A Modified Regula Falsi Method for Solving Root of Nonlinear Equations

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ABSTRACT— ***A Modified Regula Falsi method based on classic Regula Falsi method or False Position method for solving nonlinear equations is proposed in this paper. The method is verified on a number of test examples and numerical results obtained shows that the proposed method is very effective with respect to the classic Regula Falsi method and the other methods.***

Keywords— Regula falsi method, Nonlinear equations, Roots.

1. INTRODUCTION

In practical problem of finding roots of a nonlinear equation where good initial estimates of the roots are known, there are several of computationally efficient algorithms available which can be programmed for using on a computer. However, in which earlier information on the location of the root of difficult problems are poor because the methods often fail to converge. So, this paper is an attempt to investigate for a method which based on classic Regula Falsi method. We consider the nonlinear equation

$$f(x) = 0, \tag{1}$$

where f is a continuously differentiable on $[a,b] \subset \Re$ and has at least one root, r, in $[a,b]$. The well-known quadratically convergent Newton method and its variants are iterative formula generally used for finding a root of (1). But, these methods may fail to converge in case the initial point is far from root or the derivative vanishes in the vicinity of the root. One of the oldest and best well known of these methods is Regula Falsi method (RF) in [2] which for solving (1) consists of successive application of the following two statements:

(i) Setup an initial interval $[a,b]$ including the root, r , take $c = \frac{af(b)-b(f(a)}{f(b)-f(a)}$ as an approximation of r where c is the root of the linear interpolation polynomial of f in $[a,b]$.

(ii) Replace the interval $[a,b]$ with $[a,c]$ if $f(a)f(c) < 0$ or with $[c,b]$ if $f(a)f(c) > 0$.

The iterative process is repeated until one stopping criterion is attained. For example, the interaction is continued either one or both of the convergence criteria are satisfied $|c_{i+1} - c_i| < \varepsilon$ or $|f(c)| < \varepsilon$ where ε is small positive numbers. Some modifications overcoming these difficulties have been discussed in [1,2]. Dowell and Jarratt [2] proposed modifications of Regula Falsi method to increase its asymptotic rate of convergence from linear to super-linear.

2. MODIFIED METHOD

Let r be a root of the nonlinear equation (1) in $[a,b]$ and $f(a)f(b) < 0$ to guarantee that r is a simple root of (1) in $[a,b]$. We start with compute an approximation point of the root (1), $c = \frac{af(b)-b(f(a)}{f(b)-f(a)}$ which straight line, L , joining the points $(a, f(a))$ and $(b, f(b))$ intersecting the x-axis by RF. By the position of root r , we have four main cases:

case 1: Figure 1 (top left) both b,c,d are same side

case 2: Figure 1 (bottom left) both a,d are same side but b,c are different side

where point $d = \frac{(1+k)af(b)-bf(a)}{(1+k)f(b)-f(a)}$ for case 1, 2 is intersecting between line L and $y = \frac{-kf(b)}{b-a}(x-a)$,

case 3: Figure 1 (top right) both a,c,d are same side

case 4: Figure 1 (bottom right) both a, c are same side but b, d are different side
where point $d = \frac{(1+k)bf(a)-af(b)}{(1+k)f(a)-f(b)}$ for case 3, 4 is intersecting between line L and $y - f(a) = \frac{-kf(a)}{b-a}(x-a)$.

