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On Intuitionistic Fuzzy Almost Resolvable and Irresolvable Spaces

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ABSTRACT— ***The aim of this paper is to study several characterizations of intuitionistic fuzzy almost resolvable and irresolvable spaces and the conditions under which an intuitionistic fuzzy almost resolvable space becomes an intuitionistic fuzzy Baire space. The interrelations between intuitionistic fuzzy almost resolvable and other spaces are also discussed.***

Keywords— Intuitionistic fuzzy almost resolvable, intuitionistic fuzzy first category, intuitionistic fuzzy submaximal, intuitionistic fuzzy Baire space.

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1. INTRODUCTION

In order to deal with uncertainties, the idea of fuzzy sets and fuzzy set operations was introduced by Zadeh[15]. The theory of fuzzy topological space was studied and developed by C.L.Chang[7]. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then much attention has been paid to generalise the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. Atanassov and many researchers[1,2,3] worked on intuitionistic fuzzy sets in the literature. The concept of resolvability and irresolvability in topological spaces was studied by E.Hewitt in [10]. A.Geli'kin[9] gave a new approach to open hereditarily irresolvable spaces in classical topology. Almost resolvable spaces was defined by Richard Bolstein [13]. In this paper several characterizations of intuitionistic fuzzy almost resolvable and irresolvable spaces are studied and the conditions under which an intuitionistic fuzzy almost resolvable space becomes an intuitionistic fuzzy Baire space are also discussed.

2. PRELIMINARIES

Definition 2.1[2]. An intuitionistic fuzzy set (IFS, in short) A in X is an object having the form $A=\{x,\mu_A(x),\upsilon_A(x)/x\in X\}$ where the functions $\mu_A:X\to I$ and $\upsilon_A:X\to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\upsilon_A(x)$) of each element $x\in X$ to the set A on a nonempty set X and $0\le\mu_A(x)+\upsilon_A(x)\le 1$ for each $x\in X$. Obviously every fuzzy set A on a nonempty set X is an IFS's A and B be in the form $A=\{x,\mu_A(x),1-\mu_A(x)/x\in X\}$

Definition 2.2[2]. Let X be a nonempty set and the IFS's A and B be in the form $A=\{x,\mu_A(x),\upsilon_A(x)/x\in X\}$, $B=\{x,\mu_B(x),\upsilon_B(x)/x\in X\}$ and let $A=\{A_j : j\in J\}$ be an arbitrary family of IFS's in X. Then we define

(i) $A\subseteq B$ if and only if $\mu_A(x)\le\mu_B(x)$ and $\upsilon_A(x)\ge\upsilon_B(x)$ for all $x\in X$.

(ii) A=B if and only if $A\subseteq B$ and $B\subseteq A$.

(iii) $\overline{A}=\{x,\upsilon_A(x),\mu_A(x)/x\in X\}$.

(iv) $A\cap B=\{x,\mu_A(x)\cap\mu_B(x),\upsilon_A(x)\cup\upsilon_B(x)/x\in X\}$.

(v) $A \cup B = \{x, \mu_A(x) \cup \mu_B(x), \upsilon_A(x) \cap \upsilon_B(x) / x \in X\}$

(vi) $1_{\sim} = \{\langle x,1,0 \rangle x \in X\}$ and $0_{\sim} = \{\langle x,0,1 \rangle x \in X\}$.

Definition 2.3[6]. An intuitionistic fuzzy topology (IFT, in short) on a nonempty set X is a family τ of an intuitionistic fuzzy set (IFS, in short) in X satisfying the following axioms:

(i) $0_{\sim}, 1_{\sim} \in \tau$.

(ii) $A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$.

(iii) $\bigcup A_j \in \tau$ for any $A_j : j \in J \subseteq \tau$.

The complement $\overline{A}$ of intuitionistic fuzzy open set (IFOS, in short) in intuitionistic fuzzy topological space (IFTS, in short) (X, τ) is called an intuitionistic fuzzy closed set (IFCS, in short).

Definition 2.4[6]. Let (X, τ) be an IFTS and $A = \{x, \mu_A(x), \upsilon_A(x)\}$ be an IFS in X. Then the fuzzy interior and closure of A are denoted by

(i) $cl(A) = \bigcap\{$K: K is an IFCS in X and $A \subseteq K\}$.

(ii) $\text{int}(A) = \bigcup\{$G: G is an IFOS in X and $G \subseteq A\}$.

Definition 2.5[6]. Let (X, τ) be any intuitionistic fuzzy topological space (IFTS, in short). Let A be an intuitionistic sets in (X, τ). Then the intuitionistic fuzzy closure operator satisfy the following properties:

(i) $1 - IFcl(A) = IF\,\text{int}(1 - A)$

(ii) $1 - IF\,\text{int}(A) = IFcl(1 - A)$

Definition 2.6[8]. An IFS A in IFTS (X, τ) is called IF dense if there exists no IFCS B in (X, τ) such that $A \subseteq B \subseteq 1_{\sim}$.That is $IFcl(A) = 1_{\sim}$

Definition 2.7.[8]. An IFS A in an IFTS (X, τ) is called IF nowhere dense if there exists no non-zero IFOS B in (X, τ) such that $B \subseteq IFcl(A)$. That is $IF\,\text{int}(IFcl(A)) = 0_{\sim}$.

