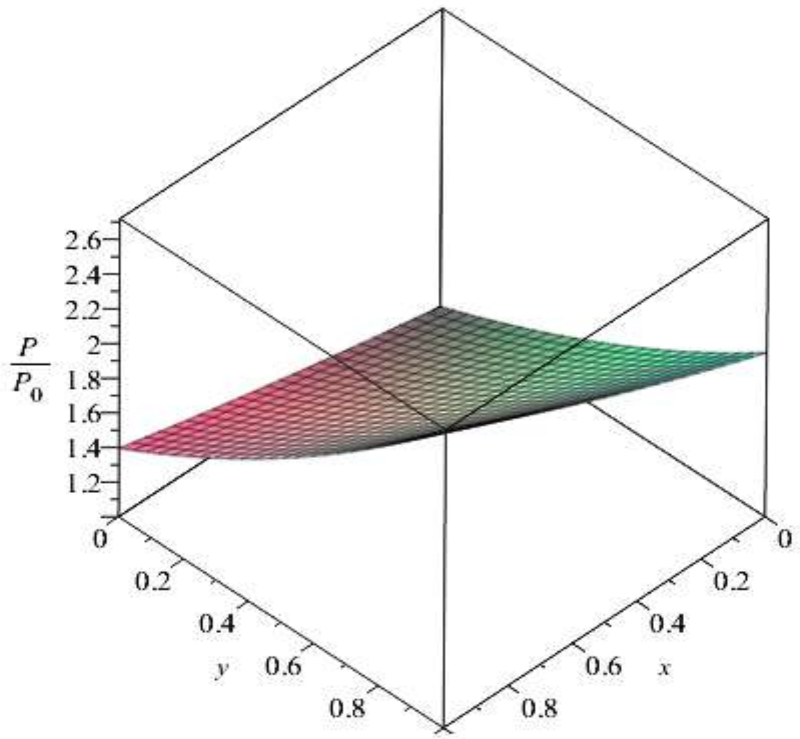

Figure 3. $\frac{P}{P_0}$ **for** ***A=1*****, and** ***a=1***

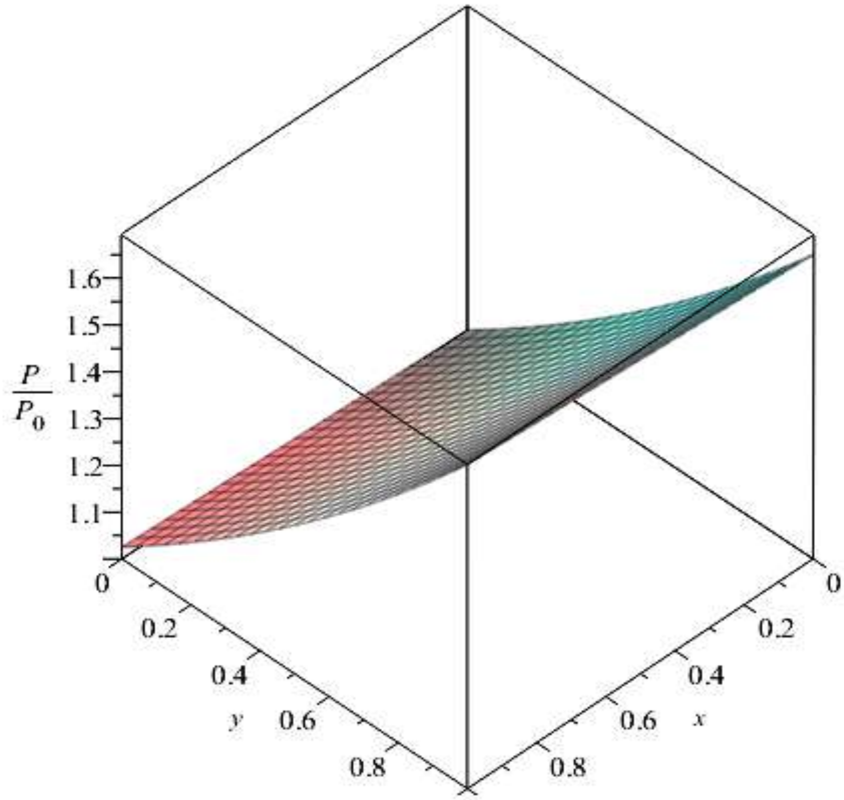

Figure 4. $\frac{P}{P_0}$ **for** ***A=1*****, and** ***a=10***

4.3. Example 3

Let $f(y) = A(\sin y + \cos y)$ where *A* is a known coefficient, and choose a variable permeability of the form

