On lower level subsets of Anti Fuzzy Γ-Ideals in Γ-CI-Algebras

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ABSTRACT— ***The purpose of this paper is to introduce the notion of a lower level subsets of anti fuzzy Γ-ideals in Γ-CI-algebras, we study lower level subsets, level subsets, upper sets, fuzzy and anti fuzzy Γ-ideals in Γ-CI-algebras. Those are extended from lower level subsets, level subsets, upper sets, fuzzy and anti fuzzy ideals in CI-algebras respectively. Some characterizations of lower level subsets, level subsets, upper sets, fuzzy and anti fuzzy fuzzy Γ-ideals in Γ-CI-algebras are obtained.***

Keywords— Γ-CI-algebra, Γ-ideal, upper set, level subset, lower level subset, fuzzy Γ-ideal, anti fuzzy Γ-ideal.

1. INTRODUCTION

In what follows, let X denote an CI-algebra unless otherwise specified. By an CI-algebra we mean an algebra $(X;*,1)$ of type (2, 0) with a single binary operation "$*$" that satisfies the following identities: for any $x, y, z \in X$,

1. $x*x=1$,
2. $1*x=x$,
3. $x*(y*z)=y*(x*z)$.

In, 2006, H. S. Kim and Y. H. Kim defined a BE-algebra. Biao Long Meng, defined notion of CI–algebra as a generation of a BE–algebra [9]. BE-algebras and CI-algebras are studied in detail be some researchers [3, 7, 9, 11] and some fundamental properties of CI–algebra are discussed.

The concept of an ordered Γ-CI-algebras was first given by Pairote Yiarayong and Phakakorn Panpho in [12] which is infect the generalization of a CI-algebras.

In this paper is to introduce the notion of a Γ-CI-algebras, we study lower level subsets, level subsets, upper sets, fuzzy and anti fuzzy Γ-ideals in Γ-CI-algebras. Those are extended from lower level subsets, level subsets, upper sets, fuzzy and anti fuzzy ideals in CI-algebras respectively. Some characterizations of lower level subsets, level subsets, upper sets, fuzzy and anti fuzzy fuzzy Γ-ideals in Γ-CI-algebras are obtained.

2. BASIC PROPERTIES

In this section we refer to [12, 13] for some elementary aspects and quote few definitions and examples which are essential to step up this study. For more detail we refer to the papers in the references.

Definition 2.1. [12, 13] Let X and Γ be any nonempty sets. If there exists a mapping $X\times\Gamma\times X\to X$ written as (x,γ,y) by $x\gamma y$, then $(\Gamma,X;1)$ is called a Γ-CI-algebra if

1. $x\gamma x = 1$ for all $x\in X$ and $\gamma\in\Gamma$,
2. $1\gamma x = x$ for all $x\in X$ and $\gamma\in\Gamma$,
3. $x\gamma(y\beta z) = y\gamma(x\beta z)$

for all $x, y, z \in X$ and $\gamma, \beta \in \Gamma$.

Lemma 2.2. [12] Let X be a Γ-CI-algebra. Then $x\gamma y = x\beta y$ for any $x, y \in X$ and $\gamma, \beta \in \Gamma$.

Proposition 2.3. [12] Any Γ-CI-algebra X satisfies for any $x, y \in X$ and $\gamma, \beta, \alpha \in \Gamma$, $y\gamma\left[\left(y\alpha x\right)\beta x\right] = 1$

Proposition 2.4. [12] Any Γ-CI-algebra X satisfies for any $x, y \in X$ and $\gamma, \beta \in \Gamma$, $\left(x\gamma 1\right)\beta\left(y\gamma 1\right) = \left(x\gamma y\right)\beta 1$.

Proposition 2.5. [12] Let X be a Γ-CI-algebra. If $x\gamma(x\beta y) = x\beta y$ for any $x, y \in X$ and $\gamma, \beta \in \Gamma$, then $x\gamma 1 = 1$.

Definition 2.6. [13] Let X be a Γ-CI-algebra and A a nonempty subset of X. A is said to be a Γ-subalgebra of X if $AA \subseteq A$.

Definition 2.7. [13] Let X be a Γ-CI-algebra and A a nonempty subset of X. A is said to be a Γ-ideal of X if it satisfies: $x \in X$ and $\gamma, \beta, \alpha \in \Gamma$

1. $X\Gamma A \subseteq A$,
2. $a, b \in A$ imply that $(a\gamma(b\beta x))\alpha x \in A$.

For any Γ-CI-algebra X, $\{1\}$ and X are trivial ideals (resp. Γ-subalgebras) of X. Obviously every ideal in a Γ-CI-algebra is a Γ-subalgebra.

Lemma 2.8. [13] Let X be a Γ-CI-algebra. Then

(1) Every Γ-ideal of X contains 1,

(2) If A is a Γ-ideal of X, then $(a\gamma x)\beta x \in A$ for all $a \in A, x \in X$ and $\gamma, \beta \in A$.

Definition 2.9. [13] Let X be a Γ-CI-algebra and $a, b \in X, \gamma, \beta \in \Gamma$. Define $A\left(a\gamma, b\beta\right)$ by

$$A\left(a\gamma, b\beta\right) = \{1\} \cup \left\{x \in X : a\gamma\left(b\beta x\right) = 1\right\}.$$

We call $A\left(a\gamma, b\beta\right)$ an upper set of a and b.

Remark.

