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Heat Transfer Effects on Pulsaltile Flow in Tubes with Slowly Varying Cross-sections

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ABSTRACT--- ***Analytic study of heat transfer effects on ~~the~~ pulsatile blood flow in tubes with slowly varying cross-sections is presented. The governing non-linear differential equations are solved using the perturbation series solutions of the type developed by [20]. Expressions for temperature and Nusselt number are obtained and presented graphically for stenosis tubes. It is observed that the amplitude of the pulse, height of constriction and Reynolds number of the flow increase the temperature and heat transfer rates. Furthermore, it is seen that the heat generated in stenos arterial system could lead to body internal heat problem.***

Keywords--- heat transfer, pulsatile, blood flow, slowly varying tubes, stenosis

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1. INTRODUCTION

Pulsatile flow in tubes of varying cross-sections has application in engineering and biological systems. It is relevance in the study of supercharging systems of reciprocating engines, surging phenomena in power plants, line loss problem associated with the transport of crude oil in pipelines, physiological flows and the likes.

Man is plagued with a number of cardiovascular diseases. Among which is atherosclerosis of the arteries. It involves lesion or hardening of the arteries due to the deposition of plagues or lipids (a generic name for acids, sterols and their esters) in the intima, the internal walls of the arteries. The sterol causes fibrosis – a thickening and scarring of connective tissues of the arteries. The local disorders in the process called fibrinolysis gives rise to blood platelets adherence, fibrin (insoluble protein) encrustation, calcification and formation of clots (thrombi) in the arterial wall.

The localized atherosclerotic constrictions in the arteries known as stenoses are found predominately in the internal carotid artery which supplies blood to the brain, the coronary artery which supplies blood to the cardiac muscles, and the femoral artery which supplies blood to the lower limbs [1]. These constrictions serve as blockages to the flow. Clinically, the closure of the arteries constitutes a health risk to the patient [2]. The complete closure of the arteries leads to stroke or heart attack. Furthermore, moderate and severe stenoses lead to head losses which can reduce blood supply to the arteries and imposes an extra load on the heart musculature [1]. In an attempt to maintain the peripheral flow, the heart enlarges itself and also increases the amplitude of the pulse or pressure wave in the arteries. Subsequently, this produces abrupt rise or hydraulic pump in the flow variables [3].

As the atherosclerosis plagues progressively encroach on the lumen, the arterial distensibility at the points of I nfection ceases. Similarly, the arterial channel loses its taperedness to other geometrical forms which subsequently changes the flow pattern. Research workers have approximated the evolving new geometries to be locally constricted when the plague is at one point only(see [4, 5]), and peristaltic when plagues affect many points (see [6, 7].

Studies have also shown that a stenos artery produces distinct sound known as 'arterial murmurs' or bruits which can be heard externally. The detection of these sounds provides a non-invasive means of screening patients with carotid artery stenos [1]. On the causal mechanism of the sound, it is theorized that the flow downstream of a constriction is similar to a flow past an obstacle (or a wake flow), and is given that vortex shedding in a wake produces the Aeolian tones [8 - 10].

A number of literatures exist on the pulsating blood flow in the arteries. For example, [11] did experimental work on the pulsating turbulent flow in pipes but with interest in heat transfer. They observed an increase in the heat flux over the steady value when the frequency is greater than a critical frequency and a decrease in flow over lower frequencies. In connection with heat transfer in the flow, much work has been done.[12] demonstrated the explicit dependence of the overall heat transfer on pulsatile frequency. They found that for a constant wall temperature boundary condition, the resulting Nusselt number showed periodic axial change which could enhance heat transfer; [13 - 15] conducted some experiments on the heat transfer rate, and showed that it increases with pulsatitility.

Moreso, [16] solved the Navier–Stokes and the energy equations using a finite difference method and developed an asymptotic series for the dynamic and thermal quantities. Their model shows the existence of an annular effect (Richardson effect) in the entry region for the pulsatile part of the velocity and temperature; [17] solved the Navier–Stokes and energy equations numerically for the laminar pulsating flow in a channel, and concluded that an appreciable heat transfer enhancement occurs in the channel.

Similarly, [18] studied numerically the heat transfer characteristics of a fully developed pulsatile flow in a channel, and found that changes in the Nusselt number are pronounced in the entrance region, but with minor changes for the downstream. They also noticed that oscillation may produce both heat transfer enhancement as well as reduction at different axial location in the channel; [19] investigated experimentally the heat transfer characteristics of a pulsating flow in pipes, and found that there is a critical frequency at which there is an increase in the steady value of heat transfer for fluids of Prandtl number near unity, but for values less than unity the Nusselt number increased as the Prandtl number decreased, whereas for Prandtl number above unity the reverse occurred.

In this paper, we shall examine the effects of variation in the amplitude of the pressure wave, height of constriction of the tube, and Reynolds number on the temperature distribution and rate of heat transfer in convergent and divergent channels, using the asymptotic series expansion developed by [20]. In fact, we shall examine the heat flow structure in the region before and after the peak of the constriction.

This paper is organized in the following format: section 2 is the physics of problem and mathematical formulation; section 3 the method of solution; section 4 the results; section 5 the discussion, and section 6 the conclusion.

2. PHYSICS OF THE PROBLEM AND MATHEMATICAL FORMULATION

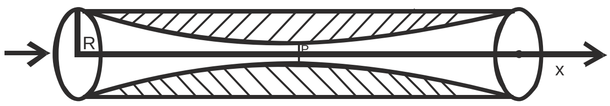

Figure 1 The locally constricted model of the sclerosis artery.

The flow is examined on the basis of the following assumptions: that blood is incompressible and Newtonian, thus allowing the use of Navier-Stokes equations; the viscosity of blood varies with temperature, the arteries are axi-symmetrical cylinders with varying cross-sections**.** Furthermore, due to the lose of distensibilty, the arteries are assumed to be rigid.

