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# Application of Decomposition Method for Natural Transform

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**ABSTRACT—** ***In this paper, Adomian decomposition method is applied to compute Natural transform. The suggested method is described and illustrated with some examples.***

**Keywords—** Natural Transform, Adomian Decomposition Method

## 1. INTRODUCTION

Adomain decomposition method (ADM) [1, 2] is a powerful decomposition methodology for the practical solution of linear or nonlinear ordinary differential equations, partial differential equations, integral equations, etc. This method has been receiving much attention in recent years in the area of series solutions. The ADM provides an approximate analytic solution of differential equations; we shall deal here only with the first order linear differential equation.

Consider the first order linear differential equations

$$\frac{dy}{dt} + p(t)y = f(ut) \tag{1}$$

with

$$y(0) = 0.$$

The analytical solution of (1) is given by

$$\int \mu(t) f(ut) dt = y(t)\mu(t), \tag{2}$$

where $\mu(t) = e^{\int p(t)dt}$ is an integrating factor.

By using the operator $L = \frac{d}{dt}$ equation (1) can be written as

$$Ly + p(t)y = f(ut) \tag{3}$$

or

$$p(t)y = f(ut) - Ly, \tag{4}$$

with

$$y(0) = 0.$$

By applying the standard Adomian decomposition method to equation (4) the solution $y$ will be obtained by the series:

$$y = \sum_{i=0}^{\infty} y_i \,. \tag{5}$$

The components $y_0, y_1, y_2, \ldots$ are calculated by

$$
\begin{aligned}
& y_0 = \frac{f(ut)}{p(t)}, \\
& y_1 = -\frac{1}{p(t)} L\left(\frac{f(ut)}{p(t)}\right), \\
& y_2 = (-1)^2 \frac{1}{[p(t)]^2} L^2\left(\frac{f(ut)}{p(t)}\right), \\
& \vdots \\
& y_i = (-1)^i \frac{1}{[p(t)]^i} L^i\left(\frac{f(ut)}{p(t)}\right),
\end{aligned} \tag{6}
$$

where $L^i = \dfrac{d^i}{dt^i}, i = 1, 2, 3, \ldots$ is $i$-fold differential operator. The convergence of the ADM was discussed by Cherruault [3, 4].

## 2. NATURAL TRANSFORM

The Natural transform of the function $f(t)$ for $t \in (-\infty, \infty)$ is defined by [5, 6]

$$\mathbb{N}[f(t)] = R(s,u) = \int_{-\infty}^{\infty} e^{-st} f(ut)dt; \quad s,u \in (-\infty, \infty), \tag{7}$$

where $\mathbb{N}[f(t)]$ is called the Natural transform of time function. Variables $s$ and $u$ are the Natural transform variables. Equation (7) can be written as

$$\mathbb{N}[f(t)] = \mathbb{N}^-[f(t)] + \mathbb{N}^+[f(t)] \tag{8}$$

where

$$\mathbb{N}^-[f(t)] = R^-(s,u) = \int_{-\infty}^{0} e^{-st} f(ut)dt; \quad s,u \in (-\infty, 0), \tag{9}$$

and

$$\mathbb{N}^+[f(t)] = R^+(s,u) = \int_{0}^{\infty} e^{-st} f(ut)dt; \quad s,u \in (0, \infty). \tag{10}$$

If the real function $f(t) > 0$ and $f(t) = 0$ for $t < 0$ is both piecewise continuous and exponential order, then the Natural transform is defined on the set

$$A = \left\{ f(t) \mid \exists M, \tau_1, \tau_2 > 0, |f(t)| < Me^{t/\tau_j} \text{ if } t \in (-1)^j \times [0, \infty) \right\}$$

as

$$\mathbb{N}^+[f(t)] = R^+(s,u) = \int_{0}^{\infty} e^{-st} f(ut)dt; \quad s,u \in (0, \infty),$$

where $H(\cdot)$ is the Heaviside function.

## 3. MAIN RESULTS

We now apply Adomian decomposition method to obtain some formulas for Natural transform.