Let A be an intuitionistic fuzzy set in (X, τ). If A is an IFCS in (X, τ) with $IF\,\text{int}(A) = 0_{\sim}$, then A is an IF nowhere dense set in (X, τ).

Definition 2.8.[8]. An IFTS (X, τ) is called intuitionistic fuzzy resolvable if there exists a IF dense set A in (X, τ) such that $IFcl(1 - A) = 1_{\sim}$. Otherwise (X, τ) is called IF irresolvable.

Definition 2.9.[8]. An IFTS (X, τ) is called an IF submaximal space if $IFcl(A) = 1_{\sim}$ for any non-zero IFS A in (X, τ).

Definition 2.10[8]. An IFTS (X, τ) is called an IF open hereditarily irresolvable if $IF\,\text{int}(IFcl(A)) \neq 0_{\sim}$ for any IFS A in (X, τ).

Definition 2.11.[8]. An IFTS (X, τ) is called intuitionistic fuzzy first category if $\bigcup_{i=1}^{\infty} A_i = 1_{\sim}$, where A_i's are IF nowhere dense sets in (X, τ). An IFTS which is not IF first category is said to be IF second category.

Definition 2.12[8]. An IFTS (X, τ) is called an IF Baire space if $IF\,\text{int}(\bigcup_{i=1}^{\infty} A_i) = 0_{\sim}$, where A_i's are IF nowhere dense sets in (X, τ).

3. INTUITIONISTIC FUZZY ALMOST RESOLVABLE SPACES

Definition 3.1[13]. An IFTS (X,τ) is called an IF almost resolvable space if $\bigcup_{i=1}^{\infty} A_i = 1$, where A_i's are IFSs in (X,τ) are such that $IF\operatorname{int}(A_i) = 0_\sim$. Otherwise (X,τ) is called IF almost irresolvable space.

Proposition 3.2. If $\bigcap_{i=1}^{\infty} A_i = 0_\sim$, A_i's are IF dense sets in (X,τ), then (X,τ) is an IF almost resolvable space.

Proof. Suppose that $\bigcap_{i=1}^{\infty} A_i = 0_\sim$, where $IFcl(A_i) = 1_\sim$ in (X,τ). Then we have $1 - \bigcap_{i=1}^{\infty}(A_i) = 1 - 0 = 1_\sim$, where $1 - IFcl(A_i) = 0_\sim$. This implies that $\bigcup_{i=1}^{\infty}(1 - A_i) = 1_\sim$, where $IF\operatorname{int}(1 - A_i) = 0_\sim$.Let $1 - A_i = B_i$, then we have $\bigcup_{i=1}^{\infty} B_i = 1_\sim$, where $IF\operatorname{int}(B_i) = 0_\sim$ in (X,τ). Hence (X,τ) is an IF almost resolvable space.

Definition 3.3. An IFTS (X,τ) is called an IF hyper connected space if every IFOS is IF dense in (X,τ). That is $IFcl(A_i) = 1_\sim$ for all $A_i \in \tau$.

Proposition 3.3. If $\bigcap_{i=1}^{\infty} A_i = 0_\sim$, where A_i's are IFOS in an IF hyper connected space (X,τ), then (X,τ) is an IF almost resolvable space.

Proof. Suppose that $\bigcap_{i=1}^{\infty} A_i = 0_\sim$, where $A_i \in \tau$. Since (X,τ) is an IF hyper connected space, the IFOS A_i is an IFdense set in (X,τ) for each i. Hence we have $\bigcap_{i=1}^{\infty} A_i = 0_\sim$, where $IFcl(A_i) = 1_\sim$ in (X,τ). Then by Proposition 3.2, (X,τ) is an IF almost resolvable space.

Definition 3.4. An IFS A in an IFTS (X,τ) is called $IF\, G_\delta$ if $A = \bigcap_{i=1}^{\infty} A_i$ where each $A_i \in \tau$.

Definition.3.5. An IFS A in an IFTS (X,τ) is called $IF\, F_\sigma$ if $A = \bigcup_{i=1}^{\infty} A_i$ where each $A_i \in \tau$.

Definition 3.6. An IFTS (X,τ) is called IF *P-space*, if countable intersection of IFOSs in (X,τ) is intuitionistic fuzzy open. That is, every non-zero IF G_δ −set in (X,τ) is intuitionistic fuzzy open in (X,τ).

Proposition 3.7. If $\bigcap_{i=1}^{\infty} A_i = 0_\sim$, A_i's are IF G_δ −sets in an IF hyper connected and *P-space* (X,τ), then (X,τ) is an IF almost resolvable space.

Proof. Let A_i's be IF G_δ −sets in IF*P-space* (X,τ). Then A_i 's are IFOSs in (X,τ). Hence, we have $\bigcap_{i=1}^{\infty} A_i = 0_\sim$, where A_i's are IFOS in an IF hyper connected space (X,τ). Therefore by Proposition 3.3, (X,τ) is an IF almost resolvable space.

Proposition 3.8. If each IFS A_i is an IF F_σ − set in an IF almost resolvable space (X,τ), then $\bigcap_{i=1}^{\infty}(1 - A_i) = 0_\sim$, where $(1 - A_i)$'s are IF dense sets in (X,τ).

