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Coalesce of Queuing theory in Multiphase Queuing system in International Airport in Kerala

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ABSTRACT--- ***This paper ambidextrously and comprehensively analyses the waiting time of Passengers in Airport Terminal departure in International Airport in Kerala. Data was collected by observation preparing questionnaires in respect of the number of Passengers being serviced and the number of arrivals. The average arrival rate, average service rate, average time spent in the queue for the International Airport in Kerala from which necessary comparisons were made. The Paper describes the analysis of passenger flow in the Airport terminal, from entrance to boarding.***

Keywords---- multiple-server model, queuing steady-state condition, estimated queue length, inter arrival time.

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1. INTRODUCTION

1.1 Background and motivation

One of life's repugnance activities, namely "linger in line", is the ecstasy subject of this paper. Waiting for one service or the other is an inevitable phenomenon of life.. This paper ambidextrously and comprehensively analyses the waiting time of Passengers flow in the Airport terminal, from the entrance to the boarding in Airport Terminal in International Airport in Kerala. This technique will be helpful for any Airport for improving their passenger's service towards competitive gain

The airport is the gateway of any country. The airline industry is important for the global economy. Airports, in particular, hub airports are the backbone of air travels. Air travels plays a vital role in the changing global economy, linking people and places, facilitating trade and tourism, and encouraging economics competition and specialization. The aviation system offers one of the most significant engines for national economic growth. If managed well, this economics advantage will become ever more important as there is continued movement toward a global economy dominated by services. The airport forms a vital part in the aviation system, because, at this point in the system, the mode of transport changes from the air mode to land mode and vice versa. Therefore, it is the point of interaction of the three major components of the air transport system: The airport (with related air traffic control), the airline and the user. This study highlights a successful application of queuing theory. There are some inevitable problems that are emerging against the work of the Airport terminal that is the problem of waiting lines. Queue causes unnecessary delay and reduces the service effectiveness. Application of the queuing theory as the development of mathematical models to study and analyses the waiting lines with the hope of reducing this social factor in our Airport terminal will lead to improve the services economy.

It is proposed to concentrate on finding real applications, but also consider the theoretical contributions to applied areas that have been developed based on real applications. Thus, this study features examples of queuing theory applications over a spectrum of areas in Air port terminal. Observation is that some of the successful queuing applications san be applied and san be ameliorated by using simple principles gained from studying the size and pattern of queues.

1.2 Define the Problem

This study deals with the modeling and analysis of certain queuing systems with a kind of synchronization and their applications in International Airports in Kerala. There are three Airports in Kerala, that is Karipur international Airport Kozhikode, Nedumbassery International Airport Kochi, and Trivandrum International Airport. **Karipur international**

Airport is the gate way of Malabar .It is the 13th busiest International Airport in India of overall passenger traffic. **cochin International Airport** also known as Nedumbassery Airport and CIAL, is the largest and busiest airport in Kerala . cochin International Airport is the first airport in the country developed under a public-private partnership(PPP) model. , it was the seventh busiest airport in India in terms of overall passenger traffic and fourth busiest in terms of international passenger traffic.
Queuing theory provides an efficient mathematical framework for the study of several congestion phenomena arising in diverse application areas such as telecommunications, production lines, etc. For the accurate description of a queuing system the following are the basis elements.

2. OBJECTIVES

The core goal of this study is to dissecting the performance of the current functional design of the passenger departure terminal at Airport terminal in International Airport in Kerala.
check-in process the passengers have entered at the terminal (checked-in at the counters in the meantime) until they passed through the security controls, in order to estimate their times through the terminal in question and the times needed for the passengers to be served. This route is one of the trickiest services that the airports provide because when the check-in process is ill executed initial undesirable delays as well as long waiting times occur and thus the service provided becomes of poor quality.
One of the prime aims of this study is to identify whether the current situation fits well with the current demand and whether it is efficient, or on the contrary, whether there are needs to make improvements through either Physical or/and Operational changes. As the future traffic increases, it would be interesting to study (and further consider) adequate changes in the passenger departure terminal in question. These changes are needed in order to accommodate that traffic growth. Another idea is to give insight view of the steady-state behavior of queuing processes**.**

