Towards an Agent-Based Modeling of the Consumer Goods Market

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ABSTRACT— ***The modeling and prediction of the human behavior in economy attract the interest of many scholars and researchers around the globe. In order to analyze it, the economists use complex mathematical equations called top-down approach; which is currently an outdating system. Instead the agent-based modeling named bottom-up analysis is focused in the characteristics of each agent. Therefore the implementation of the different behaviors of agents, allows them to interact with each other freely and the system evolves automatically.***

Our aim is to create an agent-based model of consumer goods market which will allow us to investigate the effect and the outcome of some economic policies or new product introductions.

Keywords— Multi-agent systems, agent-based modeling and simulation, complexity economics, macroeconomic behavior, Netlogo.

1. INTRODUCTION

A system is complex, if the whole transcends the parts. Complex systems consist of diverse entities or agents that [2]:

- They interact both in space (real or virtual) and time.
- They are connected.
- Theirs behaviors and actions are interdependent.
- They adapt.

In other words, complex systems are characterized by three features: integration, interaction and evolution.

Tesfatsion and Judd [4], explain that complex systems are used to analyze the behavior of individuals, but also to explain how interactions between many individuals can lead to results that emerge macroscopically, where what occurs on the macro level differs in kind, from what we see in the parts. For example, some sorts of spatial pattern or structure: flocking of birds, schooling of fish, or in physics, crystal structure are what we call self-organization phenomena.

According to Tesfatsion [4], economy is a complex system and complexity in economic systems is due to large number of economic agents involved in local interactions, which are distributed:

- Information in the two directions between the microstructure and macroeconomic patterns induced by the interactions of agents;
- Uncertain invasive behavior;
- Potential existence of multiple equilibriums;
- Critical role of institutional arrangements.

We can say that it is easy to make models of some aspects of a complex system; but it is difficult to model its entire and its emergence. However, a number of aspects of the complex system have traditionally been modeled using simplification of complex components of aggregates, for example; by using differential equations, system dynamics or Monte Carlo simulations [2].

Mathematical models rely on the identification of the key system components, often represented in a discrete manner. This limits mathematical models, because emergence present in complex systems arises as a consequence of local interactions, and could not be identified as a key system component.

Mathematical models are analogues, but cannot provide significant insight into the continuous internal process of a complex system. Moreover, they only take into consideration the global point of view, usually not explaining the reasons locally leading to the global behaviour of the modelled system [2].

In order to offer a complete analysis to resolve the complex problem of market, several researchers had to think about the possibility of linking the microscopic aspects; in other words to consider the behavior of individuals throughout their heterogeneity; their characteristics and the manner in which they interact and the macroscopic aspects of the system.

However, we can see through the literature that complex system researchers often prefer to use agent-based modelling approaches, to model and simulate complex systems.

The agent-based simulation provides solutions to the limitations of the mathematical approach [2]. It allows to:

- Represent the inter-temporal phenomena in economics;
- Represent heterogeneous behavior;
- Ignore the assumption loopback model and represent for example changes without a constraint demand or budget;
- Represent a continuously evolving system and consequently avoids the account Equilibrium;
- Consider the consequences of strategic choices on the properties of resources committed by the agent in the competitive process (integration of changes and innovations).

Unlike mathematical equations, in multi-agent approach, other properties can be given to an agent:

- Birth, death, and reproduction;
- Individual needs of resources (i.e.to eat and drink);
- Competition and fighting ability;
- Tool making ability (i.e. the possibility to grow food, hunt etc.);
- Perception;
- Curiosity, exploration behavior, ability for innovation;
- Emotions;
- Memory and future expectations;
- Mobility and carrying capacity;
- Communication;
- Learning and teaching ability;
- Possibility of trading and exchange;
- Tendency to have relationships with other agents.

This work represents a deepening of paper we have presented in the International Congress on Islamic Economics and Finance (ICISEF)-2015, titled :``The Agent-Based Modelling of the Macroeconomic Behaviour of Sharia-Compliant Companies`` [1],focused on the importance of the use of the agent-based modelling instead of computable general equilibrium models, and a proposition of a model of company's behaviour during its life cycles.