Proof. Let (X,τ) be an IF almost resolvable space. Then $\bigcup_{i=1}^{\infty} A_i = 1_\sim$, where A_i's are such that $IF\,\mathrm{int}(A_i) = 0_\sim$. This implies that $1-\bigcup_{i=1}^{\infty} A_i = 0_\sim$ and $1-IF\,\mathrm{int}(A_i) = 1_\sim$. Then $\bigcap_{i=1}^{\infty}(1-A_i) = 0_\sim$ and $IFcl(1-A_i) = 1_\sim$. Since A_i's are IF F_σ – sets, $(1-A_i)$'s are IF G_δ – sets in (X,τ). Hence we have $\bigcap_{i=1}^{\infty}(1-A_i) = 0_\sim$, where $(1-A_i)$'s are IF dense and IF G_δ – sets in (X,τ).

Definition 3.9. An IFTS (X,τ) is *called intuitionistic fuzzy nodec space*, if every non-zero IF nowhere dense set in (X,τ) is intuitionistic fuzzy closed.

Proposition 3.10. If the IFTS (X,τ) is an IF first category, then (X,τ) is an IF almost resolvable space.

Proof. Since (X,τ) is of IF first category , we have $\bigcup_{i=1}^{\infty} A_i = 1_\sim$, where A_i 's are IF nowhere dense sets in (X,τ). Now A_i is an IF nowhere dense set implies that $IF\,\mathrm{int}(A_i) = 0$. Hence $\bigcup_{i=1}^{\infty} A_i = 1_\sim$, where $IF\,\mathrm{int}(A_i) = 0$ and therefore (X,τ) is an IF almost resolvable space.

Proposition 3.11. If (X,τ) is an IF first category space and IF nodec space, then (X,τ) is an IF almost resolvable space.

Proof. Let (X,τ) is an first category space. Then we have $\bigcup_{i=1}^{\infty} A_i = 1_\sim$ where A_i's are IF nowhere dense sets in (X,τ). Since (X,τ) is an IF nodec space, the IF nowhere dense sets are IFCS in (X,τ). Hence A_i's are IFCS in (X,τ). That is, $IFcl(A_i) = A_i$. Now $IF\,\mathrm{int}(IFcl(A_i)) = 0_\sim$ implies that $IF\,\mathrm{int}(A_i) = 0_\sim$. Hence we have $\bigcup_{i=1}^{\infty} A_i = 1_\sim$, where A_i's in (X,τ) are such that $IF\,\mathrm{int}(A_i) = 0_\sim$. Hence (X,τ) is an IF almost resolvable space.

Proposition 3.12. If $IFcl(IF\,\mathrm{int}(A)) = 1_\sim$, for each IF dense set A in an IF almost resolvable space (X,τ), then (X,τ) is an IF first category space.

Proof. Let (X,τ) be an IF almost resolvable space such that $IFcl(IF\,\mathrm{int}(A)) = 1_\sim$, for each IF dense set A in (X,τ). Since (X,τ) is IF almost resolvable, $\bigcup_{i=1}^{\infty} A_i = 1_\sim$, where A_i's in (X,τ) are such that $IF\,\mathrm{int}(A_i) = 0_\sim$. Now $1-IF\,\mathrm{int}(A_i) = 1_\sim$, implies that $IFcl(1-A_i) = 1_\sim$. Then by hypothesis, $IFcl(IF\,\mathrm{int}(1-A_i)) = 1_\sim$ for the IF dense set $1-A_i$ in (X,τ). This implies that $IF\,\mathrm{int}(IFcl(A_i)) = 0_\sim$. Hence A_i's are IF nowhere dense set in (X,τ). Therefore $\bigcup_{i=1}^{\infty} A_i = 1_\sim$, where A_i's are IF nowhere dense set in (X,τ), implies that (X,τ) is an IF first category space.

Proposition 3.13. If $IFcl(IF\,\mathrm{int}(A)) = 1_\sim$ for each IF dense set (X,τ) is an IF almost resolvable space (X,τ), then (X,τ) is not an IF Baire space.

Proof. Let (X,τ) be IF almost resolvable space such that $IFcl(IF\,\mathrm{int}(A)) = 1_\sim$, for each IF dense set A in (X,τ). Then by Proposition 3.12, (X,τ) is an IF first category space. This implies that $IF\,\mathrm{int}(\bigcup_{i=1}^{\infty} A_i) = IF\,\mathrm{int}(1) = 0_\sim$. Hence (X,τ) is not an intuitionistic fuzzy Baire space.

Proposition 3.14. If (X,τ) is an IF second category space, then (X,τ) is an IF almost resolvable space.

Proof. Let (X,τ) be an intuitionistic fuzzy second category space. Then $\bigcup_{i=1}^{\infty} A_i \neq 1_{\sim}$, where A_i 's are IF nowhere dense sets in (X,τ). That is $\bigcup_{i=1}^{\infty} A_i \neq 1_{\sim}$, where $IF\operatorname{int}(IFcl(A_i)) = 0_{\sim}$. Now $IF\operatorname{int}(A_i) \subseteq IF\operatorname{int}(IFcl(A_i))$, implies that $IF\operatorname{int}(A_i) = 0_{\sim}$. Hence $\bigcup_{i=1}^{\infty} A_i \neq 1_{\sim}$, where $IF\operatorname{int}(A_i) = 0_{\sim}$ and therefore (X,τ) is an IF almost resolvable space.