The molecular motion of the viscous incompressible Newtonian fluid in an infinite axi–symmetrical cylindrical tubes of varying cross-sections is considered. At the entrance of the tube, we assume the flow to be fully developed and pulsating with a prescribed periodic frequency β and time average volume flux Q. Let (R,θ,X) and (u,v,w) represent respectively, in polar cylindrical orthogonal coordinates system, the components of displacement and velocity of the fluid. Then assuming the flow to be symmetrical about the θ - axis, the problem reduces to two-dimension with (X, 0, R) and $(u,0,w)$ as the coordinates and velocity components. The continuity, momentum and energy equations governing are then given as follows:

$$\frac{1}{\mathrm{R}}\frac{\partial}{\partial \mathrm{R}}(\mathrm{Ru})+\frac{\partial u}{\partial X} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial R}+w\frac{\partial u}{\partial X}=-\frac{1}{\rho}\frac{\partial p}{\partial R}+\upsilon\left(\frac{\partial^2 u}{\partial R^2}+\frac{1}{R}\frac{\partial u}{\partial R}+\frac{u}{R^2}+\frac{\partial^2 u}{\partial X^2}\right) \tag{2}$$

$$\frac{\partial w}{\partial \mathrm{t}}+\mathrm{u}\frac{\partial w}{\partial R}+w\frac{\partial w}{\partial X}=-\frac{1}{\rho}\frac{\partial \mathrm{p}}{\partial X}+\upsilon\left(\frac{\partial^2 w}{\partial \mathrm{R}^2}+\frac{1}{\mathrm{R}}\frac{\partial w}{\partial \mathrm{R}}+\frac{\partial^2 w}{\partial \mathrm{X}^2}\right) + \rho g\beta_1(\bar{T}-T_\infty) \tag{3}$$

$$\frac{\partial \overline{T}}{\partial t}+u\frac{\partial \overline{T}}{\partial R}+\frac{w\partial \overline{T}}{\partial X}=\frac{k_o}{\rho C_p}\left(\frac{\partial^2 \overline{T}}{\partial R^2}+\frac{1}{R}\frac{\partial \overline{T}}{\partial R}+\frac{\partial^2 \overline{T}}{\partial X^2}\right)+\frac{\mu}{\rho C_p}\left\{2\left[\left(\frac{\partial u}{\partial R}\right)^2+\left(\frac{u}{R}\right)^2+\left(\frac{\partial w}{\partial X}\right)^2\right]+\left(\frac{\partial w}{\partial X}+\frac{\partial u}{\partial R}\right)^2\right\} \quad (4)$$

where ρ is the density, p the pressure, υ the kinematic viscosity, $\overline{T}$ the temperature of the fluid, k_o the thermal conductivity of the arterial wall, β_1 the volumetric expansion of fluid due to temperature differential, C_p specific heat capacity at constant pressure of the fluid, and t is the time. Let R=0 be the centre of the symmetric axis of the tube; and $R=a_o(X,t)$, an arbitrary function of X and t ~~is~~ be the instantaneous cross-sectional radius of the tube; a_o is the characteristic radius of the tube, and t is the time, We assume that β_1 is very small such that the free convection force is insignificant. Then we have

$$\frac{1}{R}\frac{\partial}{\partial R}(Ru)+\frac{\partial w}{\partial X} = 0 \quad (5)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial R}+w\frac{\partial u}{\partial X}=-\frac{1}{\rho}\frac{\partial p}{\partial R}+\upsilon\left(\frac{\partial^2 u}{\partial R^2}+\frac{1}{R}\frac{\partial u}{\partial R}+\frac{u}{R^2}+\frac{\partial^2 u}{\partial X^2}\right) \quad (6)$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial R}+w\frac{\partial w}{\partial X}=-\frac{1}{\rho}\frac{\partial p}{\partial X}+\upsilon\left(\frac{\partial^2 w}{\partial R^2}+\frac{1}{R}\frac{\partial w}{\partial R}+\frac{\partial^2 w}{\partial X^2}\right) \quad (7)$$

$$\frac{\partial \overline{T}}{\partial t}+u\frac{\partial \overline{T}}{\partial R}+\frac{w\partial \overline{T}}{\partial X}=\frac{k_o}{\rho C_p}\left(\frac{\partial^2 \overline{T}}{\partial R^2}+\frac{1}{R}\frac{\partial \overline{T}}{\partial R}+\frac{\partial^2 \overline{T}}{\partial X^2}\right)+\frac{\mu}{\rho C_p}\left\{2\left[\left(\frac{\partial u}{\partial R}\right)^2+\left(\frac{u}{R}\right)^2+\left(\frac{\partial w}{\partial X}\right)^2\right]+\left(\frac{\partial w}{\partial X}+\frac{\partial u}{\partial R}\right)^2\right\} \quad (8)$$

The boundary conditions are:

(i) $w = 0,\ \frac{\partial u}{\partial R}=0,\ \overline{T}=0$ at R=0 (9)

(ii) that there is no tangential motion at the wall of the tube ie:

$$u(a_o,t)=0,\ T=T_w \quad \text{at} \quad R=a_o(X,t) \quad (10)$$

ii) the flux across a cross–section of the tube is prescribed as:

$$\int_0^{a_o(X,t)} dR \int_0^{2\pi} Rud\theta = 2\pi\psi_o\,(1+k\,e^{i\beta t}) \quad (11)$$

where ψ_o is a constant, k is the amplitude of the pulse which is assumed to be small, and β is the frequency of oscillation.

For simplicity, we eliminate the pressure terms from equations (6) and (7) by taking the derivatives of equations (6) and (7) with respect to R and X respectively, then subtracting the first result from the second one, we have

$$\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial R}-\frac{\partial w}{\partial X}\right)+\frac{\partial u}{\partial X}\left(\frac{\partial u}{\partial R}-\frac{\partial w}{\partial X}\right)+\frac{\partial w}{\partial R}\left(\frac{\partial u}{\partial R}-\frac{\partial w}{\partial X}\right)=$$

$$\upsilon\left[\frac{\partial^2}{\partial X^2}\left(\frac{\partial u}{\partial R}-\frac{\partial w}{\partial X}\right)+\frac{\partial^2}{\partial R^2}\left(\frac{\partial u}{\partial R}-\frac{\partial w}{\partial X}\right)+\frac{1}{R}\frac{\partial}{\partial R}\left(\frac{\partial u}{\partial R}-\frac{\partial w}{\partial X}\right)-\frac{1}{R^2}\left(\frac{\partial u}{\partial R}-\frac{\partial w}{\partial X}\right)\right] \quad (12)$$

Introducing the stream function ψ, vorticity Ω, respectively i.e.

$$u=-\frac{1}{R}\frac{\partial \psi}{\partial R},\ w=\frac{1}{R}\frac{\partial \psi}{\partial X} \quad (13)$$

$$\Omega = \frac{\partial u}{\partial R} - \frac{\partial w}{\partial X} = \frac{1}{R}\frac{\partial^2 \psi}{\partial R^2} - \frac{1}{R^2}\frac{\partial \psi}{\partial R} + \frac{1}{R}\frac{\partial^2 \psi}{\partial X^2} \tag{14}$$

into equations (12) and (8), we have

$$\frac{\partial \Omega}{\partial t} - \frac{1}{R}\frac{\partial \Omega}{\partial X}\frac{\partial \psi}{\partial R} + \frac{1}{R}\frac{\partial \Omega}{\partial R}\frac{\partial \psi}{\partial X} + \frac{\Omega}{R^2}\frac{\partial \psi}{\partial X} = \upsilon\left[\frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial R^2} + \frac{1}{R}\frac{\partial \Omega}{\partial R} - \frac{\Omega}{R^2}\right] \tag{15}$$