H. Dawid and all realized a work titled "The Eurace@Unibi Model, An Agent-Based Macroeconomic Model for Economic Policy Analysis", which aims to provide a micro-founded macroeconomic model that can be used as a unified framework for the analysis of economic policies and for the treatment of macroeconomic research questions.The Eurace@Unibimodel summarizes interactions in the labour market, consumption goods market, financial market, capital goods market and finally credit market

In the same way, our work presents a model of consumer's and producer's behaviors, which is then incorporated within a multi-agent simulation framework to illustrate precision and usefulness of such an approach in predicting market phenomena. However, what is very interesting is that the consumption of each agent will depend not only of his wealth but also the consumption of his neighbors.

We will try to discover how such models can be used to explore the future, with their ability to incorporate "what-if" scenario building techniques, and illustrate how radical changes in the market; such as new product introductions; can be modeled robustly using computational methods. Also, we will analyze interactions in the labor market, credit market and finally consumption goods market.

In section II, we summarize some works developing agent-based theory, as well as the added value of our work. In section III, we present the composition of the market and the types of agents that will be integrated in the model. In section IV, we discover the timing of the events and the activity sequences of the agents. In section V, we quote the three market interactions. In section VI, we analyze the microeconomic behavior of agents using mathematical equations, and finally in section VII, we present the Netlogo platform which will be used for the implementation of our model.

2. LITERATURE REVIEW

Consumer and producer behaviors as a field of study is highly interdisciplinary in its approach, and that is evident in the amount of literature on this topic in multiple fields of study, whether in economics, psychology, sociology, computer science or even applied mathematics.

Correspondingly, the traditional methods of analysis used by researchers in this field are numerous, however, over the last few years, new studies are being increasingly seen in the literature, which use modern computational techniques based on computer simulations.

Among one of these techniques: theAgent-Based Simulation (ABS) which is considered new fields of investigation, have led to significant results, and behavioural analysis is considered one of the most important goal of this theory [3].

Massachusetts Institute of Technology, in the United States has been the most fertile ground of the theory of complex systems (TCS) since 1940, through the work of several researchers; namely, Heinz von Foerster and J. von Neumann who first worked on the self-organization problems before formulating their famous game theory [2].

There are many scientists who contribute to the field of complex systems, which follows focuses on the two most famous scientists Murray Gell-Mann and John Henry Holland and their contributions in this area.

We cited the work of Newman [18] that divided the applications of complex systems into 5 types with principal references. It's synthesized in the following table [2]:

Tableau 1: Applications of Complex Systems Theory [2].

Applications	Principal references
Physical systems	Anderson [71]; Sethna [72]; Sander [73]
Ecosystems and biological evolution	May [74] ; Kauffman [75]; Kauffman [76]; Levin [77]; Holling [78]; Solé [79]; Nowak[80]
Human societies	Jacobs [81]; Wasserman [82]; Scott [83]; Watts [84]; Batty [85]; Bettencourt [86]; Batty [87]; Bettencourt [88].
Economics & markets	Mandelbrot [89]; Jackson [90]; Easley [91].
Pattern formation and collective motion	Turing [92]; Nagel [93]; Vicsek [94]; D. Helbing [95]; Winfree [96]; Ballerini [97]

We quote some interesting work that summarizes the evolution of complex systems and agent-based simulation over the time:

Tableau 2: Evolution of Complex Systems and Agent-Based Simulation Over the Time.