3. LITERATURE REVIEW

This literature review reveals various decision support techniques for the analysis of an airport. The motivation for this study is to determine an appropriate model to solve the queuing problem in International Airports in Kerala Numerous studies and work have been done on the subject of airport.
Sao, Nsakanda and Pressman(2003) Pressman carried out a simulation study of the passenger check-in system at Ottawa International Airport. Later, **in 2005, Van der Sluis and again Van Dijk(in 2005,**) investigated more in detail the check-in problem. They fixed their attention at the daily level and used simulation to determine: the minimal number of counters in order to meet a service level for each separate flight in term of waiting times, . Next, they provided some integer programming formulations to minimize the total number of counters and the total number of counter hours under the realistic constraint that counters for one and the same flight should be adjacent **Shang and Yang** (**In 2007,)**studied specifically the case of the kiosks, or automated self-service check-in machines, and analysis has been done for the improving of service quality of self-service kiosks, but also assisted the industry in developing a SUSS (Common-use self-service) standard that would enable airlines to share kiosks**, de Barros, Somasundaraswaran and Wirasinghe**[**In 2007**] analyzed transfer passengers' views on the quality of services at the terminal building, using data collected at Bandaranaike International Airport in Sri Lanka.In which Regression analysis was used to identify the transfer passenger facilities and services with the strongest effect on the overall perception of level of service
The check-in process of the airport terminal has been modeled and examined by Appelt et al, (2007).The goal of their study was to determine delays and solutions to improve efficiency. There were peak times associated with aircraft operations. It was established that delays occur during these peak hours
Valentin et al,(2002) the time spent by passengers to reach the end of an area san be determined by simulation modeling. They also argued that the use of simulation modeling for determining the separation between infrastructures makes the modeling of complex airports easy.
A study made by James, (2009) addressed facility related decision problems of airports. In order to solve this problem, it was categorized all the airport aspects involved in the decision making into three categories, namely, fixed facilities, variable facilities which could be deployed to where it was needed and transient entities whish were items that moved through the system.

4. METHODOLOGY

- The data collected was primary and was collected for an arrival time of each passenger in one week by using the questionnaire predetermine
- The observations for number of Passengers in a queue, their arrival-time and departure-time were taken without distracting the employees
- The whole procedure of the service unit each day was observed and recorded using a time-watch during the same time period for each day.
- In the Airport Terminal queue there was no balk or renege

5. MODELING

Performing any Mathematical analyses, it is absolutely necessary to describe adequately the process being modeled .It is extremely important to use the correct model or at least the model that describes the real situation. The ability to model and analysis system of queues help to minimize their inconveniences and maximize the use of limited resources.
On the other hand queuing model is used to overcome the congestion of the traffic; This traffic can be of any form. This replica chiefly used in stipulation where passengers' are in queue. Then , these models are used to understand the

situation related to Airport terminal waiting line and to find some alternative to overcome this problem by suggesting certain alternatives.
The Multi Server approach of modeling was adopted in this study to develop a mathematical model to solve problem of queuing of air transport passengers at the International Airports in Kerala. A mathematical queuing model was developed in this study using data obtained from three international Airport in Kerala.
The elementary probability theory to predict average waiting time ,average queue length, distribution of queue length etc on the basis of

*Arrival pattern of passengers to the terminal,
*Service pattern of the passengers' and
* The scheduling algorithm, or the manner in which the next customers to be served is chosen

Arrival Process

The customers arrival process may be described in two ways

*By categorizing the number of arrivals per units time (the arrival rate)
* By categorizing the time between the successive arrivals (inter arrival time)

If λ is defined as the no: of arrivals per unit time then $1 / \lambda$ will be the successive arrival. If μ is the rate of service then $1 / \mu$ is the service time.
The average rate of customers' entering the queuing system as λ and the average rate of serving customers' as μ a measure of traffic congestion for s server system is $\rho = \lambda / s\mu$. When $\rho > 1$, the average no: of arrivals in to the system exceeds the maximum average service rate of the system. To attain a steady state system ρ must less than one. When $\rho = 1$ arrivals and service are deterministic therefore the average arrival rate and average service rate guaranteed a steady state solution only if $\rho < 1$.
Our first aspiration is to cram the assets of a replica of the arrival process to a system and to compare its features to the Poisson process.
In determining whether the Poisson process is a reasonable model for arrivals in a specifies service system, this study deem its three crucial properties:
i. Customers arrive one at a time.
ii. The probability that a customers arrives at any time is independent of when other customers' arrived.
iii. The probability that a customers arrives at a given time is independent of the time.
The events occur successively in time , so that the intervals between successive events are independently and identically distributed according to an exponential distribution. Consider an arrival counting process $\{ N(t) \mid t \geq 0\}$ denotes the total number of arrivals up to time t ,with $N(t) = 0$ and whish satisfies the following assumptions:

i The probability that an arrival occurs between time t and time $t + \Delta t$ is equal to . $\lambda \Delta t + o(\Delta t)$.we write this Pr(arrival occurs between t and $t + \Delta t$) where λ is a constants independent of N(t) , Δt is an incremental element ,and $o(\Delta t)$ denotes a quantity that becomes negligible when compared to Δt as $\Delta t \to 0$ that is $\lim_{\Delta t \to 0} \frac{0(\Delta t)}{\Delta t} = 0$

ii Pr (more than one arrival between t and $t + \Delta t$) $= o(\Delta t)$
iii The number of arrivals in non overlapping intervals is statistically independent; i.e., the process has independent increments.