$$k(x,y) = \frac{2[\cos y - \sin y]}{aAC_2[\sin y \cos y] + [\sin y - \cos y]} . \qquad \ldots(61)$$

Provided that $a^2 A^2 = 1$, C_2 can be any constant that is tied to the permeability through equation (61). Under these conditions, the pressure and viscosity distributions are given, for selected values of C_2, *a*, and *A*, respectively by

$$P = C_4 \exp[\frac{C_2}{2}\{x + aA(\cos y - \sin y)\}] \qquad \ldots(62)$$

$$\eta = a\rho C_4 \exp[\frac{C_2}{2}\{x + aA(\cos y - \sin y)\}]. \qquad \ldots(63)$$

If we impose the condition $P(0,0) = P_0$ then equation (62) yields $C_4 = P_0 \exp[\frac{-aAC_2}{2}]$, and the pressure and viscosity distributions take the form:

$$P = P_0 \exp[\frac{C_2 x}{2} + \frac{aAC_2}{2}(\cos y - \sin y - 1)] \qquad \ldots(64)$$

$$\eta = a\rho \exp[\frac{C_2 x}{2} + \frac{aAC_2}{2}(\cos y - \sin y - 1)].. \qquad \ldots(65)$$

The flow quantities of streamfunction, vorticity and velocity components are determined from equations (17), (21), (22) and (23) as:

$$\psi = A(\sin y + \cos y) \qquad \ldots(66)$$

$$u = A(\cos y - \sin y) \qquad \ldots(67)$$

$$v = 0 \qquad \ldots(68)$$

$$\omega = A(\sin y + \cos y) . \qquad \ldots(69)$$

For the sake of illustration, the permeability distribution, equation (61), is graphed in **Figures 5, 6, 7** and **8** for $C_2 = 1$, 2, 5 and 10, respectively. With increasing C_2, the location of the jump in permeability shifts closer to the plane *y = 0*.

The effect of C_2 on $\frac{P}{P_0}$ is illustrated in **Figures 9** to **10**, which demonstrate the relative shift of the location of maximum and minimum pressure towards the line *y = 0* as C_2 increases from 1 to 10.