Authors	***Titles***	***Years***
T. Schelling	"Dynamicmodels of segregation,"	1971
-T. S. Ray, C. Taylor, J. D. Farmer, and S. Rasmussen. - R. G. Palmer, W. B. Arthur, J. H. Holland, B. LeBaron, and P.Tayler	- "An approach to the synthesis of life," :"Artificial Life II," volume XI of Santa Fe Institute Studies in the Sciences of Complexity -"Artificial economic life: a simple model of a stock market,"	1991 1994
J. M. Epstein and R. L. Axtell	"Growing Artificial Societies: Social Science from the Bottom Up",	1996
M. W. Macy and R. Willer	"From factors to actors: Computational sociology and agent based modeling,",	2002
N. Gilbert	"Agent-BasedModels"	2007
H. Dawid, S. Gemkow,P. Harting, S. V. Hoog, M. Neugart.	"The Eurace@Unibi Model, An Agent-Based Macroeconomic Model for Economic Policy Analysis"	2012

THE COMPOSITION OF THE MARKET AND THE TYPES OF AGENTS

2.1. Composition of the Market:

Economy is composed of several markets related by interactions; which creates liveliness to the level of economic activity. A market is an articulated system around formal procedures or market rules.

So we can identify three major components in a market [6]:

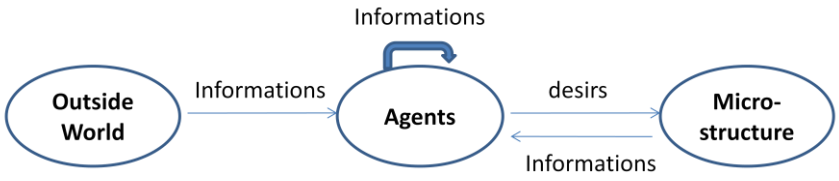

Figure 1: the composition of a market [6]

- Market structure: a market is structured around a set of rules that describe its operation and govern the details of the exchange (quotation schedule auction process setting prices, existence of intermediaries, etc.). This set of rules is called market microstructure.

- Economic agents: they want to invest their money capital. Around these investors there are a number of other agents such as consultants, brokers, representatives of brokerage company, information operators, which can be both in the market and outside.

- Information: is used by agents to make decisions and choose their investments. It can be of two types: endogenous, whether it comes from inside or exogenous market if it is from the world outside that market.

In this paper we are interested in the consumer goods market, but there will be inevitably interactions with other markets.

2.2. Types of Agents Used in the Model:

Agents are able to act and make decisions to reach their goals, they are of four types:

- Households: their functions are work, consumption and savings. To be able to consume they must have a salary, so they have to work or to own a firm or a bank. Households are the most numerous agents.
- Firms: their main function is the production; in addition a firm hires, borrows, produces, sells, repays her loans and pays dividends to its owner.
- Banks: in an economy with endogenous money, banks are supposed to be accommodative. We can assume the presence of a single representative bank in the banking sector, and indeed the disintegration of the latter is of secondary importance which is the macroscopic dynamic study of the modeled economy.
- Government: it sets the economic policies and regulation of the market.

3. TIMING OF THE EVENTS AND ACTIVITY SEQUENCES OF AGENTS

3.1. Timing:

For the time unit in the model, there are event-based activities and calendar driven. Many decisions are taken monthly or weekly. Each agent has a particular day to act once a year/month/week for each decision, where these days to act might differ between agents of the same type [7].

Therefore, there is in general no synchronization between different actors, although for each type of decision or action the time spans between subsequent days to act are identical for all firms and households.

Only few activities in the model are event-based, like the application for unemployment benefits at the point in time when an employee becomes unemployed.

3.2. Activity Sequences and Calendar Driven or Event-Based Activities of Households and Firms:

3.2.1. Firms:

The economic activity sequences followed by a company [7]:

1. During the activation date; the firm begins its production cycle by a production planning; it plans its output based on historical observations of market research results; and according to its production schedule; the firm determines its input in terms of capital and labor.

2. The firm's financial management: the firm calculates its cost of production and the cost of financing commitments, if the resources are insufficient; the firm tries to provide external financing by means of credit.

3. The bank grants loans according to first come first served; it requires credit conditions depending on the financial situation of the firm; if credit is refused totally or partially; the firm must reduce its planned production.