Now to calculate $P_n(t)$, the probability of n arrivals in a time interval of length t, n being an integer ≥ 0 .For $n \geq 1$ we have

$$
\begin{aligned}
P_n(t + \Delta t) = {} & \Pr(\text{ n arrivals in t and none in } \Delta t) \\
& + \Pr(\text{ n-1 arrivals in t and one in } \Delta t) \\
& + \Pr(\text{ n} - 2 \text{ arrivals in t and two in } \Delta t) + \ldots\ldots\ldots\ldots\ldots \\
& + \Pr(\text{ no arrivals in t and n in } \Delta t) \qquad \to (1)
\end{aligned}
$$

Using assumption I,ii,iii and equation (1) becomes

$$P_n(t + \Delta t) = P_n(t) \ [1 - \lambda\Delta t - o(\Delta t)] + P_{n-1}(t) [\lambda\Delta t + o(\Delta t)] + o(\Delta t)] \qquad \to (2)$$

Where the last term,$o(\Delta t)$, represents the term Pr { n – j arrivals in t and j in (Δt); $2 \leq j \leq n$}
When n = 0 ,we have

$$P_0(t + \Delta t) = P_0(t) [1 - \lambda\Delta t - o(\Delta t)] \qquad \to (3)$$

Rewriting equation (2) and (3) and combining all $o(\Delta t)$ terms , we have

$$P_0(t + \Delta t) - P_0(t) = -\lambda\Delta t \, P_0(t) + o(\Delta t) \qquad \to (4)$$

And

$$P_n(t + \Delta t) - P_n(t) = -\lambda \Delta t \, P_n(t) + \lambda \Delta t \, P_{n-1}(t) + o(\Delta t) \qquad (n \geq 1) \qquad \to (5)$$

dividing equation (4) and (5) by Δt, and take the limit as $\Delta t \to 0$,and obtain the differential – difference equation

$$\lim_{\Delta t \to 0} \left[\frac{P_0[t + \Delta t - P_0(t)}{\Delta t} \right] = \lim_{\Delta t \to 0} -\lambda P_0(t) + \lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t}$$

$$\lim_{\Delta t \to 0}\left[\frac{P_n(t+\Delta t - P_n(t)}{\Delta t}\right] = \lim_{\Delta \to 0}\left[\{-\lambda P_n(t) + \lambda P_{n-1}(t)\} + \lim_{\Delta \to 0}\frac{o(\Delta t)}{\Delta t}\right]$$

This reduces to

$$\frac{dP_0(t)}{dt} = -\lambda P_0(t) \qquad \to (6)$$

And
$$\frac{dP_n(t)}{dt} = -\lambda P_n(t) + \lambda P_{n-1}(t) \qquad (n \geq 1) \qquad \to (7)$$

Now to solve an infinite set of linear first order ordinary differential equations .Equation (6) clearly has the general solution $P_0(t) = Ce^{-\lambda t}$,where the constants S is equal to one ,since $P_0(0) = 1$ let n = 1 in equation (7) then

$$\frac{dP_1(t)}{dt} = -\lambda P_1(t) + \lambda P_0(t)$$

Or
$$\frac{dP_1(t)}{dt} + \lambda P_1(t) = \lambda P_0(t) = \lambda e^{-\lambda t}$$

The solution to this equation is given by

$$P_1(t) = Ce^{-\lambda t} + \lambda t e^{-\lambda t}$$

Using the boundary condition $P_n(0) = 0$ for all n > 0 yields S – 0 and gives

$$P_1(t) = \lambda t e^{-\lambda t}$$

Continuing sequentially to n = 2,3,4, ……………..in equation (7) and proceeding similarly we get

$$P_2(t) = \frac{(\lambda t)^2}{2} e^{-\lambda t}, \qquad P_3(t) = \frac{(\lambda t)^3}{3!} \qquad \textit{and in general}$$

$$P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}$$

This is the formula for a Poisson distribution probability distribution with mean λt. Thus the independently identically distributed random variable defined as the number of arrivals to a queuing system by time t, this random variable has the Poisson distribution with mean λt. Arrivals or a mean arrival rate of λ. Now test if the arrivals are described by a Poisson probability distribution

In order to show that if the arrival process is Poisson, an associated random variable defined as the time between successive arrivals follows exponential distribution. Let T be the random variable " time between successive arrivals " ;then

$$P_r(T \geq t) = P_r(\text{no arrivals in time } t) = P_0(t) = e^{-\lambda t}$$

Therefore the cumulative distribution function of T can be written as

$$C(t) = P_r(T \leq t) = 1 - e^{-\lambda t}$$

With corresponding density function

$$c(t) = \frac{dC(t)}{dt} = \lambda e^{-\lambda t}$$

Thus T has the exponential distribution with mean 1 /λ if the inter arrival time are independent and have the same exponential distribution then the arrival follows the Poisson distribution. So in this study arrival of passengers are independently and identically distributed random variable so the arrival follows the Poisson distribution and service rate are exponentially distributed .