Definition 3.15. An IFTS (X,τ) is called *intuitionistic fuzzy volterra space,* if $IFcl(\bigcap_{i=1}^{N} A_i) = 1_{\sim}$ where A_i's are IF dense and IF G_δ $-$sets in (X,τ).

Definition 3.16. An IFTS (X,τ) is called *intuitionistic fuzzy weakly volterra space,* if $IFcl(\bigcap_{i=1}^{N} A_i) \neq 0_{\sim}$,where A_i's are IF dense and IF G_δ $-$sets in (X,τ).

Proposition 3.17. If an IFTS (X,τ) is not an IF weakly volterra space, then (X,τ) is an IF almost resolvable space.

Proof. Let (X,τ) be an IF non-weakly volterra space. Then, we have $IFcl(\bigcap_{i=1}^{N} A_i) = 0_{\sim}$, where A_i's are IF dense and IF G_δ $-$sets in (X,τ).

Now $IFcl(\bigcap_{i=1}^{N} A_i) = 0_{\sim}$, implies that $IF\operatorname{int}(\bigcup_{i=1}^{N}(1-A_i)) = 1_{\sim}$ and $IFcl(A_i) = 1_{\sim}$, implies that $IF\operatorname{int}(1-A_i) = 0_{\sim}$. Let B_j's be such that $IF\operatorname{int}(B_j) = 0_{\sim}$ and take the first N B_j's as $(1-A_i)'s$. Now $\bigcup_{i=1}^{N}(1-A_i) \subseteq \bigcup_{j=1}^{\infty} B_j$, implies that $IF\operatorname{int}(\bigcup_{i=1}^{N}(1-A_i)) \subseteq IF\operatorname{int}(\bigcup_{i=1}^{\infty} B_j) \subseteq \bigcup_{i=1}^{\infty} B_j$. Then, we have $1_{\sim} \subseteq \bigcup_{i=1}^{\infty} B_j$. That is, $\bigcup_{i=1}^{\infty} B_j = 1_{\sim}$, where B_j's in (X,τ) are such that $IF\operatorname{int}(B_j) = 0_{\sim}$. Hence (X,τ) is an IF almost resolvable space.

4. INTER-RELATIONS BETWEEN IF ALMOST RESOLVABLE SPACES AND IRRESOLVABLE SPACES WITH OTHER SPACES

Proposition 4.1. If the IF almost resolvable space (X,τ) is an intuitionistic fuzzy submaximal space, then (X,τ) is an IF first category space.

Proof. Let (X,τ) be an IF almost resolvable space. Then $\bigcup_{i=1}^{\infty} A_i = 1_{\sim}$, where A_i's in (X,τ) are such that $IF\operatorname{int}(A_i) = 0_{\sim}$. Then we have $\bigcap_{i=1}^{\infty}(1-A_i) = 0_{\sim}$, where $IFcl(1-A_i) = 1_{\sim}$. Since the space (X,τ) is an IF submaximal space, the IF dense sets $(1-A_i)'s$ are IFOS in (X,τ). Then A_i's IFCS in (X,τ) and hence $IFcl(A_i) = A_i$. Now $IF\operatorname{int}(IFcl(A_i)) = IF\operatorname{int}(A_i) = 0_{\sim}$. Then A_i's IF nowhere dense sets in (X,τ). Hence $\bigcup_{i=1}^{\infty} A_i = 1_{\sim}$, where A_i's are IF nowhere dense sets in (X,τ) implies that (X,τ) is an IF first category space.

Remark 4.2. In view of the above Proposition, we have the following result, If the IF almost resolvable space (X,τ) is an IF submaximal space, then (X,τ) is an IF second category space.

Proposition 4.3. If the IF almost irresolvable space (X,τ) is an IF submaximal space, then (X,τ) is an IF second category space.

Proof. Let (X,τ) be an IF almost irresolvable space. Then $\bigcup_{i=1}^{\infty} A_i \neq 1_\sim$, where A_i's in (X,τ) are such that $IF\operatorname{int}(A_i) = 0_\sim$. Now $IF\operatorname{int}(A_i) = 0_\sim$, implies that $IFcl(1-A_i) = 1_\sim$. Since (X,τ) is an IF submaximal space, the IF dense sets $(1-A_i)'s$ are IFOSs in (X,τ). Then A_i's are IFCSs in (X,τ) and hence $IFcl(A_i) = A_i$. Now $IF\operatorname{int}(A_i) = 0_\sim$ implies that $IF\operatorname{int}(IFcl(A_i)) = 0_\sim$. Then A_i's IF nowhere dense sets in (X,τ). Hence $\bigcup_{i=1}^{\infty} A_i \neq 1_\sim$, where A_i's are IF nowhere dense sets in (X,τ). Therefore (X,τ) is an IF second category space.

Proposition 4.4. If the IF almost irresolvable space (X,τ) is an IF submaximal space, then (X,τ) is not an IF Baire space.