Suppose that $p(t) = -s$ where $s$ is a positive constant. Then equation (2) becomes

$$\int e^{-st} f(ut)dt = e^{-st} y. \tag{11}$$

Integrating from zero to infinity of equation (11) makes the left hand side of this equation become the Natural transform of $f(t)$. Decompose $y$ as in equation (5) to obtain

$$\square^{+}[f(t)] = \int_0^\infty e^{-st} f(ut)dt = \left[ e^{-st} \sum_{i=0}^{\infty} y_i \right]_{t=0}^{\infty}. \tag{12}$$

**Example 1.** Let $f(t) = 1$, then by the Natural transform (12) we have

$$\square^{+}[1] = \int_0^\infty e^{-st} dt = \left[ e^{-st} \sum_{i=0}^{\infty} y_i \right]_{t=0}^{\infty}.$$

According to scheme (6), we have

$$y_0 = \frac{f(ut)}{p(t)} = -\frac{1}{s},$$
$$y_1 = y_2 = y_3 = \cdots.$$

Therefore,

$$\square^{+}[1] = \left[ e^{-st} \left( -\frac{1}{s} \right) \right]_{t=0}^{\infty} = \frac{1}{s}.$$

**Example 2.** Let $f(t) = e^{at}, s > au.$ From equation (12), we have

$$\square^{+}[e^{at}] = \int_0^\infty e^{-st} e^{aut} dt = \left[ e^{-st} \sum_{i=0}^{\infty} y_i \right]_{t=0}^{\infty}.$$

The components $y_i$ can be computed as follows:

$$\begin{aligned}
y_0 &= \frac{f(ut)}{p(t)} = -\frac{1}{s} e^{aut}, \\
y_1 &= -\frac{1}{p(t)} L\left( \frac{f(ut)}{p(t)} \right) = -\frac{au}{s^2} e^{aut}, \\
y_2 &= (-1)^2 \frac{1}{[p(t)]^2} L^2 \left( \frac{f(ut)}{p(t)} \right) = -\frac{a^2u^2}{s^3} e^{aut}, \\
&\vdots \\
y_n &= (-1)^n \frac{1}{[p(t)]^n} L^n \left( \frac{f(ut)}{p(t)} \right) = -\frac{a^n u^n}{s^{n+1}} e^{aut}, \quad n = 1, 2, 3, \ldots
\end{aligned}$$

Then,

$$\square^{+}[e^{at}] = \frac{1}{s} + \frac{au}{s^2} + \frac{a^2u^2}{s^3} + \cdots = \frac{1}{s - au}.$$

**Example 3.** Let $f(t) = t.$ From equation (12) we have

$$\square^{+}[t] = \int_0^\infty e^{-st} ut dt = \left[ e^{-st} \sum_{i=0}^{\infty} y_i \right]_{t=0}^{\infty}.$$

The have

$$
\begin{aligned}
y_0 &= \frac{f(ut)}{p(t)} = -\frac{1}{s}ut, \\
y_1 &= -\frac{1}{p(t)} L\left(\frac{f(ut)}{p(t)}\right) = -\frac{1}{s^2}u, \\
y_2 &= y_3 = y_4 = \cdots = 0.
\end{aligned}
$$

Therefore

$$
\square^{+}[t] = \left[ e^{-st}\left(-\frac{ut}{s} - \frac{u}{s^2}\right)\right]_{t=0}^{\infty} = \frac{u}{s^2}.
$$

**Example 4.** Let $f(t) = t^n$. From equation (12) we have

$$
\square^{+}[t^n] = \int_0^{\infty} e^{-st}(ut)^n \, dt = \left[ e^{-st} \sum_{i=0}^{\infty} y_i \right]_{t=0}^{\infty}.
$$