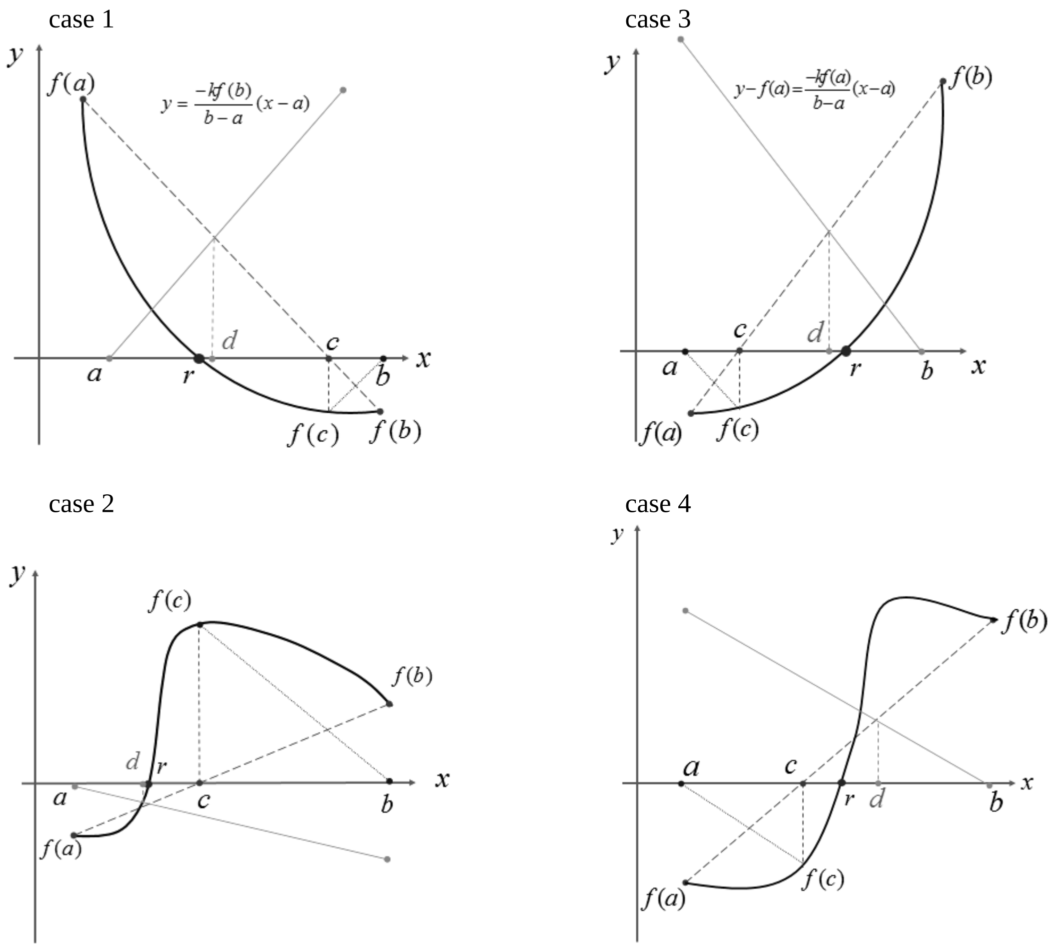

Figure 1: Four cases of the new approximation point, d which are case 1 (top left), case 2 (bottom left), case 3 (top right) and case 4 (bottom right).

3. ALGORITHM

A modified Regula Falsi will have four steps:

(i) Set $[a, b]$ is an initial interval which has at least one root in the interval,

(ii) compute $c = \frac{af(b)-b(f(a)}{f(b)-f(a)}$

(ii) if $f(a)f(c) < 0$ set $k = \left|\frac{2f(c)}{b-c}\right|$, compute $d = \frac{(1+k)af(b)-bf(a)}{(1+k)f(b)-f(a)}$ and

if $f(d)f(a) < 0$ set $b = d$ (case 1)

else set $b = c$ and $a = d$ (case 2)

else if $f(a)f(c) > 0$ set $k = \left|\frac{0.5f(c)}{b-c}\right|$, compute $d = \frac{(1+k)bf(a)-af(b)}{(1+k)f(a)-f(b)}$ and

if $f(d)f(a) > 0$ set $a = d$ (case 3)

else set $a = c$ and $b = d$. (case 4)

(iv) if $|f(d)| < \varepsilon$ then stop

else set $i = i + 1$ and go to step (ii).

4. RESULTS

Table 1 presents comparison of the iteration number between Regula Falsi (RF), IRF from [1] and the present work (MRF) with $\varepsilon = 1\times 10^{-10}$.

Table 1: The number of iteration of RF, IRF and MRF methods for specifics precision

Equation	Initial interval	Iteration of			x_n
		RF	IRF	MRF	
$xe^x - 1 = 0$	$[-1,1]$	22	7	6	0.5671433083
$11x^{11} - 1 = 0$	$[0.1, 0.9]$	33	9	8	0.8041330975
$e^{x^2+7x-30} - 1 = 0$	$[2.8, 3.1]$	32	7	6	3.0000000000
$\frac{1}{x} - \sin(x) + 1 = 0$	$[-1.3, -0.5]$	14	6	6	-0.6294463362
$x^3 - 2x - 5 = 0$	$[2,3]$	21	6	5	2.0945515532
$\frac{1}{x} - 1 = 0$	$[0.5, 1.5]$	33	6	5	1.0000000000
$x^3 + 2x^2 + 10x - 20 = 0$	$[0,2]$	14	7	6	1.3688081078

5. CONCLUSION

We presented a class of Regula Falsi methods for finding simple zeros of nonlinear equations. In this paper, an algorithm MRF is developed for computational purposes. The algorithm is tested on a number of numerical examples and the results obtain up to the desire accuracy $\varepsilon = 1\times 10^{-10}$ are compared with the other methods. It is observed that our method takes less number of iterations and more effective in comparison with these methods.

6. ACKNOWLEDGEMENT

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7. REFERENCES

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