4. Bankruptcy: if the firm has credit difficulties to the point that it is unable to meet its financial commitments; it becomes illiquid and illiquidity bankruptcy is declared. But if at the end of the production cycle; net income is negative; the firm is declared insolvent.In both cases the firm stops all production activities and employees lose their jobs; and the company is inactive for a period before it is active again.

5. The labor market opens when firms that have vacancies are suitable for unemployed workers. The correspondence is based on salary and the level of employee competence.

6. Production begins on the day of activation; and as soon as it is completed; the output must be distributed at malls. The firm offers a displayed price goods with annual revision of prices.

7.When the production cycle finishes; the firm calculates its revenues and updates its income statement and balance sheet; it pays taxes; dividends; Taxes and payments on the debt. It ensures if net income is negative; whether it declares bankruptcy or it starts a new production cycle.

Tableau 3: Firm sequences of economic activities [7]

Yearly :

- determine price

Monthly :

- Production planning : decide planned output
- Determine input demand for capital and labor
- Financial management
- Credit market interaction
- Capital goods market interaction
- Labor market interaction
- Production of output and distribution to malls
- Accounting : revenues ,
- Pay taxes ,

Event-based :

- Bankruptcy protocol: entry and exit (if equity <0)

3.2.2. Households:

The economic activity sequences of a household follow that order[7]:

1. Households receive wages in the first day of the production cycle. They also receive dividends from the first day of the monthly calendar.

2. After the payment of taxes; the household determines the budget earmarked for consumption; indeed consumption decision affects the portfolio of assets planned. Households fall into the financial market before the consumer goods market opens.

3. Households are doing shopping every week; they choose consumption goods available in malls; they spend their wages evenly over four weeks.

4. In case of unemployment; the household receives unemployment benefits by the government.

Tableau 4: Household Sequences of Economic Activities [7]

Monthly :

- Receive labor income (unemployment benefits if unemployed)
- pay taxes
- consumption

Weekly :

- consumption goods market interaction

Event-based :

- labor market interaction (if unemployed)

4. MARKET INTERACTIONS

We summarize the agents, markets, and main messages that we have in the model:

Tableau 5: Agents,Markets and Messages in the Model [7].

Agent	***Context***	***Role***	***Messages***
Household	Consumption goods Market	Buyer	Units demanded
	Labour Market	Worker	application, accept/reject job
Firm	Labour Market	Employer	vacancy, job offer
	Credit Market	Borrower	Loan request
	public sector		Tax payments
Bank	Credit Market	lender	Credit conditions
Government	Public sector		Tax payments

4.1. Credit Market Interactions

The bank accepts credit demand taking into consideration the risk of their loan portfolios according to bale II standards; if the credit demand is refused; firms must reduce its planned production quantity until the plan can be financed by the bank.

4.2. Labor Market Interactions

According to the procedures described in the household sequence of economic activities, consumption goods producers' review once a month whether to post vacancies for production workers. Job seekers check for vacancies[7]:

Step 1: the firms post vacancies including wage offers.

Step 2: every job seeker extracts from the list of vacancies those postings to which he fits in terms of his reservation wage. He sends an exogenous determined number of applications to randomly chosen firms.

Step 3: if the number of applicants is smaller or equal to the number of vacancies the firms send job offers to every applicant. If the number of applicants is higher than the number of vacancies firms send job offers to as many applicants as they have vacancies to fill.

Step 4: each worker ranks the incoming job offers according to the wages net. Each worker accepts the highest ranked job offer at the advertised wage rate. After acceptance a worker ignores all other job offers and outstanding applications.

Step 5: vacancy lists are adjusted for filled jobs and the labor force is adjusted for new employees.

Step 6: if the number of unfilled vacancies exceeds some threshold the firm raises the base wage offer. If an unemployed job seeker did not find a job he reduces his reservation wage. Go to step 1.

4.3. Consumption Market Interactions

We suppose that there is the only one mall where the households can buy their needs in consumption; and firms can sell their products.