Proof. Let the IF almost irresolvable space (X,τ) be an IF submaximal space. Then by Proposition 4.1, (X,τ) is an IF first category space and hence $\bigcup_{i=1}^{\infty} A_i = 1_\sim$, where A_i's are IF nowhere dense sets in (X,τ). Now $IF\operatorname{int}(\bigcup_{i=1}^{\infty} A_i) = IF\operatorname{int}(1) = 1 \neq 0_\sim$. Hence (X,τ) is not an IF Baire space.

Theorem 4.5. Let (X,τ) be IFTS. If (X,τ) is be IF open hereditarily irresolvable then $IF\operatorname{int}(A) = 0_\sim$ for any nonzero IF dense set A in (X,τ) implies that $IF\operatorname{int}(IFcl(A)) = 0_\sim$.

Proof. Let A be an IFS in (X,τ), such that $IF\operatorname{int}(A) = 0_\sim$. We claim that $IF\operatorname{int}(IFcl(A)) = 0_\sim$. Suppose that $IF\operatorname{int}(IFcl(A)) = 0_\sim$. Since (X,τ) is IF open hereditarily irresolvable, we have $IF\operatorname{int}(A) \neq 0_\sim$, which is a contradiction to $IF\operatorname{int}(A) = 0_\sim$. Hence $IF\operatorname{int}(IFcl(A)) = 0_\sim$.

Proposition 4.6. If the IF almost irresolvable space (X,τ) is an IF open hereditarily irresolvable space, then (X,τ) is an IF second category space.

Proof. Let (X,τ) is an IF almost irresolvable space. Then $\bigcup_{i=1}^{\infty} A_i \neq 1_\sim$, where A_i's in (X,τ) are such that $IF\operatorname{int}(A_i) = 0_\sim$. Since (X,τ) is an IF open hereditarily irresolvable space, $IF\operatorname{int}(A_i) = 0_\sim$, implies that $IF\operatorname{int}(IFcl(A_i)) = 0_\sim$. Then A_i's are IF nowhere dense sets in (X,τ). Hence $\bigcup_{i=1}^{\infty} A_i \neq 1_\sim$, where A_i's are IF nowhere dense sets in (X,τ) implies that (X,τ) is an IF second category space.

Proposition 4.7. Let (X,τ) be an IFTS, then the following are equivalent:

(i) (X,τ) is an IF Baire space.

(ii) $IF\operatorname{int}(A) = 0_\sim$ for every IF first category set A in (X,τ).

(iii) $IFcl(B) = 1_\sim$ for every IF residual set B in (X,τ).

Theorem 4.8. If the IFTS (X,τ) is an IF Baire space, then (X,τ) is an IF almost irresolvable space.

Proof. Since (X,τ) is an intuitionistic fuzzy Baire space, $IF\operatorname{int}(\bigcup_{i=1}^{\infty} A_i) = 0_\sim$, where A_i 's are IF nowhere dense sets in (X,τ). Now A_i is an IF nowhere dense set implies that $IF\operatorname{int}(IFcl(A_i)) = 0_\sim$. Since $IF\operatorname{int}(\bigcup_{i=1}^{\infty} A_i) = 0_\sim$, where $IF\operatorname{int}(A_i) = 0_\sim$. Suppose that (X,τ) is an IF almost resolvable space. Then $\bigcup_{i=1}^{\infty} A_i = 1_\sim$, where $IF\operatorname{int}(A_i) = 0_\sim$.

Now $IF\ \text{int}(\bigcup_{i=1}^{\infty} A_i) = IF\ \text{int}(1) = 1_{\sim}$, which implies that 0 =1~, a contradiction. Hence we must have $\bigcup_{i=1}^{\infty} A_i \neq 1_{\sim}$, where $IF\ \text{int}(A_i) = 0_{\sim}$. Therefore (X,τ) is an IF almost resolvable space.

The following proposition establishes under what conditions an IF Baire space becomes an intuitionistic fuzzy almost resolvable space.

Proposition 4.9. If $\bigcap_{i=1}^{\infty} A_i = 0_{\sim}$, where A_i's are IF residual sets in an IFBaire space (X,τ), then (X,τ) is an IF almost resolvable space.

Proof. Let (X,τ) be an IF Baire space in which $\bigcap_{i=1}^{\infty} A_i = 0_{\sim}$, where A_i's are IF residual sets in (X,τ). Since (X,τ) is an IF Baire space and A_i's are IF residual sets in (X,τ). By Proposition 4.6, $IFcl(A_i) = 1_{\sim}$. Then $1 - IFcl(A_i) = 0_{\sim}$ and hence $IF\ \text{int}(1 - A_i) = 0_{\sim}$. Now $\bigcap_{i=1}^{\infty} A_i = 0_{\sim}$, implies that $\bigcup_{i=1}^{\infty}(1 - A_i) = 1_{\sim}$. Let $1 - A_i = B_i$. Hence $\bigcup_{i=1}^{\infty} A_i = 1_{\sim}$, where $IF\ \text{int}(A_i) = 0_{\sim}$. Therefore (X,τ) is an IF almost resolvable space.

Theorem 4.10. If A is an IF dense and IFOS in an IFTS (X,τ), then 1-A is an intuitionistic fuzzy nowhere dense set in (X,τ).