$$\frac{\partial \bar{T}}{\partial t} + \frac{1}{R}\frac{\partial \psi}{\partial X}\frac{\partial \bar{T}}{\partial R} - \frac{1}{R}\frac{\partial \psi}{\partial R}\frac{\partial \bar{T}}{\partial X} = \frac{\upsilon}{\Pr}\left[\frac{\partial^2 \bar{T}}{\partial R^2} + \frac{1}{R}\frac{\partial \bar{T}}{\partial R} + \frac{\partial^2 \bar{T}}{\partial X^2}\right] +$$
$$\frac{4\mu}{C_p R}\left[\left(\frac{\partial^2 \psi}{\partial X \partial R}\right)^2 + \frac{1}{R^2}\left(\frac{\partial \psi}{\partial X}\right)^2 - \frac{1}{R}\frac{\partial \psi}{\partial X}\frac{\partial^2 \psi}{\partial X \partial R}\right] + \frac{\mu}{C_p R^2}\left[\left(\frac{\partial^2 \psi}{\partial X^2}\right)^2 + \frac{2}{R}\frac{\partial \psi}{\partial R}\frac{\partial^2 \psi}{\partial X^2} - 2\frac{\partial^2 \psi}{\partial X^2}\frac{\partial^2 \psi}{\partial R^2}\right] +$$
$$\frac{\mu}{C_p R^2}\left[\frac{\partial^2 \psi}{\partial R^2} + \frac{1}{R^2}\frac{\partial \psi}{\partial R} - 2\frac{\partial \psi}{\partial R}\frac{\partial^2 \psi}{\partial R^2}\right] \tag{16}$$

with the boundary conditions as

$$\psi = 0,\ \frac{1}{R}\frac{\partial \psi}{\partial X} = 0,\ \frac{1}{R}\frac{\partial^2 \psi}{\partial R^2} - \frac{1}{R^2}\frac{\partial \psi}{\partial R} = 0,\ \bar{T} = 0 \quad \text{at} \quad R = 0 \tag{17}$$

$$\frac{\partial \psi}{\partial R} = 0,\ \psi = \psi_o\left(1 + ke^{i\beta t}\right),\ \bar{T} = T_w \ \text{at} \ R = a\left(X, t\right) \tag{18}$$

Furthermore, we assume that the cross–section of the tube in the model vary in the axial direction, and for which we take a(X, t) = $a_o s$ (ε X/a_o , t), $r = R_0(1 + \varepsilon f(x,t))$, $0 \le r \le 1$, $0 \le x \le 1$ where s is an arbitrary function of X; a is the variation in the r along the axial direction; $0 < \varepsilon = \frac{a_o}{L} << 1$ is a small dimensionless parameter that characterizes the slow variation in the channel radius; L defines the channel characteristic length. $\varepsilon = 0$ corresponds to a tube with constant radius. As ε increases from zero the variation of ψ in the axial direction depends upon ε X instead of X.

Similarly, introducing the following non-dimensionalized variables

$$r = \frac{R}{a_o},\ x = \varepsilon\frac{X}{a_o},\ \mathrm{T} = \beta t,\ \phi\ (r, x, T) = \frac{\Psi}{\Psi_o},\ \omega(r, x, T) = \frac{\Omega a_o^3}{\Psi_o},\ \mathrm{Re} = \frac{\Psi_o}{a_o \upsilon}$$
$$\Pr = \frac{k_o}{\rho C_p},\quad \eta = \frac{\beta}{a_o^2 \upsilon},\ a_o = \frac{a}{s},\quad \Theta = \frac{\bar{T} - T_\infty}{T_w - T_\infty},\ H = \frac{\psi_o^2}{k_o(T_w - T_\infty)a_o^4}$$

where Re is the Reynolds number of the flow, Pr is the Prandtl number, η is a dimensionless number for the frequency of oscillation, T is the dimensionless time, ε is the height of constriction, ϕ, ω and Θ are the dimensionless stream function, vorticity and temperature, respectively; into equations (14) - (18) respectively, we have

$$\omega = \frac{1}{r}\frac{\partial^2 \phi}{\partial r^2} - \frac{1}{r^2}\frac{\partial \phi}{\partial r} \tag{19}$$

$$\eta\frac{\partial \omega}{\partial T} + \frac{\mathrm{Re}}{r}\varepsilon\left(\frac{\partial \omega}{\partial r}\frac{\partial \phi}{\partial x} + \frac{\partial \omega}{\partial x}\frac{\partial \phi}{\partial r} + \frac{\omega}{r}\frac{\partial \phi}{\partial x}\right) = \frac{1}{r^2}\frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r}\frac{\partial \omega}{\partial r} - \frac{1}{r}\omega \tag{20}$$

$$\eta\frac{\partial \Theta}{\partial T} + \frac{\mathrm{Re}\Pr}{r}\varepsilon\left(\frac{\partial \phi}{\partial x}\frac{\partial \Theta}{\partial r} - \frac{\partial \phi}{\partial r}\frac{\partial \Theta}{\partial x}\right) = \frac{\partial^2 \Theta}{\partial r^2} + \frac{1}{r}\frac{\partial \Theta}{\partial r} + \frac{H}{r^2}\left[\left(\frac{\partial \phi}{\partial r^2}\right)^2 + \frac{1}{r^2}\left(\frac{\partial \phi}{\partial r}\right)^2 - \frac{2}{r}\frac{\partial \phi}{\partial r}\frac{\partial^2 \phi}{\partial r^2}\right] \tag{21}$$

where the $O(\varepsilon)^2$ terms are neglected for being small.
and the boundary conditions:

$$\phi = 0 \;,\; \frac{\partial \phi}{\partial x} = 0 \;,\; \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \phi}{\partial r}\right) = 0,\; \Theta = 0 \qquad \text{at } r = 0 \tag{22}$$

$$\phi = 1 + k\,e^{iT},\; \frac{\partial \phi}{\partial r} = 0,\; \Theta = 1 \qquad \text{at } r = s(x, T) \tag{23}$$