Households go shopping once a week and then they try to spend their entire weekly consumption budget for one good.

Households have asynchronized shopping days and thus on each day of the months there may be shopping activities in the mall. If the total demand in the mall for one good exceeds its mall inventory level, then the mall has to ration the demand. In this case the mall sets a rationing quota corresponding to the percentage of the total demand that can be satisfied with the available goods. Each household receives then the indicated percentage of her requested consumption.

The production of the consumption goods firm follows a fixed time schedule with fix production and delivery dates. Even if the mall stock is completely sold out, it can only be refilled at the fixed delivery date. Consequently, all the demand that exceeds the expected value of the monthly sales and the additional buffer cannot be satisfied [7].

5. DESCRIPTION OF THE MICROECONOMIC BEHAVIOUR OF THE AGENTS

5.1. Firms choose to create some new posts for this period

The agents firms and household (worker) communicate and interact between them in a decentralized way. The fate of the meetings: 'the agent A postulates for a job offer of the firm B' is modelled in the program by random selection [5].

At every step, the agents adapt their behaviour according to their new state and changes of the environment.

We suppose that the economy consists of K firms noted i ∈ [1,…..K]; and N workers noted : j∈ [1,…..N].

Each of these firms employs workers to produce the same consumer good noted g.

Given the state of the system in time t-1, the stage in time t takes place in a following way [8]:

$$v_{it} = \frac{p_{t-1} q_{it-1}}{w_{it-1}}$$

Companies choose to create a number of new v_{it} posts for this period, where:

- p_{t-1} : the price of the good g in the previous stage
- q_{it-1}: the quantity of produced good.
- $w_{it\,1}$: the salary of an employee in the firm i in the time t-1.

This is the firm's calculation of turnover at time t- 1 reduced the salary of an employee. It is a measure of the maximum possible increase in payroll while maintaining positive expected profits in the future.

5.2. Workers Look and Postulate for Job Offers

Every worker j is characterized in time t by the salary which he wishes to obtainw_{it}^{s} and by a base salaryw_{i}^{R} supposed constant and below which he will not agree to choose an employment.

Also every firm is characterized in time t by a level of salary w_{it}^{s} which it agrees to overturn to her future employees.

In every stage of simulation, the worker j postulates from a company i with a proportional probability for the last starting salary w_{it-1} that she proposed in the previous stage. If the firm still has available places, the worker takes place in the list of the firm for this tour and asks for a salary equal to the salary which he wishes w_{jt}^{s} [8].

5.3. Process of Matching and Wage Negotiation

When all the workers postulated for job offers, the recruitment process itself is going to take place.

Let us suppose that the firm i receive in time t, m_{it} workers' demands.

It calculates the average wage asked by these workers [8]:

$$\bar{w}_{it} = \frac{1}{m_{it}} \sum_{h=1}^{m_{it}} w_{j_h t-1}^{s}$$

The company defines then a starting salary in this way:

$$w_{it} = \beta\, w_{it-1}^{s} + (1-\beta)\bar{w}_{it}$$

Where :

β : is a parameter which characterizes the trend of the company to be negotiated. When the company defined its level of starting salary, she proposes this salary to the workers who accept only if this salary is upper to their base salary w_j^R.

5.4. Production and Consumption of Goods

5.4.1. Production

In time t the firm i produce q_{it} units of the good g, with the pricep_t, following the equation [8]:

$$q_{it} = \alpha_{it} n_{it}$$

Where

α_{it} : is the productivity of the work in the firm i andn_{it} is the number of employees hired in time t by the same firm.

The price of the good g is given by the equation:

$$W_t = p_t Q_t$$

Where Q_t is the total production in the system and W_t is the global salary perceived by all the employees. This relation translates simply the fact that in this model the employees spend all of their income to buy the good g every period.

5.4.2. Consumption:

In a complex system, agent's behaviour is determined by two types of variables:

- Independent variables: are variables determined outside the system, they are exogenous to all agents.
- Relative variables: are variables describing the behaviour of local neighbours of each agent.