Proof. Since A be an IFdense set, $IFcl(A) = 1_{\sim}$ and let A be an IFOS, then 1-A be IFCS. Hence $IFcl(1 - A) = 1 - A$ and $1 - IFcl(A) = 1 - 1 = 0_{\sim}$, which implies that $IFcl(1 - A) = 1 - A$ and $IF\ \text{int}(1 - A) = 0_{\sim}$. Therefore $IF\ \text{int}(IFcl(1 - A)) = 0_{\sim}$, which implies 1-A is an *IF nowhere dense set* in (X,τ).

Proposition 4.11. If each IF G_δ – set is intuitionistic fuzzy open and IF dense set in an IFTS (X,τ), then (X,τ) is an IF almost irresolvable space.

Proof. Let A be an IF G_δ – set in (X,τ) such that A is an IF dense an intuitionistic fuzzy open in (X,τ). Then $A = \bigcap_{i=1}^{\infty} A_i$, where A_i's in (X,τ) are intuitionistic fuzzy open in (X,τ). Now $1 - A = 1 - (\bigcap_{i=1}^{\infty} A_i)$. Then $IFcl(1 - A) = IFcl(1 - (\bigcap_{i=1}^{\infty} A_i)) = IFcl(\bigcup_{i=1}^{\infty}(1 - A_i))$ and hence $IFcl(1 - A) = IFcl(\bigcup_{i=1}^{\infty}(1 - A_i)) \supseteq \bigcup_{i=1}^{\infty} IFcl(1 - A_i)$. Since A_i's are IFOSs in (X,τ), (1-A_i)'s are IFCS in (X,τ) and hence $IFcl(1 - A_i) = 1 - A_i$. $IFcl(1 - A) \supseteq \bigcup_{i=1}^{\infty}(1 - A_i)$, which implies that

$$IF\ \text{int}\ IFcl(1 - A) \supseteq IF\ \text{int}(\bigcup_{i=1}^{\infty}(1 - A_i)) \ldots\ldots\ldots\ldots\ldots\ldots \text{I}$$

Since $\bigcup_{i=1}^{\infty} IF\ \text{int}(1 - A_i) \subseteq IF\ \text{int}(\bigcup_{i=1}^{\infty}(1 - A_i))$, we have $IF\ \text{int}\ IFcl(1 - A) \supseteq \bigcup_{i=1}^{\infty} IF\ \text{int}(1 - A_i) \ldots\ldots\ldots\ldots\ldots\ldots II$

Since A is an IFdense set in (X,τ). By theorem 3.9, the IFS 1-A is an IF nowhere dense set in (X,τ). Then , we have $IF\ \text{int}\ IFcl(1 - A) = 0_{\sim}$. Hence from II, $0_{\sim} \supseteq \bigcup_{i=1}^{\infty}(IF\ \text{int}(1 - A_i))$. That is $\bigcup_{i=1}^{\infty}(IF\ \text{int}(1 - A_i)) = 0_{\sim}$. Then $IF\ \text{int}(1 - A_i) = 0_{\sim}$, for each $(i = 1\ to\ \infty)$. Hence $IF\ \text{int}(IFcl(1 - A_i)) = 0_{\sim}$. (Since $IFcl(1 - A_i) = 1 - A_i$). This implies that (1-A_i)'s are IF nowhere dense sets in (X,τ). From I, we have

$0_{\sim} \supseteq IF\ \text{int}(\bigcup_{i=1}^{\infty}((1-A_i))$. That is , $IF\ \text{int}(\bigcup_{i=1}^{\infty}((1-A_i)) = 0_{\sim}$. Hence (X,τ) is an IF Baire space. Therefore, by theorem 4.8 is an IF almost irresolvable space.

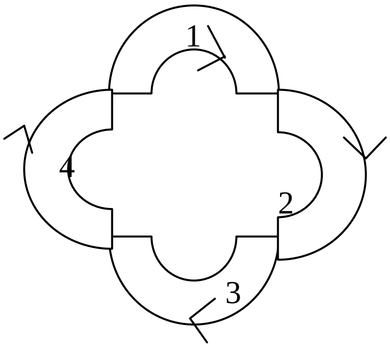
Proposition 4.12. If $\bigcup_{i=1}^{\infty} A_i = 1_{\sim}$, where A_i 's are non-zero intuitionistic fuzzy open sets in an intuitionistic fuzzy topological space (X,τ), then (X,τ) is an intuitionistic fuzzy almost irresolvable space.

Proof. Suppose that $\bigcup_{i=1}^{\infty} A_i = 1_{\sim}$, where the intuitionistic fuzzy sets A_i's are non-zero IFOSs in (X,τ). Since A_i 's are non-zero IFOSs $IF\ \text{int}(A_i) = A_i \neq 0_{\sim}$. Hence we have $\bigcup_{i=1}^{\infty} A_i = 1_{\sim}$, where $IF\ \text{int}(A_i) \neq 0_{\sim}$ for all $(i = 1\ to\ \infty)$.Therefore (X,τ) is an IF almost irresolvable space.

5. LEVELS OF INTUITIONISTIC FUZZY IRRESOLVABILITY

Theorem 5.1. If (X,τ) is an intuitionistic fuzzy irresolvable space if and only if $IF\ \text{int}(A) \neq 0_{\sim}$ for all intuitionistic fuzzy dense sets A in (X,τ).