3. METHOD OF SOLUTION

A close look at equations (20) and (21) shows that they are nonlinear and coupled. To linearize them, we seek for perturbation series solutions about a small parameter ε. In particular, we shall use the form developed by Rao and Devanathan (1973), and which is of the form:

$$f = f^{(o)} + ke^{iT}\bar{f}^{(o)} + \varepsilon\left(f^{(1)} + ke^{iT}\bar{f}^{(1)}\right) + \ldots \tag{24}$$

where $f^{(n)}$ represents ω, ϕ and Θ.
Substituting equation (24) into equations (19) – (23), we have:
For zeroth order

$$\omega^{(o)} = \frac{1}{r}\left(\frac{\partial^2 \phi^{(o)}}{\partial r^2} - \frac{1}{r}\frac{\partial \phi^{(o)}}{\partial r}\right) \tag{25}$$

$$\varpi^{(o)} = \frac{1}{r}\left(\frac{\partial^2 \bar{\phi}^{(o)}}{\partial r^2} - \frac{1}{r}\frac{\partial \bar{\phi}^{(o)}}{\partial r}\right) \tag{26}$$

$$\frac{\partial^2 \omega^{(o)}}{\partial r^2} + \frac{1}{r}\frac{\partial \omega^{(o)}}{\partial r} - \frac{1}{r^2}\,\omega^{(o)} = 0 \tag{27}$$

$$\frac{\partial^2 \varpi^{(o)}}{\partial r^2} + \frac{1}{r}\frac{\partial \varpi^{(o)}}{\partial r} - \left(\lambda^2 + \frac{1}{r}\right)\varpi^{(o)} = 0 \tag{28}$$

$$\frac{\partial^2 \Theta^{(o)}}{\partial r^2} + \frac{1}{r}\frac{\partial \Theta}{\partial r} = \frac{-H}{r^2}\left[\left(\frac{\partial^2 \phi^{(o)}}{\partial r^2}\right)^2 + \frac{1}{r^2}\left(\frac{\partial \phi^{(o)}}{\partial r}\right) - \frac{2}{r}\frac{\partial \phi^{(o)}}{\partial r}\frac{\partial^2 \phi^{(o)}}{\partial r^2}\right] \tag{29}$$

$$\frac{\partial^2 \bar{\Theta}^{(o)}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{\Theta}^{(o)}}{\partial r} - \eta_1 \bar{\Theta}^{(o)} = \frac{-H}{r^2}\left[\frac{\partial^2 \phi^{(o)}}{\partial r^2}\frac{\partial^2 \bar{\phi}^{(o)}}{\partial r^2} + \frac{1}{r^2}\frac{\partial \phi^{(o)}}{\partial r}\frac{\partial \bar{\phi}^{(o)}}{\partial r} - \frac{1}{r}\left(\frac{\partial \phi^{(o)}}{\partial r}\frac{\partial^2 \bar{\phi}^{(o)}}{\partial r^2} + \frac{\partial \phi^{(o)}}{\partial r}\frac{\partial^2 \bar{\phi}^{(o)}}{\partial r^2}\right)\right] \tag{30}$$

where $\lambda^2 = i\eta$
and the boundary conditions are:

$$\left.\begin{aligned} &\phi^{(o)} = \bar{\phi}^{(o)} = 0 \\ &\Theta^{(o)} = \bar{\Theta}^{(o)} = 0 \\ &\frac{1}{r}\frac{\partial \phi^{(o)}}{\partial x} = \frac{1}{r}\frac{\partial \bar{\phi}^{(o)}}{\partial x} = 0 \\ &\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \phi^{(o)}}{\partial r}\right) = \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \bar{\phi}^{(o)}}{\partial r}\right) = 0 \end{aligned}\right\} \quad \text{at } r = 0 \tag{31}$$

$$\left.\begin{aligned}&\frac{\partial\phi^{(o)}}{\partial r}=\frac{\partial\bar{\phi}^{(o)}}{\partial r}=0\\&\phi^{(o)}=\bar{\phi}^{(o)}=1\\&\Theta^{(o)}=1,\bar{\Theta}^{(o)}=1\end{aligned}\right\}\quad \text{at}\quad r=s \tag{32}$$

and for the first terms

$$\omega^{(1)}=\frac{1}{r}\left(\frac{\partial^2\phi^{(1)}}{\partial r^2}-\frac{1}{r}\frac{\partial\phi^{(1)}}{\partial r}\right)$$

(33)

$$\varpi^{(1)}=\frac{1}{r}\left(\frac{\partial^2\bar{\phi}^{(1)}}{\partial r^2}-\frac{1}{r}\frac{\partial\bar{\phi}^{(1)}}{\partial r}\right)$$

(34)

$$\frac{\partial^2\omega^{(1)}}{\partial r^2}+\frac{1}{r}\frac{\partial\omega^{(1)}}{\partial r}-\frac{1}{r}\omega^{(1)}=\frac{\mathrm{Re}}{r}\left[\frac{\partial\phi^{(o)}}{\partial r}\frac{\partial\omega^{(o)}}{\partial x}+\frac{\partial\phi^{(o)}}{\partial x}\left(\frac{\omega^{(o)}}{r}-\frac{\partial\omega^{(o)}}{\partial r}\right)\right] \tag{35}$$

$$\frac{\partial^2\varpi^{(1)}}{\partial r^2}+\frac{1}{r}\frac{\partial\varpi^{(1)}}{\partial r}-\left(\lambda^2+\frac{1}{r^2}\right)\varpi^{(1)}=\frac{\mathrm{Re}}{r}\left[\frac{\partial\phi^{(o)}}{\partial r}\frac{\partial\omega^{(o)}}{\partial x}+\frac{\partial\bar{\phi}^{(o)}}{\partial r}\frac{\partial\omega^{(o)}}{\partial x}\right.$$

$$\left.-\frac{\partial\varpi^{(o)}}{\partial x}\left(\frac{\partial\varpi^{(o)}}{\partial r}-\frac{\varpi^{(o)}}{r}\right)-\frac{\partial\bar{\phi}^{(o)}}{\partial x}\left(\frac{\partial\omega^{(o)}}{\partial r}-\frac{\omega^{(o)}}{r}\right)\right] \tag{36}$$

$$\frac{\partial^2\Theta^{(1)}}{\partial r^2}+\frac{1}{r}\frac{\partial\Theta^{(1)}}{\partial r}=\frac{\mathrm{Re\,Pr}}{r}\left[\frac{\partial\phi^{(o)}}{\partial x}\frac{\partial^2\Theta^{(o)}}{\partial r^2}-\frac{\partial\phi^{(o)}}{\partial r}\frac{\partial\Theta^{(o)}}{\partial x}\right]$$

$$\frac{-2H}{r^2}\left[\frac{\partial^2\phi^{(o)}}{\partial r^2}\frac{\partial^2\phi^{(1)}}{\partial r^2}+\frac{1}{r^2}\frac{\partial\phi^{(o)}}{\partial r}\frac{\partial\phi^{(1)}}{\partial r}-\frac{1}{r}\left(\frac{\partial\phi^{(o)}}{\partial r}\frac{\partial^2\phi^{(1)}}{\partial r^2}+\frac{\partial^2\phi^{(o)}}{\partial r^2}\frac{\partial\phi^{(1)}}{\partial r}\right)\right] \tag{36}$$