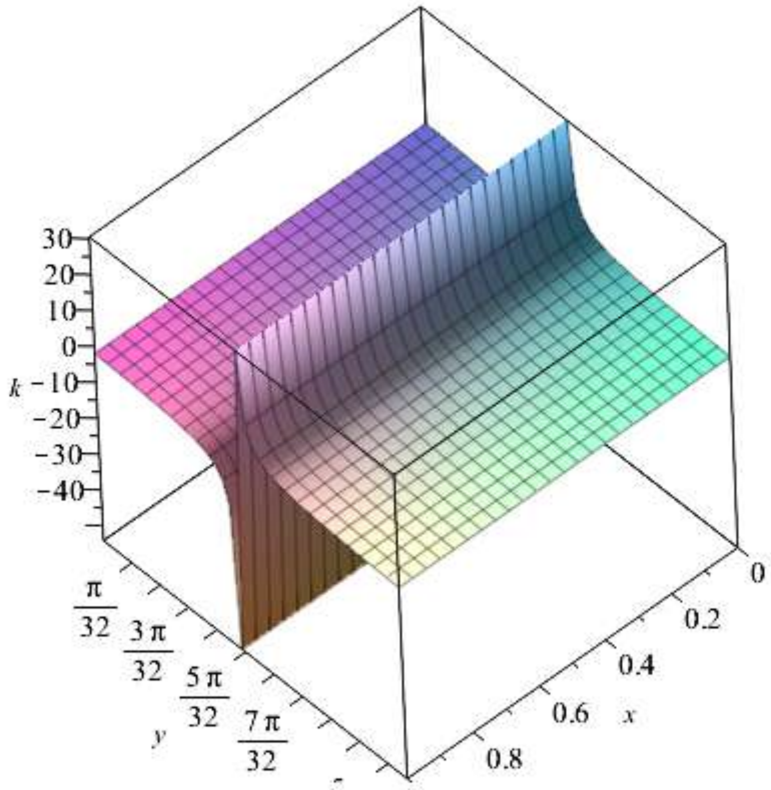

Figure 5. Permeability function for $C_2 = 1, A = a = 1$

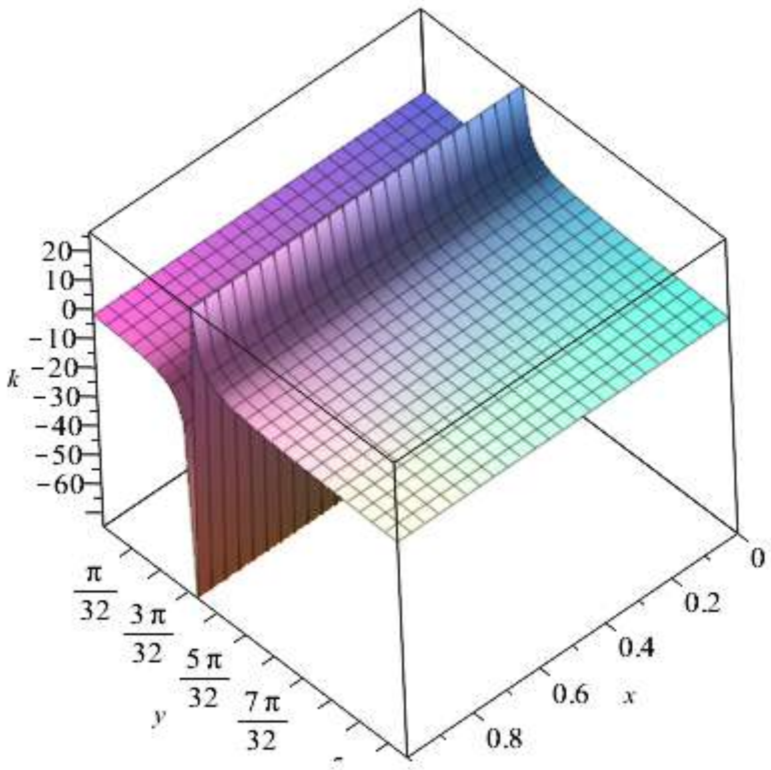

Figure 6. Permeability function for $C_2 = 2, A = a = 1$

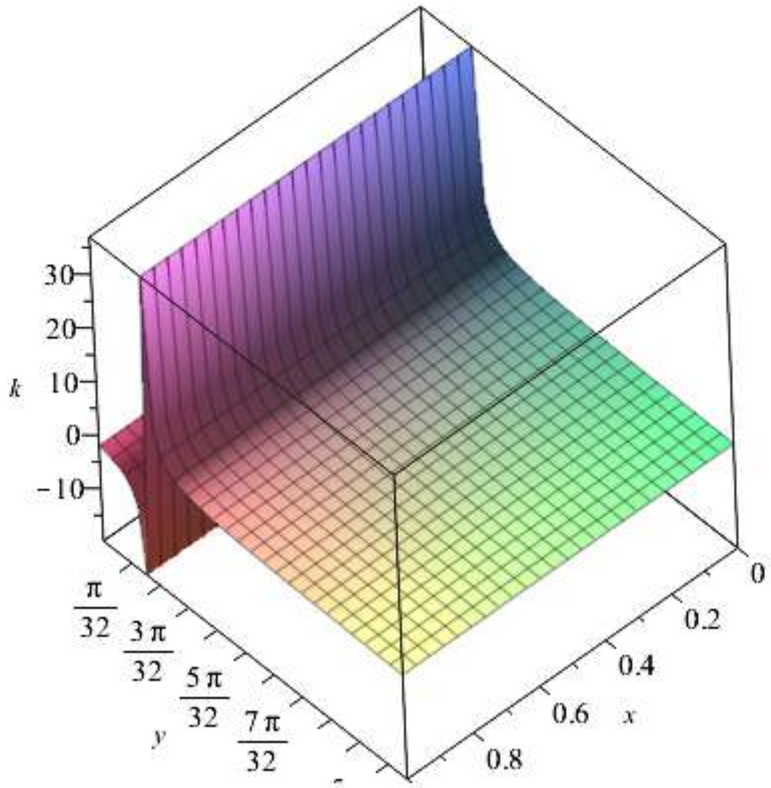

Figure 7. Permeability function for $C_2 = 5, A = a = 1$

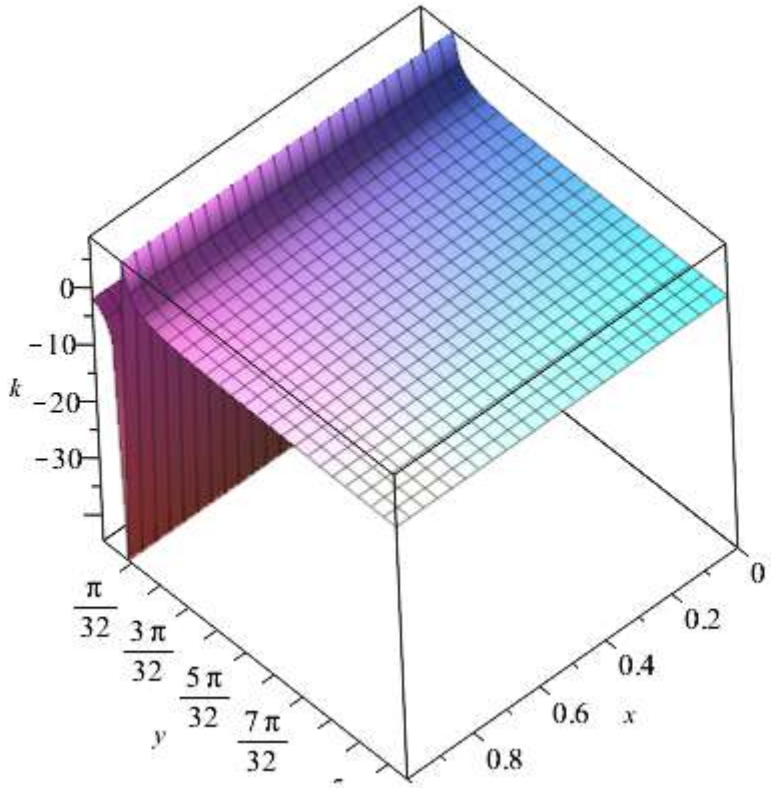

Figure 8. Permeability function for $C_2 = 10, A = a = 1$

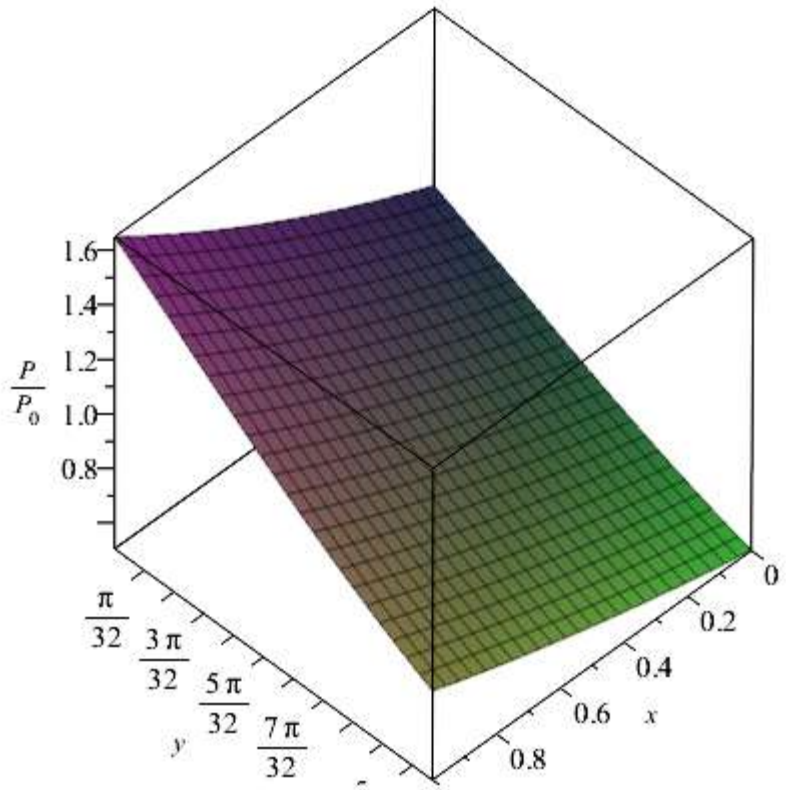

Figure 9. $\frac{P}{P_0}$ for $C_2 = 1, A = a = 1$