By following the scheme in (6), we have

$$
\begin{aligned}
y_0 &= \frac{f(ut)}{p(t)} = -\frac{1}{s}u^n t^n, \\
y_1 &= -\frac{1}{p(t)} L\left(\frac{f(ut)}{p(t)}\right) = -\frac{n}{s^2}u^n t^{n-1}, \\
y_2 &= (-1)^2 \frac{1}{[p(t)]^2} L^2\left(\frac{f(ut)}{p(t)}\right) = -\frac{n(n-1)}{s^2}u^n t^{n-2}, \\
&\vdots \\
y_n &= (-1)^n \frac{1}{[p(t)]^n} L^n\left(\frac{f(ut)}{p(t)}\right) = -\frac{n!}{s^{n+1}}u^n, \\
y_{n+1} &= y_{n+2} = y_{n+3} = \cdots = 0.
\end{aligned}
$$

Then,

$$
\square^{+}[t] = \left[ e^{-st}\left(-\frac{1}{s}u^n t^n - \frac{n}{s^2}u^n t^{n-1} - \frac{n(n-1)}{s^3}u^n t^{n-2} - \cdots - \frac{n!}{s^{n+1}}u^n\right)\right]_{t=0}^{\infty} = \frac{n!}{s^{n+1}}u^n.
$$

**Example 5.** Let $f(t) = \sin(at)$. From equation (12) we have

$$
\square^{+}[\sin(at)] = \int_0^{\infty} e^{-st} \sin(aut) dt = \left[ e^{-st} \sum_{i=0}^{\infty} y_i \right]_{t=0}^{\infty}.
$$

where

$$
\begin{aligned}
y_0 &= \frac{f(ut)}{p(t)} = -\frac{1}{s}\sin(aut), \\
y_1 &= -\frac{1}{p(t)} L\left(\frac{f(ut)}{p(t)}\right) = -\frac{au}{s^2}\cos(aut), \\
y_2 &= (-1)^2 \frac{1}{[p(t)]^2} L^2\left(\frac{f(ut)}{p(t)}\right) = \frac{a^2u^2}{s^3}\sin(aut), \\
y_3 &= (-1)^3 \frac{1}{[p(t)]^3} L^3\left(\frac{f(ut)}{p(t)}\right) = \frac{a^3u^3}{s^4}\cos(aut), \\
&\vdots
\end{aligned}
$$

Then,

$$\mathbb{N}^{+}[\sin(at)] = \frac{au}{s^2} - \frac{a^3u^3}{s^4} - \frac{a^5u^5}{s^6} - \cdots = \frac{au}{s^2 + (au)^2}.$$

**Example 6.** Let $f(t) = \cosh(at)$. From equation (12) we have

$$\mathbb{N}^{+}[\cosh(at)] = \int_0^\infty e^{-st}\cosh(aut)dt = \left[ e^{-st}\sum_{i=0}^{\infty} y_i \right]_{t=0}^{\infty}.$$

where

$$\begin{aligned}
y_0 &= \frac{f(ut)}{p(t)} = -\frac{1}{s}\cosh(aut), \\
y_1 &= -\frac{1}{p(t)} L\left(\frac{f(ut)}{p(t)}\right) = -\frac{au}{s^2}\sinh(aut), \\
y_2 &= (-1)^2 \frac{1}{[p(t)]^2} L^2\left(\frac{f(ut)}{p(t)}\right) = -\frac{a^2u^2}{s^3}\cosh(aut), \\
y_3 &= (-1)^3 \frac{1}{[p(t)]^3} L^3\left(\frac{f(ut)}{p(t)}\right) = -\frac{a^3u^3}{s^4}\sinh(aut), \\
&\vdots
\end{aligned}$$

Then,

$$\mathbb{N}^{+}[\cosh(at)] = \frac{1}{s} + \frac{a^2u^2}{s^3} + \frac{a^4u^4}{s^5} + \cdots = \frac{s}{s^2 - (au)^2}.$$

## 4. CONCLUSION

The decomposition method is simple for solving the linear first order differential equation. In this work, this method has been applied to compute Natural transform for some functions. It has been observed that, the suggested method is very efficient for Natural transform, which requires simple differentiation.

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