$$\frac{\partial^2\bar{\Theta}^{(1)}}{\partial r^2}+\frac{1}{r}\frac{\partial\bar{\Theta}^{(1)}}{\partial r}-\eta_1^{\,2}\bar{\Theta}^{(1)}=\frac{\mathrm{Re\,Pr}}{r}\left[\frac{\partial\phi^{(o)}}{\partial r}\frac{\partial\bar{\Theta}^{(o)}}{\partial r}+\frac{\partial\bar{\Theta}^{(o)}}{\partial x}\frac{\partial\Theta^{(o)}}{\partial r}-\frac{\partial\phi^{(o)}}{\partial r}\frac{\partial\bar{\Theta}^{(o)}}{\partial x}-\frac{\partial\bar{\phi}^{(o)}}{\partial r}\frac{\partial\Theta^{(o)}}{\partial x}\right]$$

$$\frac{-2H}{r^2}\left[\frac{\partial^2\phi^{(o)}}{\partial r^2}\frac{\partial^2\bar{\phi}^{(1)}}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2\bar{\phi}^{(o)}}{\partial r^2}\frac{\partial^2\phi^{(1)}}{\partial r^2}-\frac{1}{r^2}\left(\frac{\partial\phi^{(1)}}{\partial r}\frac{\partial\phi^{(o)}}{\partial r}+\frac{\partial\phi^{(o)}}{\partial r}\frac{\partial\bar{\phi}^{(1)}}{\partial r}\right)\text{x}\right.$$

$$\left.-\frac{1}{r}\left(\frac{\partial\phi^{(o)}}{\partial r}\frac{\partial^2\bar{\phi}^{(1)}}{\partial r^2}+\frac{1}{r^2}\frac{\partial\bar{\phi}^{(o)}}{\partial r^2}\frac{\partial^2\phi^{(1)}}{\partial r^2}\right)+\frac{1}{r}\left(\frac{\partial\phi^{(1)}}{\partial r}\frac{\partial^2\bar{\phi}^{(o)}}{\partial r^2}+\frac{\partial\bar{\phi}^{(1)}}{\partial r}\frac{\partial^2\phi^{(o)}}{\partial r^2}\right)\right] \tag{37}$$

with boundary conditions

$$\left.\begin{aligned}
&\phi^{(1)} = \bar{\phi}^{(1)} = 0\\
&\Theta^{(1)} = \bar{\Theta}^{(1)} = 0\\
&\frac{1}{r}\frac{\partial\phi^{(1)}}{\partial x} = \frac{1}{r}\frac{\partial\bar{\phi}^{(1)}}{\partial x} = 0\\
&\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi^{(1)}}{\partial r}\right) = \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\bar{\phi}^{(1)}}{\partial r}\right) = 0
\end{aligned}\right\} \text{ at } r = 0 \tag{38}$$

$$\left.\begin{aligned}
&\frac{\partial\phi^{(1)}}{\partial r} = \frac{\partial\bar{\phi}^{(1)}}{\partial r} = 0\\
&\phi^{(1)} = \bar{\phi}^{(1)} = 0\\
&\Theta^{(1)} = \bar{\Theta}^{(1)} = 0
\end{aligned}\right\} \text{ at } r = s \tag{39}$$

The analyses of the solutions show that the non-steady part is very insignificant.

After exhaustive algebraic operations, the solution of the zeroth order equations (25) - (31) are:

$$\phi^{(o)} = 2\left(\frac{r}{s}\right)^2 - \left(\frac{r}{s}\right)^4 \tag{40}$$

$$\bar{\phi}^{(o)} = \frac{r^2}{\lambda s^2 I_2(\lambda s)}\left[I_o(\lambda s) - I_o(\lambda r) + I_2(\lambda r)\right] \tag{41}$$

$$\omega^o = -\frac{8}{s^3}\left(\frac{r}{s}\right) \tag{42}$$

$$\bar{\omega}^{(O)} = -2\frac{\lambda I_1(\lambda r)}{s^2 I_2(\lambda s)} \tag{43}$$

$$\Theta^{(o)} = 1 + \frac{4H}{s^4}\left[1 + \left(\frac{r}{4}\right)^4\right] \tag{44}$$

$$\bar{\Theta}^{(o)} = \frac{-16H}{s^6 I_2(\lambda s)}\left[\left(\frac{r^2}{5} - \frac{s^2}{5} + \frac{\lambda s}{15}\frac{I_1(\lambda s)}{I_o(\lambda s)}\right)I_o(\lambda r) + \frac{\lambda r I_1(\lambda s)}{15}\right] \tag{45}$$

while those of the first order equations (32) - (39) are:

$$\phi^{(1)} = -\frac{\mathrm{Re}}{s}\frac{\partial s\varepsilon}{\partial x}\left[\frac{1}{9}\left(\frac{r}{s}\right)^8 - \frac{2}{3}\left(\frac{r}{s}\right)^6 + \left(\frac{r}{s}\right)^4 - \frac{4}{9}\left(\frac{r}{s}\right)^2\right] \tag{46}$$

$$\bar{\phi}^{(1)} = \frac{4\,\mathrm{Re}}{\lambda^4 s^4}\frac{\partial s}{\partial x}\left\{\frac{I_1(\lambda r)}{s\lambda^2 I_2(\lambda s)}\left[\left(\frac{2}{3}s^4\lambda^4 - 5s^2\lambda^2 - 32\right)\lambda I_1^2(\lambda s)\right.\right.$$

$$\left. + \left(\frac{1}{2}s^2\lambda^2 + 16\right)\lambda^2 s I_o(\lambda s) I_1(\lambda s) + \left(\frac{2}{3}\lambda^4 s^4 + 12\lambda^2 s^2 - 128\right)\frac{I_1(\lambda s)}{s}\right.$$

$$+\left(64-6\lambda^2 s^2-\frac{1}{4}\lambda^4 s^4\right)\lambda I_o(\lambda s)\Bigg]\Bigg\} \quad (47)$$

$$\omega^{(1)}=\frac{8\,\mathrm{Re}}{s^4}\frac{\partial s}{\partial x}\left[\frac{2}{3}\left(\frac{r}{s}\right)^5-2\left(\frac{r}{s}\right)^3+\left(\frac{r}{s}\right)\right] \quad (48)$$

$$\bar{\omega}^{(1)}=4\frac{\mathrm{Re}}{\lambda^4 s^4}\frac{\partial s}{\partial x}\left\{\frac{1}{\lambda^2 I^2(\lambda s)}\left[\left(\frac{2s^4\lambda^4}{3}-5s^2\lambda^2-32\right)\lambda I_1^2(\lambda s)\right]\right.$$

$$\left.+\left[I_o(\lambda r)\left(\lambda s^2 r-\frac{\lambda r^2}{3}\right)I_1(\lambda s)-\frac{r^3}{s}+2sr\right]\right\} \quad (49)$$