Proposition 5.2. For any intuitionistic fuzzy topological space (X,τ) we have the following relations:



1. Intuitionistic fuzzy submaximality
2. Intuitionistic fuzzy hereditary irresolvability
3. Intuitionistic fuzzy strong irresolvability
4. Intuitionistic fuzzy irresolvability.

Proof. Let (X,τ) be an intuitionistic fuzzy submaximal space. Then, $IFcl(A) = 1_{\sim}$ implies that $A \in \tau$. Suppose that $IF\ \text{int}(A) = 0_{\sim}$ for any non-zero intuitionistic fuzzy set A in (X,τ).
Then $1 - IF\ \text{int}(A) = 1 - 0 = 1_{\sim}$ implies that $IFcl(1-A) = 1_{\sim}$. Since (X,τ) is intuitionistic fuzzy submaximal, $(1-A) \in \tau$. Then A is an intuitionistic fuzzy closed set in (X,τ). Hence $IF\ \text{int}(A) = IF\ \text{int}(IFcl(A))$. $IF\ \text{int}(A) = 0_{\sim}$ implies that $IF\ \text{int}(IFcl(A)) = 0_{\sim}$. Therefore (X,τ) is an intuitionistic fuzzy open hereditarily irresolvable space. Hence intuitionistic fuzzy submaximality implies intuitionistic fuzzy open hereditarily irresolvable space.

Let A be an intuitionistic fuzzy dense set in (X,τ). Then $IFcl(A) = 1_{\sim}$ implies that $IF\ \text{int}(1-A) = 0_{\sim}$. Since (X,τ) is intuitionistic fuzzy open hereditarily irresolvable, $IF\ \text{int}(1-A) = 0_{\sim}$. Now we claim that $IFcl(1-A) \neq 1_{\sim}$. Suppose $IFcl(1-A) = 1_{\sim}$. Then $IF\ \text{int}\ IFcl(1-A) = IF\ \text{int}(1) = 1_{\sim}$ implies that 0 = 1, a contradiction. Hence we must have $IFcl(1-A) \neq 1$. Therefore $IFcl(A) = 1_{\sim}$ implies that $IFcl(1-A) \neq 1_{\sim}$, which means that (X,τ) is an intuitionistic fuzzy irresolvable space. Then $IFcl(A_i) = 1_{\sim}$ implies that $IFcl(1-A) \neq 1_{\sim}$. Then $IF\ \text{int}(A_i) \neq 0_{\sim}$. Now $IFcl(A_i) = 1_{\sim}$ implies that $IF\ \text{int}(1-A_i) = 0_{\sim}$. We claim that

$\bigcup_{i=1}^{n}(1-A_i)\neq 1_{\sim}$. Suppose that $\bigcup_{i=1}^{n}(1-A_i)=1_{\sim}$. Then $1-\bigcap_{i=1}^{n}(1-A_i)=1_{\sim}$ implies that $\bigcap_{i=1}^{n}(A_i)=0$. Hence there must be atleast two non-zero disjoint fuzzy sets A_i and A in (X,τ). Then $A_i+A\subseteq 1_{\sim}$ which implies that $A_i\subseteq 1-A$. Hence $IFcl(A_i)\subseteq IFcl(1-A)$. Then $1\subseteq(1-A)$. That is $IFcl(1-A_j)=1_{\sim}$. Then $IF\,\mathrm{int}(A_j)=0_{\sim}$, a contradiction to $IF\,\mathrm{int}(A_i)\neq 0_{\sim}$. Hence $\bigcup_{i=1}^{n}(1-A_i)\neq 1_{\sim}$, where $IF\,\mathrm{int}(1-A_i)=0_{\sim}$. Therefore (X,τ) is intuitionistic fuzzy almost irresolvable space. Hence intuitionistic fuzzy irresolvability implies intuitionistic fuzzy almost irresolvability.

6. FUNCTIONS AND INTUITIONISTIC FUZZY ALMOST RESOLVABLE SPACES

Definition 6.1. Let $f:(X,\tau)\to(Y,\kappa)$ be an intuitionistic fuzzy topological space (X,τ) to (Y,κ). If the function is somewhat intuitionistic fuzzy continuous and one-one and if $IF\,\mathrm{int}(A)=0_{\sim}$ for any intuitionistic fuzzy set A in (X,τ), $IF\,\mathrm{int}(f(A))=0_{\sim}$ in (Y,κ).

Definition 6.2.A function $f:(X,\tau)\to(Y,\kappa)$ from an intuitionistic fuzzy topological space (X,τ) to (Y,κ) is called somewhat intuitionistic fuzzy open if $A\in(X,\tau)$ and $A\neq 0_{\sim}$ then there exists a $B\in\kappa$, such that $B\neq 0_{\sim}$ and $B\subseteq f(A)$.

Theorem 6.3. Let (X,τ) and (Y,κ) be any two intuitionistic fuzzy topological spaces. If the function $f:X\to Y$ is somewhat intuitionistic fuzzy open and if $IF\,\mathrm{int}(A)=0_{\sim}$ for any nonzero intuitionistic fuzzy set A in (Y,κ), then $IF\,\mathrm{int}(f^{-1}(A))\neq 0_{\sim}$ in (X,τ). Then there exists an intuitionistic fuzzy open set A in (Y,κ), then $IF\,\mathrm{int}(f^{-1}(A))=0_{\sim}$ in (X,τ).