There are s parallel service channels and a customers can go to any of the free counter for his service ,where the service time is identical and have the same probability density function s(t) with mean service rate μ per unit of time per busy server. Thus, overall service rate when there are n units in the system is given by:

1. If n≤s, all the customers' may be served simultaneously and in such cases there will be no queue.(s-n) servers may remain idle and then mean service rate is nμ, for n= 0,1,…..,s-1,s.
2. If n≥s, all the servers will remain busy, number of customers' waiting in the queue will be (n-s)and then mean service rate is sμ ,i.e., mean service rate is given by

The mean service time μ_n is given time by

$$\mu_n = \begin{bmatrix} n\mu & if\ 0 \le n < s \\ s\mu & if\ n \ge s \end{bmatrix}$$

6. STEADY STATE DIFFERENCE EQUATIONS

Let Pn (t) be the probability that there are n units in the system at time t and Pn (t+Δt) is the probability that there are n units in the system at time (t+Δt) then

Pn(t+Δt)= Pn(t).P[no arrival in the system during Δt].P[no service during Δt]
+Pn-1(t).P[1 arrival during Δt].P[no service during Δt]
+Pn+1(t).P[no arrival during Δt].P[1 service during Δt]+o(Δt);1≤n≤s-1

{Since cases are mutually exclusive and exhaustive and arrival and service are distributed independently}

i.e

$$Pn(t+\Delta t) = Pn(t)[1-\lambda\Delta t+o(\Delta t)][1-n\mu\Delta t + o(\Delta t)]$$
$$+Pn-1(t)[\lambda\Delta t+o(\Delta t)][1- (n-1)\mu\Delta t+o(\Delta t)]$$
$$+Pn+1(t)[1-\lambda\Delta t+o(\Delta t)][(n+1)\mu\Delta t+o(\Delta t)]+o(\Delta t) ; \quad 1 \le n \le s-1$$

$$Pn(t+\Delta t)= Pn(t) - (\lambda+n\mu)Pn(t)\Delta t + \lambda Pn-1(t)\Delta t+(n+1)\mu Pn+1(t)\Delta t+o(\Delta t)$$

$1 \le n \le s-1$

{Combining all the terms of o(Δt)}

$$\lim_{\Delta t\to 0}\left[\frac{P_0[t+\Delta t - P_0(t)}{\Delta t}\right] = (\lambda + n\mu)Pn(t) + \lambda Pn-1(t) + (n+1)\mu Pn+1(t)+$$

$$\lim_{\Delta t\to 0}\frac{o(\Delta t)}{\Delta t}$$

i.e $\quad P_n'(t) = (\lambda + n\mu)Pn(t) + \lambda Pn-1(t) + (n+1)\mu Pn+1(t) \rightarrow 1$

since

$$\lim_{\Delta t\to 0}\frac{o(\Delta t)}{\Delta t} \rightarrow 0$$

For n=0

$$Po(t+\Delta t)=P_o(t).P[\text{no arrival during } \Delta t]$$
$$+P_1(t).P[\text{no arrival during } \Delta t]+P1(t).P[\text{one service during } \Delta t] +o\Delta t$$
$$Po(t+ \Delta t) = P_o(t) [1- \Delta t+o(\Delta t)]+P1(t)[1-\lambda\Delta t+o(\Delta t)][\mu\Delta t+o(\Delta t)]+o(\Delta t)$$
$$= P_o(t) -\lambda P_o(t) \Delta t\ \mu\ P1(t) \Delta t + o(\Delta t)$$
$$Po'(t) = -\lambda P_o(t) + \mu P1(t) \rightarrow 2$$

And for n≥s

$$P_n(t+\Delta t)=P_n(t).P[\text{no arrival in the system during } \Delta t].P[\text{no service during } \Delta t]$$
$$+P_{n-1}(t).P[\text{1 arrival during } \Delta t].P[\text{no service during } \Delta t]$$
$$+P_{n+1}(t).P[\text{no arrival during } \Delta t].P[\text{1 service during } \Delta t]+o(\Delta t)$$
$$=P_n(t)[1-\lambda\Delta t+o(\Delta t)][1-s\mu\Delta t + o(\Delta t)]$$
$$+P_{n-1}(t)[\lambda\Delta t+o(\Delta t)][1- s\mu\Delta t+o(\Delta t)]$$
$$+P_n+1(t)[1-\lambda\Delta t+o(\Delta t)][s\mu\Delta t+o(\Delta t)]+o(\Delta t) ; n\ge s$$

i.e

$$P_n(t+\Delta t)= Pn(t) - (\lambda+s\mu)P_n(t)\Delta t + \lambda P_{n-1}(t)\Delta t+s\mu P_{n+1}(t)\Delta t+o(\Delta t) ; n\ge s$$

i.e

$$\lim_{\Delta t\to 0}\left[\frac{P_0[t+\Delta t - P_0(t)}{\Delta t}\right] = (\lambda + s\mu)Pn(t) + \lambda Pn-1(t) + s\mu Pn+1(t)+$$

$$\lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t}$$

i.e. $P_n'(t) = (\lambda + s\mu)Pn(t) + \lambda Pn\text{-}1(t) + s\mu Pn\text{+}1(t) \rightarrow 3$

Under steady state conditions Pn(t) → Pn for large t

And $P_n'(t) \rightarrow 0$ for all n

Thus equation 1 , 2 and 3 reduces to

$0 = -(\lambda + n\mu)Pn + \lambda Pn\text{-}1 + (n+1)\mu Pn\text{+}1 \quad 1 \le n \le s\text{-}1 \rightarrow 4$

$$\text{i.e } Pn{+}1 = \frac{1}{(n+1)\mu} \{ (\lambda + n\mu)Pn - \lambda Pn{-}1\}$$