$$\Theta^{(1)}=\frac{\mathrm{Re}\,H}{9s^5}\frac{\partial s}{\partial x}(20-101\,\mathrm{Pr})+\frac{64\,\mathrm{Re}\,\mathrm{Pr}\,H}{s^9}\frac{\partial s}{\partial x}\left(\frac{r^2}{4}-\frac{r^4}{16s^2}-\frac{r^6}{36s^4}-\frac{r^8}{64s^6}\right)$$

$$+128\frac{\mathrm{Re}\,H}{s^9}\frac{\partial s}{\partial x}\left(\frac{r^4}{16}-\frac{r^6}{18s^2}-\frac{r^8}{96s^4}\right) \quad (50)$$

$$\bar{\Theta}^{(1)}=\frac{32\,\mathrm{Re}}{s^{11}I_2^2(\lambda s)}\frac{\partial s}{\partial x}\left[\left(\frac{2}{3}s^4\lambda^4-5s^2\lambda^2-32\right)I_1^2(\lambda s)+\left(\frac{1}{2}s^2\lambda^2+16\right)\lambda s I_o(\lambda s)\right.$$

$$\left.+\left(\frac{3}{2}s^4\lambda^4-12s^2\lambda^2-128\right)\frac{I_1(\lambda s)}{\lambda s}+\left(64-6s^2\lambda^2-\frac{1}{4}s^4\lambda^4\right)I_o(\lambda s)\right]\mathrm{x}$$

$$\left[\frac{r^2}{5}I_o(\lambda r)-\frac{\lambda r}{15}I_1(\lambda r)-\frac{s^2}{5}I_o(\lambda s)+\frac{\lambda s}{15}I_1(\lambda s)\right] \quad (51)$$

where I_o (λr), $I_1(\lambda r)$, I_2 (λr), $I_o(\lambda s)$, $I_1(\lambda s)$ and I_2 (λs) are the modified Bessel function of order zero, one, two, respectively.

RATE OF HEAT TRANSFER (NUSSELT NUMBER, Nu)

The rate of heat transfer to the wall of the system given in the non-dimensionalized form is expressed as:

$$Nu=-\left.\frac{\partial\Theta}{\partial r}\right|_{r=s}$$

$$Nu=\frac{-16H}{s^5}\frac{-16Hke^{iT}}{s^6 I_2(\lambda s)}\left(\frac{2}{3}sI_o(\lambda s)-\frac{\lambda^2 s I_1^2(\lambda s)}{15 I_o(\lambda s)}-\frac{\lambda I_1(\lambda s)}{15}-\frac{\lambda^2 s I_o(\lambda s)}{15}\right)+\frac{40}{3}\frac{\mathrm{Re}\,\mathrm{Pr}}{s^6}\frac{\partial s}{\partial x}\varepsilon$$

$$+\frac{32\,\mathrm{Re}\,\mathrm{Pr}}{s^{11}I_2^2(\lambda s)}\frac{\partial s}{\partial x}\varepsilon k e^{iT}\left[\left(\frac{2}{3}s^4\lambda^4-5s^2\lambda^2-32\right)I_1^2(\lambda s)+\left(\frac{1}{2}s^2\lambda^2+16\right)\lambda s I_o(\lambda s)I_1(\lambda s)\right.$$

$$\left.+\left(\frac{3}{2}s^4\lambda^4-12s^2\lambda^2-128\right)\frac{I_1(\lambda s)}{\lambda s}+\left(64-6s^2\lambda^2-\frac{1}{4}s^4\lambda^4\right)I_o(\lambda s)I_1(\lambda s)\right]\mathrm{x}$$

$$\left[\frac{2sI_1(\lambda s)}{5}-\frac{\lambda^2 s I_o(\lambda s)}{15}-\frac{\lambda s^2 I_1(\lambda s)}{5}-\frac{\lambda I_1(\lambda s)}{15}\right] \quad (52)$$

The arbitrary flux in the axial direction H(x) is given as $H(x) = h_1+(1- h_1)e^{-h_2 x}$, where h_1 is a non-dimensionalized asymptotic flux, h_2 is the wall absorption parameter. h_1 =1 gives the H(x) for the impermeable wall (see [21].

Furthermore, the geometries under consideration are: the convergent channel $s = e^{-x/2}$, and divergent channel $s = e^{x/2}$

4. RESULTS

Using Maple 12 computational programme, we computed with the following physically realistic parameters to obtain the results below:

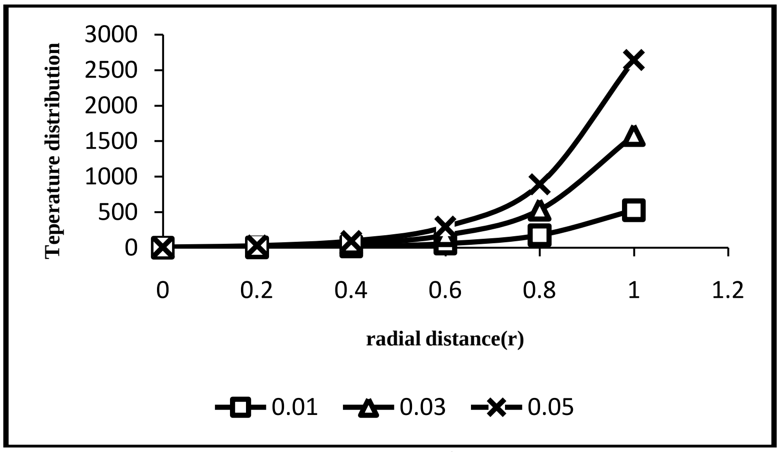

Figure 2 Temperature-amplitude (k) profiles in a convergent channel

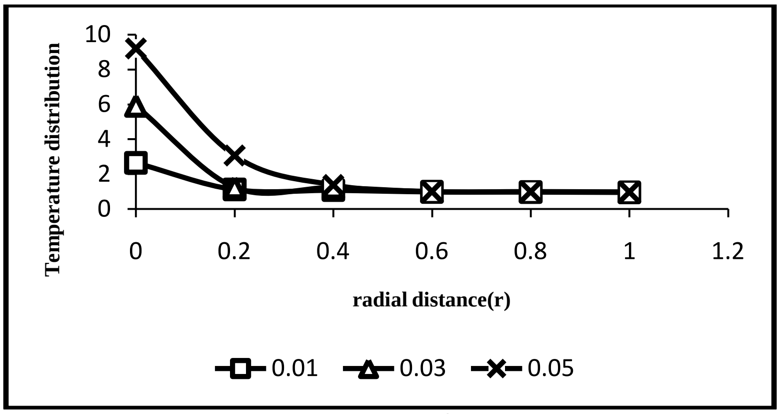

Fig 3 Temperature-amplitude (k) profile in a divergent channel

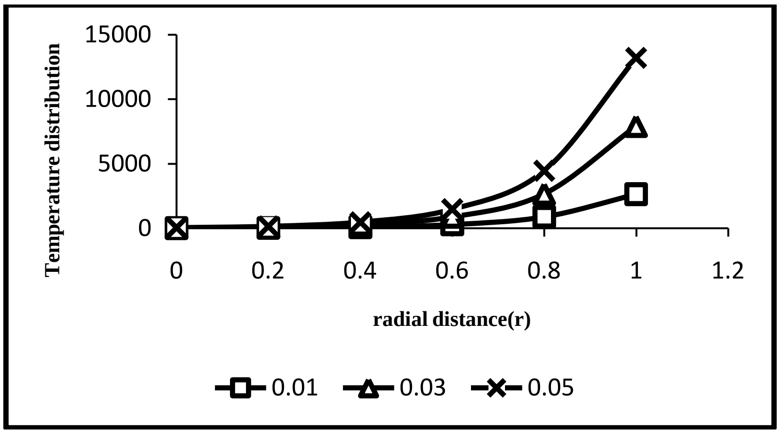

Fig 4 Temperature-height of constriction (ε) profile in a convergent channel

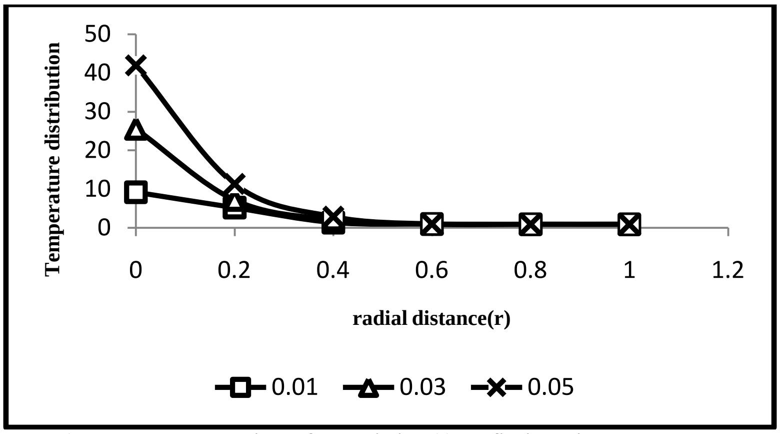

Fig 5 Temperature-height of constriction (ε) profile in a divergent channel

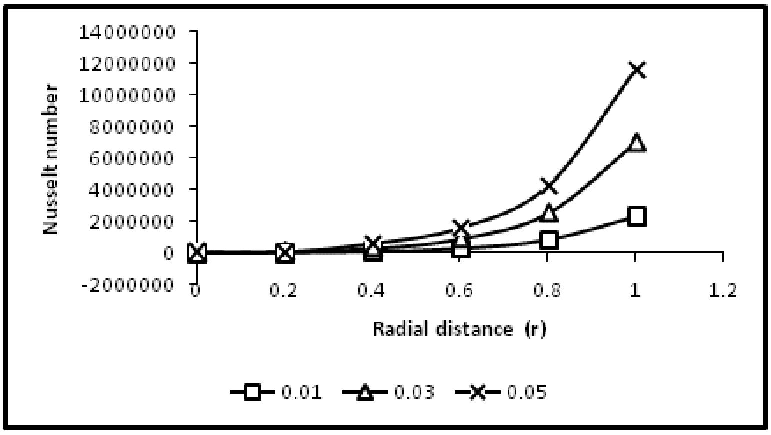

Fig 6 Nusselt number-height of constriction (ε) profile in a convergent channel

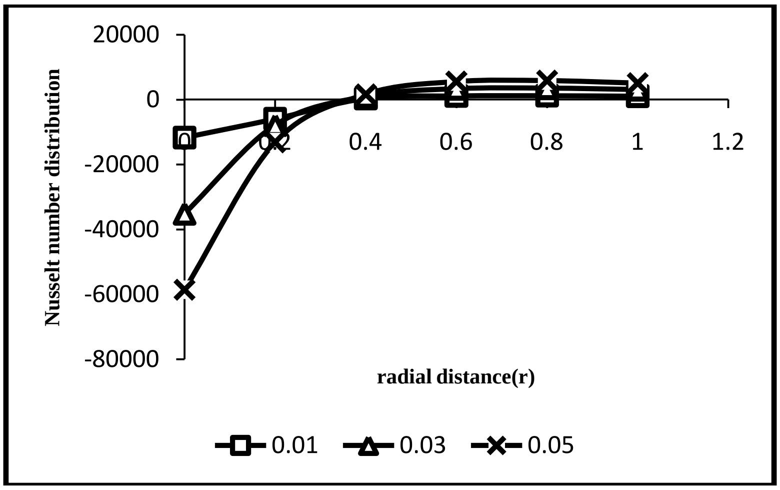

Fig 7 Temperature-height of constriction (ε) profile in a divergent channel

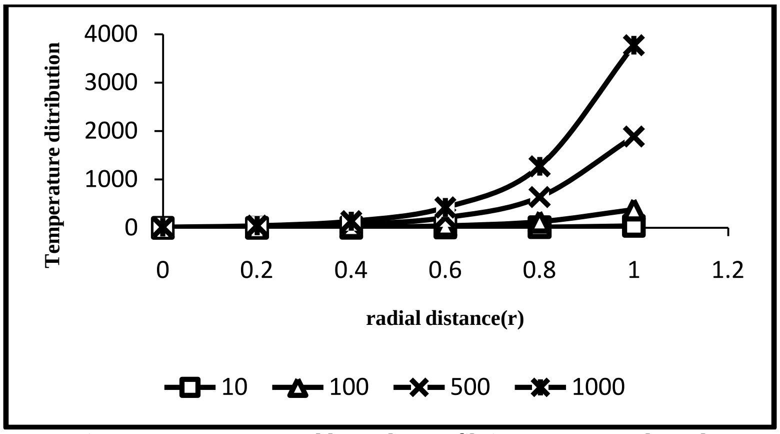

Fig 8 Temperature-Reynolds number profile in a convergent channel

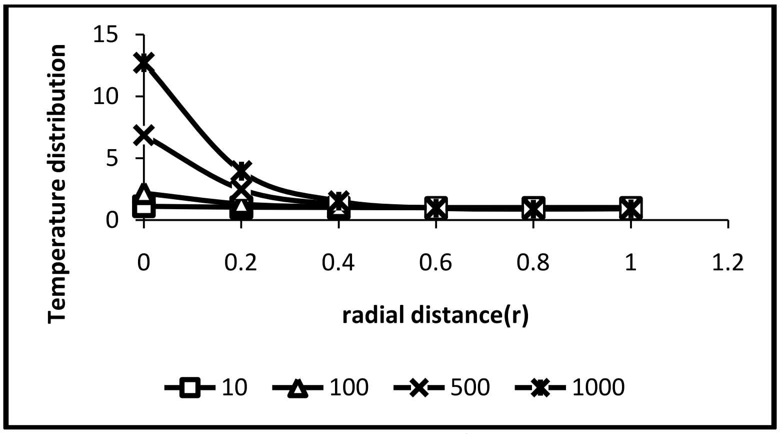

Fig 9 Temperature-Reynolds number profile in a divergent channel

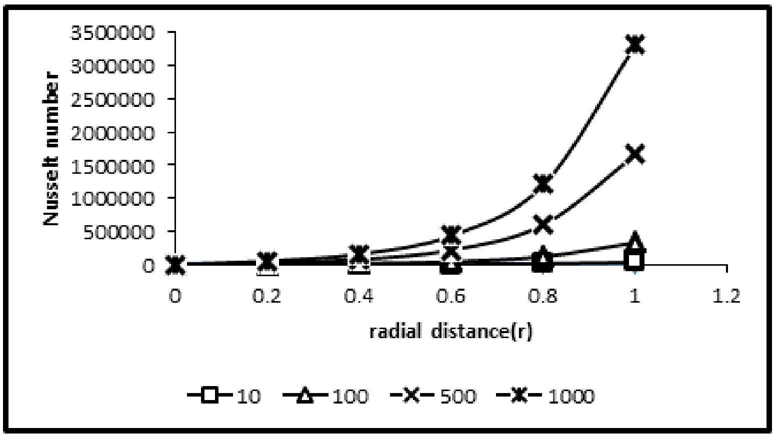

Fig 10 Nusselt number - Reynolds number profile in a convergent channel

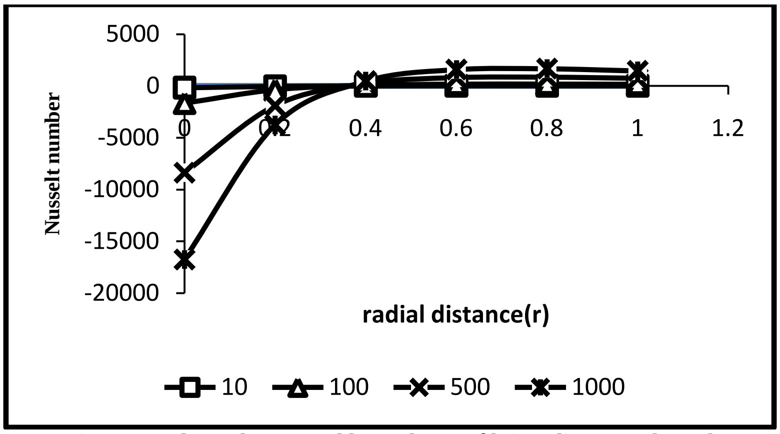

Fig 11 Nusselt number-Reynolds number profile in a divergent channel

5. DISCUSSION

In sections 2, 3 and 4, we formulated and solved the problem of heat transfer effects on the pulsatile blood flow in tubes of slowly varying cross-sections. The model in Fig.1 shows the transport of blood through a stenos artery. In the analyses, we considered the heat flow behavior in the artery before and after the peak of the plague. Therefore, we examined the heat flow in the convergent and divergent channels. In the convergent region, the geometry changes and thins down exponentially towards the peak of the constriction, while in the divergent region the geometry enlarges exponentially. The problem shows that the flow structure depends mostly on the variations in the amplitude of the pulse, height of constriction and Reynolds number (Re). To this end, Fig 2 – Fig 11 show the profiles of the computational results for the temperature and Nusselt number for various values of the amplitude k, height of constriction ε and Reynolds number Re, using Maple 12 computational software. For realistic values of $H(x)=1$; $Pr=0.7$; $T=\pi/4$; $\lambda=2$; $k = 0.01, 0.03, 0.05, 0.07$; $\varepsilon = 0.01, 0.03, 0.05, 0.07$; $Re= 10, 100, 500, 1000$ the profiles show that the temperature and Nusselt number increase with the increase in the amplitude, height of constriction and Reynolds number. The increase in the pulse wave results in increase in the size of the amplitude, which consequently, increases the strength of the flow variables, as seen in Fig 2 and Fig 3. In extreme cases, this leads to abrupt rise in the flows variables, otherwise called shocks. Furthermore, blood flow in the artery is laminar and Poiseuille. It moves with the pressure and temperature with which it is released from the right ventricle of the heart. As it gets to the region of constriction, the flow pattern changes. In particular, for the convergent