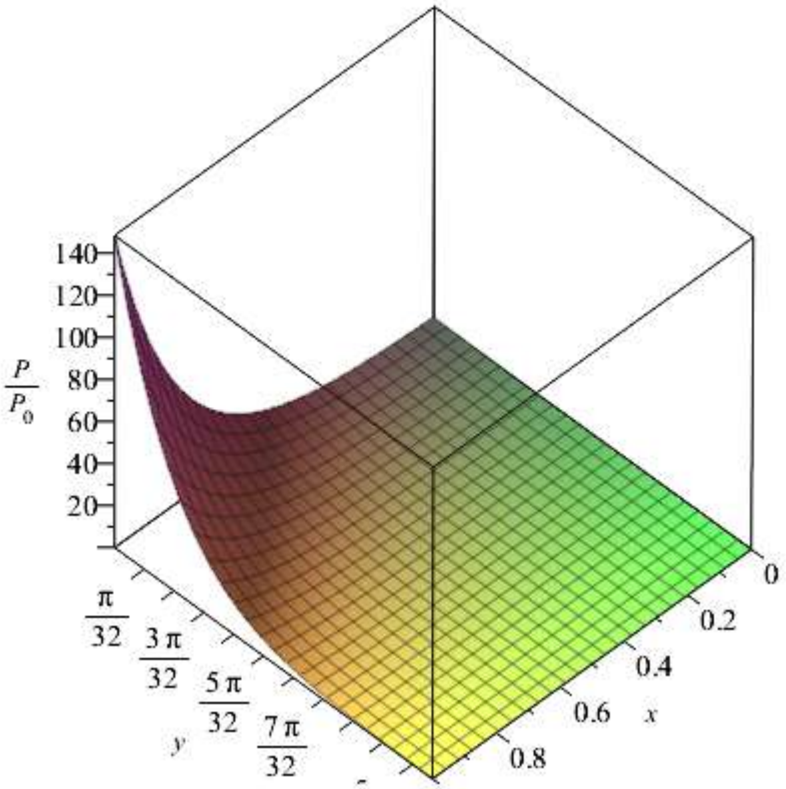

Figure 10. $\frac{P}{P_0}$ for $C_2 = 10, A = a = 1$

5. CONCLUSION

In this work, we considered the two-dimensional flow of a fluid with pressure-dependent viscosity through a porous medium with variable permeability, in general. The permeability function was selected so that the governing equations are satisfied and the arbitrary constants appearing in the pressure distribution can be determined. The variable fluid viscosity was taken to be proportional to the pressure, and the effect of the constant of proportionality on the pressure distribution was illustrated using pressure distribution graphs. We have illustrated in this work how to handle and solve the governing equations of flow through variable permeability. The same methodology can be applied when other forms of viscosity-pressure relations are employed.

6. ACKNOWLEDGEMENT

The authors would like to thank the referee of this article for comments that enhanced this contribution.

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