In our model, each agent earns an income $w_i$and decides on consumption ci. Consumption is affected by income, but we add one assumption: consumption is also affected by behaviour of local neighbours. That is [5]:

$$\boldsymbol{c_{i,t} = f(w_{i,t-1}, \bar{c}_{-i,t-1})}$$

Where $\bar{\boldsymbol{c}}_{-\boldsymbol{i}}$ is the average consumption of local neighbours of agent i.

Each agent sets his consumption level as a weighted average of his income and average consumption of his local neighbours [5]:

$$\boldsymbol{c_{i,t} = w_{i,t}} \qquad \text{for t=1}$$

$$\boldsymbol{c_{i,t} = (1-\lambda)\, w_{i,t-1} + \lambda \bar{c}_{-i,t-1}} \qquad \text{for t>1}$$

While $\boldsymbol{\lambda}$ is the weight of relative consumption.

Firms make profits at time t following the equation [8]:

$$\pi_{it} = \sum_i c_{i,t} - w_{it} n_{it}$$

5.5. Disappearance of the Firms which do not Make Profits.

The firms which make profits $\pi_{it} < 0$ disappear and are replaced by newcomers.

5.5.1. Technological Evolutions.

The technological progress in the economy is modelled by a factor of productivity of the workers which evolves according to a stochastic process [8]:

$$\alpha_{it} = \alpha_{it-1}(1+Z)$$

Where Z, conditionally Z >0, is a normal law centred by variance σ_Z^2.

5.5.2. Updates of reference salaries

When a worker accepted an employment, he updates the salary that he wishes for the next stage at the level of the salary that he perceived in the previous stage:

$$w_{jt-1}^s = w_{it}$$

Also for a company which recruited at least an employee in this stage:

$$w_{it-1}^{s} = w_{it}$$

6. AGENT-BASED SIMULATION: NETLOGO

Netlogo is a programming language based on an agent-based system. It is used for simulating different natural and social phenomena [14], where modellers can give instructions to hundreds or thousands of independent agents all operating concurrently, in order to analyze connections between micro-level behaviours of individuals and macro level patterns that emerge from their interactions [2].

Our agent-based system is implemented from the microeconomic behaviour described above using bottom-up approach [10]. The computing agents update their internal data at every step of time t.

- For a "household" agent j these data are the level of satisfactory salary w_{jt}^{s} and base salary w_{j}^{R}.
- For an agent "firm" i these data are the level of salary satisfying w_{it}^{s}, the number of employeesn_{it}, the produced quantity $q_{it}$and the generated profit π_{it}.
- The variables of the environment are the factor of productivity α_{it} and the price of the good produced p_t.
- The parameters of the model are: β the factor that characterizes the tendency of companies to negotiate, the variance σ_Z^2 of the law of evolution of the productivity factor, F the number of firms in the simulation and N the number of household (workers).

When the ABS converge after a large number of stages of the simulation, we can measure macroeconomic variables included from the microeconomic data of the individuals.

7. CONCLUSION

Mathematical approaches are limited to model complex system, and agent-based modelling approaches presents a very efficient method of analysis and simulating the interactions between different agents and explain phenomena that emerges in the system.

In this paper we have seen the different types of agent that will be integrated in the model, and also the importance of timing in the determination of activity sequences of the agents.

We detailed tree market interactions': credit market interaction, labour market interaction and finally consumption market interaction, actually this interactions represents all agent`s activities that we supposed in the model, and in the next work we have to integrate also the financial market interaction to have a more realistic model.

We have also defined the macroeconomic variables that characterise the agents and that we will use in the simulation to facilitate the implementation of the model in the Netlogo platform. So we construct an agent-based model of consumption goods market that we will try to adapt as much as possible to the reality to be able to explore the impact of some economic policies.

The model presented above can also be applicable to the analysis of opinion dynamics social networks; it may help to understand the mechanisms and dynamics of collective behaviors at the group from the decisions taken at the micro level [19].

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