$0 = -\lambda P_o + \mu P1$

$$\text{i.e } P_1 = \frac{\lambda}{\mu} P_0 \quad \rightarrow 5 \quad \text{and}$$

$$Pn{+}1 = \frac{1}{c\mu} \{ (\lambda + s\mu)Pn - \lambda Pn{-}1\} \quad ;n \ge s \quad \rightarrow 6$$

Putting n=1 in equation.4 we get

$$P_2 = \frac{1}{2\mu} \{ (\lambda + \mu)P_1 - \lambda P_0\}$$

$$= \frac{1}{2\mu} \{ (\lambda + \mu) \frac{\lambda}{\mu} P_0 - \lambda P_0\}$$

$$= \frac{1}{2}\left(\frac{\lambda}{\mu}\right)^2 P_0$$

n=2 in eq. 4 gives

$$P_3 = \frac{1}{3\mu} \{ (\lambda + 2\mu)P_2 - \lambda P_1\}$$

$$= \frac{1}{3\mu} \{ (\lambda + 2\mu) \frac{1}{2}\left(\frac{\lambda}{\mu}\right)^2 P_0 - \lambda P_1\}$$

$$P_3 = \frac{1}{3!}\left(\frac{\lambda}{\mu}\right)^3 P_0$$

and so on. In general

$$P_n = \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0$$

n= s-1 gives

$$P_s = \left(\frac{\lambda}{\mu s}\right) P_{s-1}$$

$$= \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s P_0$$

Similarly n=s in equation 6 gives

$$P_{S+1} = \frac{1}{s\,\mu} \{ (\lambda + s\,\mu)P_s - \lambda P_{s-1}\}$$

$$= \frac{1}{s\,\mu} \{ (\lambda + s\,\mu) \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s P_0 - \lambda \frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^{s-1} P_0 \}$$

$$= \frac{1}{s\,s!}\left(\frac{\lambda}{\mu}\right)^{s+1} P_0$$

Similarly n=s+1 in equation 6 gives

$$P_{s+2} = \frac{1}{s^2\,s!}\left(\frac{\lambda}{\mu}\right)^{s+2} P_0$$

And so on.
In general for n≥s

$$P_n = P_{s+(n-s)} = \frac{1}{s^{n-c}\,s!}\left(\frac{\lambda}{\mu}\right)^n P_0$$

We know that,

$$\sum_{n=0}^{\infty} P_n = 1$$

i.e
$$\left[\sum_{n-0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s}^{\infty} \frac{1}{s!s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n\right] P_0 = 1$$

i.e
$$\left[\sum_{n-0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s}^{\infty} \frac{s^s}{s!}\left(\frac{\lambda}{s\mu}\right)^n\right] P_0 = 1$$

i.e
$$\left[\sum_{n-0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{s^s}{s!}\left(\frac{\lambda}{s\mu}\right)^s \frac{1}{1-\frac{\lambda}{\mu s}}\right] P_0 = 1$$

Thus it follows that

$$\left[\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!\left(1-\frac{\lambda}{\mu s}\right)}\left(\frac{\lambda}{\mu}\right)^s\right] P_0 = 1$$

Hence $P_0 = \dfrac{1}{\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!\left(1-\frac{\lambda}{s\mu}\right)}\left(\frac{\lambda}{\mu}\right)^s}$

Thus steady state equation for the model is given by

$$P_n = \begin{pmatrix} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0, \; n = 1,2,3,\ldots\ldots\ldots c-1. \\ \frac{1}{c!c^{n-c}}\left(\frac{\lambda}{\mu}\right)^n P_0, \; n = c, c+1, c+2\ldots\ldots\ldots \end{pmatrix}$$

7. PERFORMANCE MEASURES ----- MEASURES OF EFFECTIVENESS

When analysis a queuing system, wish to find

- Number of customers' in the system;
- Waiting time for a customer;
- The length of a busy or idle period;
- The current work backload; (in units of time).

These are called measures of effectiveness. They are all random variable.

Number of customers'

Let N denote the random variable that describes the number of customers' in the system at steady state. The probability that at steady state the number of customers' present in the system is n denoted by P_n

$$P_n = \text{Prob}\,[\,N = n]$$

And average number in the system at steady state is

$$\mathbf{L = E\,[N]} = \sum_{n=0}^{\infty} n p_n$$

With in the queuing system, customer may be present in the queue waiting for the turn to receive service or they may be receiving service. Let $\mathbf{N_q}$ be the random variables to describe the number of customers' waiting in the queue and its mean is denoted by $\mathbf{L_q = E\,[N_q\,]}$

System time and queuing time

The time the customer spends in the systems, from the time of its arrival to the queue to the time of its departure from the server, is called the response time or sojourn time. Let **R** denote the random variable that describes response time and its mean by **E [R].**

The response time composed of time the customer spends waiting in the queue, called the waiting time , plus the customers spends receiving service, called the service time .Let $\mathbf{W_q}$ be the random variables to describe the time the customers spends waiting in the queue and its mean is denoted by $\mathbf{E\,[W_q\,]}$

This study analyses the application of queuing theory in International Airport passenger's departures in the international Airport in Kerala. In the Airport terminals, there are several servers in several queues. Therefore the apt queuing model is the multiple servers in multiple queues of infinite capacity .Thus this study is a queuing models of **(M /M / S) : (∞ / FIFO)**