Proposition 6.4. If $f:(X,\tau)\to(Y,\kappa)$ is a somewhat intuitionistic fuzzy open function from an intuitionistic fuzzy topological space (X,τ) onto an intuitionistic fuzzy almost resolvable space (Y,κ) then (X,τ) is an intuitionistic fuzzy almost resolvable space.

Proof . Since (Y,κ) is an intuitionistic fuzzy almost resolvable space, $\bigcup_{i=1}^{n}A_i=1_{\sim}$, where A_i 's are intuitionistic fuzzy sets in (Y,κ), such that $IF\,\mathrm{int}(A_i)=0_{\sim}$. Then $f^{-1}(\bigcup_{i=1}^{n}A_i)=f^{-1}(1)=1_{\sim}$ implies that $\bigcup_{i=1}^{n}f^{-1}(A_i)=1_{\sim}$. Now since f is somewhat intuitionistic fuzzy open and $IF\,\mathrm{int}(A_i)=0_{\sim}$, by theorem 6.3, we have $IF\,\mathrm{int}(f^{-1}(A_i))=0_{\sim}$. Hence $\bigcup_{i=1}^{n}f^{-1}(A_i)=1_{\sim}$, where $IF\,\mathrm{int}(f^{-1}(A_i))=0_{\sim}$ in (X,τ) is an intuitionistic fuzzy almost resolvable space.

Proposition 6.5. If $f:(X,\tau)\to(Y,\kappa)$ is a somewhat intuitionistic fuzzy continuous function from an intuitionistic fuzzy almost resolvable space (X,τ) onto an intuitionistic fuzzy topological space (Y,κ) then (Y,κ) is an intuitionistic fuzzy almost resolvable space.

Proof. Since (X,τ) is an intuitionistic fuzzy almost resolvable space, $\bigcup_{i=1}^{n}A_i=1_{\sim}$, where the A_i 's are intuitionistic fuzzy sets in (X,τ) such that $IF\,\mathrm{int}(A_i)=0_{\sim}$. Then $f(\bigcup_{i=1}^{n}A_i)=f(1)=1_{\sim}$. Then $f(\bigcup_{i=1}^{n}A_i)\subseteq\bigcup_{i=1}^{n}f(A_i)$ implies that $1\subseteq\bigcup_{i=1}^{n}f(A_i)$. That is $\bigcup_{i=1}^{n}f(A_i)=1_{\sim}$. Now since the function f is somewhat intuitionistic fuzzy continuous and $IF\,\mathrm{int}(A_i)=0_{\sim}$, by theorem 6.4, we have $IF\,\mathrm{int}(f(A_i))=0_{\sim}$. Hence $\bigcup_{i=1}^{n}f(A_i)=1_{\sim}$, where $IF\,\mathrm{int}(f(A_i))=0_{\sim}$ in (Y,κ). Therefore (Y,κ) is an intuitionistic fuzzy almost resolvable space.

REFERENCES

[1] K. Atanassov, "Intuitionistic fuzzy sets", Fuzzy Sets and Systems., vol. 20, no. 1, pp. 87-96, 1986.

[2] K. Atanassov, "Intuitionistic fuzzy sets past, present, and future", CLBME-Bulgarian Academy of Science, Sofia, 2003.

[3] K. Atanassov and Stoeva, "Intuitionistic fuzzy sets", Polish Syrup, On interval and fuzzy mathematics, Poznan, vol. 8, pp. 23-26, 1983.

[4] K. Atanassov and Stoeva, "Intuitionistic L-fuzzy sets, Cybernetics and System research", Elsevier, Amsterdam, pp. 539-540, 1984.

[5] G. Balasubramanian, "Maximal Fuzzy Topologies", Kybernetika, vol.5, no. 31, pp. 459-464, 1995.

[6] D. Coker , "An introduction to intuitionistic fuzzy topological spaces", Fuzzy Sets and Systems, vol. 88, pp. 81-89, 1997.

[7] C.L. Chang, "Fuzzy topological spaces", J. Math. Anal. Appl. vol.24, pp. 182-190, 1984.

[8] R. Dhavaseelan, E.Roja and M.K.Uma, "Intuitionistic fuzzy resolvable and intuitionistic fuzzy irresolvable spaces", Scientia Magna, 2, **7**(2011), 59-67.

[9] A. Gel'kin, "Ultrafilters and undecomposable spaces", Vestnik, Mosk. Univ. Mat, vol. 24, no.5, pp.51, 1969.

[10] E. Hewit, "A Problem in set Theoretic Topology", Duke. Math. Jour, vol.10, pp. 309-333, 1943,

[11] Karl R.Gentry and Hughes B. Hoyle III, "Somewhat Continuous Function", Czech.Math,Journal, vol. 21, no. 96, pp. 5-12, 1971

[12] S. J. Lee and E. P. Lee, "The category of intuitionistic fuzzy topological spaces", Bull. Korean Math. Soc. vol.37, no.1, pp.63-76, 2000.

[13] Richard Bolstein, "Sets of points of discontinuity", Proc. Amer. Math. Society, vol. 38, no. 1, pp. 193-197, 1973

[14] L.A. Zadeh, Fuzzy sets, Information and Control, vol.8, pp. 338-353, 1965.