channel, as the geometry thins down the pressure increases, and subsequently, the velocity increases. The increase in the velocities increases the kinetic energy of the system, which invariably increases the temperature of the system. Also, it is seen in the case of the divergent channel that the temperature and Nusselt number increase. This could possibly be due to the influence of the flow situations in the convergent region on it. These account for what is seen in Fig 4 and Fig 5

In the upstream, that is, the region before the plague, the flow is laminar and Poiseuille, therefore, the Reynolds number is moderate. But toward the peak of the constriction\critical point, the inertial force rises; hence the Reynolds number increases with subsequent increase in the momentum. On the other hand, in the case of the divergent region, as the channel widens away exponentially from the peak, the inertial force drops, resulting in a drop in the Reynolds number and momentum, and subsequently the velocities decrease. This is what is supposed to be. But the analyses show that an increase in the Reynolds number brings about increase in the temperature and Nusselt number. Possibly, this could be due to the influence of the flow situations in the convergent region on those of the divergent region. These account for what is seen in Fig 8 and Fig 9. It is also observed that the temperature increases towards the wall in the convergent region, but decreases towards the wall in the divergent region.

More so, the analyses show that heat is generated in the system. Therefore, to maintain equilibrium, the excess heat is transferred to the wall (see Fig 6 and Fig 10.) and finally ejected from the system. Also, a special feature called flow separation is seen in the Nusselt number especially in the divergent region (Fig 7 and Fig 11). Here, initially, the Nusselt number decreases with the increase in the height of constriction and Reynolds number. It drops at $r \leq 0.4$, picks up such that the Nusselt number increasing with both height of constriction and Reynolds number. The flow separation could be due to adverse flow situations experienced at that point.

The heat generated in the system has some clinical or health implications on the stenos patient. As the temperature\heat level rises (a hyperthermia situation), the patient experiences some sort of body internal heat sensation.

6. CONCLUSIONS

The overall analysis of the results shows that the increase in the amplitude of the pulse, height of the constriction, and the Reynolds number increase the temperature and Nusselt number of the flow. Therefore, heat is generated in the arterial flow system. The rise in the heat level leads to a hyperthermia situation, which constitutes a health risk to the stenos patient. Furthermore, the analyses presented in this model aid our understanding of the effects of stenosis on the arterial blood flow.

7. ACKNOWLEDGEMENT

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