This model deals with a queuing system having several servers arranged in parallel, each of which has identically and independently distributed and service time is distributed exponentially. The arrivals are assumed to follow Poisson distribution. The system is assumed to have an infinite capacity and the customers' are served as First in First Out basis
Hence there are 'S' number of servers each with a mean service rate of μ and let the mean arrival rate of passengers be λ
The steady state probabilities for a Poisson queue system are given by

$$P_n = \frac{\lambda_0\, \lambda_1\, \lambda_2 \,\ldots\ldots\ldots\ldots\ldots \lambda_{n-1}}{\mu_1\, \mu_2\, \mu_3 \,\ldots\ldots\ldots\ldots\ldots \mu_n} \quad P_0 \quad where\ \ n \geq 1 \qquad \rightarrow 1$$

Using the fiction that $\sum_{n=0}^{\infty} P_n = 1$

We can write $P_0 + \sum_{n=1}^{\infty} P_n = 1$

There fore

$$P_o = \frac{1}{1 + \sum_{n=1}^{\infty} \frac{\lambda_0\, \lambda_1 \lambda_2 \lambda_3 \ldots\ldots\ldots\ldots \lambda_{n-1}}{\mu_1\, \mu_2\, \mu_3 \ldots\ldots\ldots\ldots \mu_{n-1}}} \qquad \rightarrow 2$$

If there is a single server, the mean service rate $\mu_n = \mu$ for all n.But for the given model , there are s servers working independently of each other .When there are n customers' in the system, the mean service rate , $\mu_{n,}$ san be calculated in two different situations:

1) If n < s, only n of the s servers will be busy and others will be idle .Hence, the mean service rate will be nμ.
2) If n ≥ s, all the servers will be busy .Hence, the mean service rate will be sμ.

Thus, the following can be assumed.

i) The mean arrival time $\lambda_n = \lambda$ for all n
ii) The mean service time μ_n is given time by

$$\mu_n = \begin{bmatrix} n\mu & if\ \ 0 \leq n < s \\ s\mu & if\ \ n \geq s \end{bmatrix} \qquad \rightarrow 3$$

iii) The mean arrival rate is less than sμ ,i.e λ < sμ

If 0 ≤ n ≤ s, then substituting equation 3 in equation 1 we get

$$P_n = \frac{\lambda}{1.\mu.2\mu.3.\mu\ldots\ldots\ldots n\mu} P_0 = \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0 \qquad \rightarrow 4$$

If n ≥ s, then substituting equation 3 in equation 1 we get

$$P_n = \frac{\lambda^n}{(1.\mu.2\mu.3\mu\ldots\ldots\ldots(.s-1)\,\mu)(s\mu\, s\mu \ldots\ldots\ldots\ldots\ldots(n-s+1)\ times)}$$

$$P_n = \frac{\lambda^n}{(s-1)!\,\mu^{s-1}(s\mu)n-s+1} = \frac{1}{s!\,s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n P_0 \qquad \rightarrow 5$$

To find the value of P_0 we use the fast that $\sum_{n=0}^{\infty} P_n = 1$

i.e

$$\left[\sum_{n-0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s}^{\infty} \frac{1}{s!\,s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n\right] P_0 = 1$$

i.e

$$\left[\sum_{n-0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s}^{\infty} \frac{s^s}{s!}\left(\frac{\lambda}{s\mu}\right)^n\right] P_0 = 1$$

i.e
$$\left[\sum_{n-0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{s^s}{s!}\left(\frac{\lambda}{s\mu}\right)^s \frac{1}{1-\frac{\lambda}{\mu s}}\right]P_0 \quad = 1$$

using the assumption (iii) that $\frac{\lambda}{s\mu} < 1$ and the expansion is given by

$$\sum_{n=s}^{\infty}\left(\frac{\lambda}{s\,\mu}\right)^n = \left(\frac{\lambda}{s\mu}\right)^s\left[1+\frac{\lambda}{\mu\, s}+\left(\frac{\lambda}{\mu\, s}\right)^2+\ldots\ldots\ldots\right] = \left(\frac{\lambda}{\mu\, s}\right)^s \frac{1}{1-\frac{\lambda}{\mu\, s}}$$

Thus it follows that

$$\left[\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!\left(1-\frac{\lambda}{\mu s}\right)}\left(\frac{\lambda}{\mu}\right)^s\right] P_0 \quad = 1$$

Hence $\quad P_0 \quad = \quad \dfrac{1}{\displaystyle\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!\left(1-\frac{\lambda}{s\mu}\right)}\left(\frac{\lambda}{\mu}\right)^s}$ $\rightarrow 6$

Characteristics of the Model

1. Average or expected number of customers' in the queue or queue length (L_q). this model has a total of s servers ,the expected queue length is given by

$$L_q = E\,(N_q) = E\,(N - s) = \sum_{n=s}^{\infty}(n-s)P_n = \sum_{x=0}^{\infty} x\,P_{x+s} \qquad \textit{putting} \quad x = n - s$$

Thus we have

$$L_q = \sum_{x=0}^{\infty}\frac{x}{s!\,s^x}\left(\frac{\lambda}{\mu}\right)^{s+x} P_0 \quad = \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s P_0 \sum_{x=1}^{\infty} x\left(\frac{\lambda}{s\mu}\right)^x$$

it follows that

$$L_q = \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s P_0 \frac{\lambda}{\mu\, s}\frac{1}{\left(1-\frac{\lambda}{\mu\, s}\right)^2}$$

Using the fact that

$$\sum_{x=1}^{\infty} x\left(\frac{\lambda}{s\mu}\right)^x = \frac{\lambda}{\mu s}\sum_{x=1}^{\infty} x\left(\frac{\lambda}{\mu\, s}\right)^{x-1} = \frac{\lambda}{\mu\, s}\left[1-\frac{\lambda}{s\mu}\right]^{-2}$$

Hence we deduce that

$$L_q = \frac{1}{s\,s!}\frac{\left(\frac{\lambda}{\mu}\right)^{s+1}}{\left(1-\frac{\lambda}{\mu s}\right)^2} P_0$$

I. **Average or expected number of customers' in the system (L_s)**

By little's formula we have

$$L_s = L_q + \frac{\lambda}{\mu} = \frac{1}{s\,s!}\frac{\left(\frac{\lambda}{\mu}\right)^{s+1}}{\left(1-\frac{\lambda}{\mu s}\right)^2} P_0 + \frac{\lambda}{\mu}$$

where $$L_s = E[N] = \sum_{n=0}^{\infty} n P_n$$

II. **Average waiting time of a customer in the system (W_s)** By Little's formula

$$W_s = \frac{L_s}{\lambda} = \frac{1}{\lambda} * \frac{1}{s\,s!}\frac{\left(\frac{\lambda}{\mu}\right)^{s+1}}{\left(1-\frac{\lambda}{\mu s}\right)^2} P_0 + \frac{1}{\mu}$$

i.e $$W_s = \frac{1}{\mu} + \frac{1}{\mu}\frac{1}{s\,s!}\frac{\left(\frac{\lambda}{\mu}\right)^{s}}{\left(1-\frac{\lambda}{\mu s}\right)^2} P_0$$

III. **The average waiting time of a customer in the queue (W_q)**

By Little's formula $$W_q = \frac{L_q}{\lambda} = \frac{1}{\lambda}\frac{1}{s\,s!}\frac{\left(\frac{\lambda}{\mu}\right)^{s+1}}{\left(1-\frac{\lambda}{\mu s}\right)^2} P_0$$

$$W_q = \frac{1}{\mu}\frac{1}{s\,s!}\frac{\left(\frac{\lambda}{\mu}\right)^{s}}{\left(1-\frac{\lambda}{\mu s}\right)^2} P_0$$

IV. **The probability that an arrival has to wait for service**

The arrival has to wait for service if and only if $T_s > 0$ where T_s denote the waiting time of a customer in the system ,i.e if and only if there are s or more customers' in the system .Thus the required probability is equal to

$$P(T_s > 0) = P(N \geq s) = \sum_{n=s}^{\infty} P_n = \sum_{n=s}^{\infty}\frac{1}{s!s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n P_0 = \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^n P_0 \sum_{n=s}^{\infty}\left(\frac{\lambda}{\mu s}\right)^{n-s}$$

i.e $$P(T_s > 0) = P(N \geq s) = \frac{\left(\frac{\lambda}{\mu}\right)^s P_0}{s!\left(1 - \frac{\lambda}{s\mu}\right)}$$

V. The probability that an arrival enters the servise without waiting

$$1 - P(T_s > 0) = 1 - \frac{\left(\frac{\lambda}{\mu}\right)^s P_0}{s!\left(1 - \frac{\lambda}{s\mu}\right)}$$

VI. The mean waiting time in the queue for those who need to wait :

$$E[T_q | T_s > 0] = \frac{E[T_q]}{P[T_s > 0]} = \frac{W_q}{P[T_s > 0]}$$

Substituting the values of W_q and $P[T_s > 0]$ in the above equation we get

$$E[T_q | T_s > 0] = \frac{1}{\mu}\,\frac{1}{s\,s!}\,\frac{\left(\frac{\lambda}{\mu}\right)^s}{\left(1 - \frac{\lambda}{\mu s}\right)^2} P_0 \times \frac{s!\left(1 - \frac{\lambda}{\mu s}\right)}{\left(\frac{\lambda}{\mu}\right)^s P_0}$$

Simplifying we get

$$E[T_q | T_s > 0] = \frac{1}{\mu s\left(1 - \frac{\lambda}{\mu s}\right)} = \frac{1}{\mu s - \lambda}$$

VII. The average or expected number of customers' in non empty queues (Ln)

If N denote s the number of customers' in the system and N_q denote the number of customers' in the queue ,then L_n is the conditional expectation defined by

$$L_n = E[N_q \mid N_q \geq 1] = \frac{E[N_q]}{P[N \geq s]} = \frac{L_q}{P[N \geq s]}$$

Since there are s servers , substituting the values of L_q and $P[N \geq s]$ we get

$$L_n = \frac{1}{s\,s!}\,\frac{\left(\frac{\lambda}{\mu}\right)^{s+1}}{\left(1 - \frac{\lambda}{\mu s}\right)^2} P_0 \times \frac{s!\left(1 - \frac{\lambda}{\mu s}\right)}{\left(\frac{\lambda}{s\mu}\right)^s P_0} = \frac{\left(\frac{\lambda}{s\mu}\right)}{1 - \frac{\lambda}{s\mu}}$$

VIII. The probability that ther will be some one waiting:

As there are s servers, the required probability is given by

$$P[N \geq s + 1] = P[N \geq s] - P[N - s] = P[N \geq s] - P_s$$

$$P[N \geq s+1] = \frac{\left(\frac{\lambda}{\mu}\right)^s P_0}{s!\left(1-\frac{\lambda}{s\mu}\right)} - \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s P_0$$

$$= \frac{\left(\frac{\lambda}{\mu}\right)^s P_0 - \left(\frac{\lambda}{\mu}\right)^s}{}$$

8. DATA COLLECTION AT KOZHIKODE

The path related to passengers departing from Terminal 1 consists of two floors: on the ground floor we have the check-in desks, and on the first floor, assessable by escalators, the security control facilities immigration, security and customs and some shops and food services are located. On the other side of the ground floor besides the path related to arriving passengers there are the ticket office, the two currency exchange offices, several car rentals, and so forth. Therefore correct data is essential to get valid and valuable results about bottlenecks and to define relevant scenarios.

Data Analysis

The most problematic phase of this study has been that on data acquisition. Sampling took place in the passenger terminal Karipur International airport. After a thorough inspection airport it same out that passenger traffic peaks during Wednesday Therefore the focus data collection phase was on three days: Monday, Wednesday and Friday. Data was grouped in the spreadsheets that show some information such as basis information flight (the airline and destination), time of departure, time of opening and closing its check-in and finally the flight code. This information is important because from the flight code it is possible to know the type of the aircraft used for that flight and then the total number of aircraft available seats. . In order to carry out a correct analysis of collected data it was necessary to make an appropriate data stratification for highlighting some key aspects.

DATA ANALYSIS AND COMPUTATION

By observation the following data was collect at exactly 7**.00am** to **1.00Pm** on daily basis. **Table** shows the arrival, inter-arrival, service for a certain group of random Passengers that used the facilities in Airport Terminal as at the time of observation and data collection The queuing at the each server at some point was at least 11 Passengers at peak periods.

Table .1 Data Analysis of Karipur International Airport

Dubai				
	Baggage	Immigration	Customs	Security
P_o	0.01475	0.03774	0.16667	0.17266
L_q	5.16465	1.52847	4.16675	2.17074
W_q	10.3293	4.58541	16.66667	6.51222
Saudi				
	Baggage	Immigration	Customs	Security
P_o	0.01475	0.0008029	0.06667	0.044944
L_q	5.1647	2.5125	5.71695	3.51125
W_q	10.3294	2.513	22.87	17.56
Dubai				
	Baggage	Immigration	Customs	Security
P_o	0.0598	0.00812	0.09091	0.0562
L_q	3.027	12.2020	3.788	2.5
W_q	9.081	12.2020	11.364	12.5
Saudi				
	Baggage	Immigration	Customs	Security
P_o	0.9091	0.009140	0.14285	0.0747
L_q	3.788	1.265	1.928	1.7033
W_q	11.364	2.53	7.712	6.813
Sharjah				
	Baggage	Immigration	Customs	Security

P_o	0.053697	0.00585	0.052632	0.0641
L_q	14.4	7.13451	7.673454	2.14
W_q	57.6	35.40699	38.36727	6.4298

Elucidation of results for queuing model 1
The interpretation of the Airport terminal Analysis of karipur International Airport
From above table that the probability for servers to be busy in dubai flight- baggage screening is 0.1475 i.e. 14.75%. The average number of customers waiting in a queue is Lq = 5.16465 customers per minutes. The waiting time in a queue per server is Wq = 10.33 minutes .

9. DISCUSSION

This paper reviews a queuing model for multiple queue with multi servers. The average queue length can be estimated simply from raw data from questionnaires by using the collected number of customers waiting in a queue in each minute. Then compare this average with that of queuing model. The models are used to reckon a queue length is multiple queue multi-server model. In multiple queue, customers' in any queue swap to succinct queue (jockey behavior of queue).
The pragmatic analysis of queuing system of Airport terminal is that they may not be very incompetent in terms of wherewithal utilization. Queues stature and customers' linger even though servers may be idle much of the time. The blemish is not in the model or core assumptions. It is a candid upshot of the volatility of the arrival and service processes. If variability should be eliminated, system should be designed economically so that there would be little or no waiting, and hence no need for queuing models. With the escalating number of passengers' coming to Airport terminal, there is a trained employee serving at each service unit.. Increasing more than sufficient number of servers may not be the solution to increase the efficiency of the service by each service unit. When servers are dissecting with one queue for two parallel servers, the results are estimated as per server whereas when each server is analyzed with its individual queue.

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