1. By Definition 2.9, we have $A\left(a\gamma, b\beta\right) = A\left(b\gamma, a\beta\right)$ for all $a, b \in X$ and $\gamma, \beta \in \Gamma$.
2. By Definition 2.9, we have

$$\begin{aligned} a\gamma\left(b\alpha a\right) &= b\gamma\left(a\alpha a\right) \\ &= b\gamma 1 \\ &= b\gamma\left(1\alpha 1\right) \\ &= \left(b\gamma 1\right)\alpha\left(b\gamma 1\right) \\ &= 1. \end{aligned}$$

Then $a \in A\left(a\gamma, b\alpha\right)$.

3. LOWER LEVEL SUBSETS OF ANTI FUZZY Γ-IDEALS IN Γ-CI-ALGEBRAS

The results of the following lemmas seem to play an important role to study Γ-CI-algebra; these facts will be used frequently and normally we shall make no reference to this definitions.

Definition 3.1. Let X be a non-empty set**.** A fuzzy subset μ of the set X is a mapping $\mu : X \to [0,1]$. The complement of μ denoted by μ^c, is the fuzzy set in X given by $\mu^c(x) = 1 - \mu(x)$, for all $x \in X$.

Definition 3.2. Let X be a Γ-CI-algebra. A fuzzy set μ in X is called a fuzzy Γ-ideal of X if

1. $\mu(x\gamma y) \geq \mu(y)$, for all $x, y \in X$ and $\gamma \in \Gamma$.
2. $\mu((x\gamma(y\,\alpha z))\beta z) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y, z \in X$ and $\gamma, \alpha, \beta \in \Gamma$.

Theorem 3.3. A nonempty subset I of a Γ-CI-algebra X is a Γ-ideal of X if it satisfies

1. $1 \in I$
2. if $x\alpha(y\gamma z) \in I$, then $x\alpha z \in I$, for all $x, y \in X, \gamma, \alpha \in \Gamma$ and $y \in I$.

Proof. Let $x \in X, \gamma, \alpha \in \Gamma$ and $a \in I$. Since $x\gamma((x\alpha a)\alpha a) = 1 \in I$, we have $x\gamma a \in I$. To show that $(a\gamma(b\alpha x))\beta x \in I$, for all $x \in X, \gamma, \alpha, \beta \in \Gamma$ and $a, b \in I$. Now since

$$(a\gamma(b\alpha x))\beta(((a\gamma(b\alpha x))\delta x)\delta x) = ((a\gamma(b\alpha x))\delta x)\beta((a\gamma(b\alpha x))\delta x) = 1 \in I,$$

we get $(a\gamma(b\alpha x))\beta x \in I$. Hence I is a Γ-ideal of X.

Definition 3.4. Let μ be a fuzzy set of Γ-CI-algebra X. For a fixed $t \in [0,1]$, the set $\mu_t = \{x \in X \mid \mu(x) \geq t\}$ is called the level subset of μ.

Remark. Clearly if $t_1 \leq t_2$, then $\mu_{t_2} \subseteq \mu_{t_1}$.

Theorem 3.5. Let X be a Γ-CI-algebra and $u_t \neq \varnothing$. Then μ is a fuzzy Γ-ideal of X if and only if u_t is a Γ-ideal of X.

Proof. $\Rightarrow$ Assume that μ is a fuzzy Γ-ideal of X. Let $t \in [0,1], x \in X$ and $y \in \mu_t$. Then $\mu(y) \geq t$ and so $\mu(x\gamma y) \geq \mu(y) \geq t$, for all $x, y \in X$ and $\gamma \in \Gamma$. By Definition 3.4, we have $x\gamma y \in \mu_t$. To show that $(a\gamma(b\beta x))\alpha x \in \mu_t$ for all $x \in X, \gamma, \alpha, \beta \in \Gamma$ and $a, b \in u_t$. Let $x \in X, \gamma, \alpha, \beta \in \Gamma$ and $a, b \in u_t$. Then $\mu(a) \geq t$ and $\mu(b) \geq t$. Since $\mu((a\gamma(b\beta x))\alpha x) \geq \min\{\mu(a), \mu(b)\} \geq t$, we get $(a\gamma(b\beta x))\alpha x \in \mu_t$. Hence u_t is a Γ-ideal of X.

$\Leftarrow$ Assume that u_t is a Γ-ideal of X. If $\mu(x\gamma y) < \mu(y)$ for some $x, y \in X, \gamma \in \Gamma$, then

$$\mu(x\gamma y) < \frac{\mu(x\gamma y) + \mu(y)}{2} < \mu(y)$$

by taking $t_0 = \dfrac{\mu(x\gamma y)+\mu(y)}{2}$. Thus $x\gamma y \notin \mu_{t_0}$ and $y \in \mu_{t_0}$ which is a contradiction. X. Let $x, y, z \in X$ and $\gamma, \alpha, \beta \in \Gamma$ such that $\mu((x\gamma(y\beta z))\alpha z) < \min\{\mu(x),\ \mu(y)\}$. Then

$$\mu((x\gamma(y\beta z))\alpha z) < \frac{\min\{\mu(x),\ \mu(y)\} + \mu((x\gamma(y\beta z))\alpha z)}{2} < \min\{\mu(x),\ \mu(y)\}.$$

Taking $t_1 = \dfrac{\min\{\mu(x),\ \mu(y)\} + \mu((x\gamma(y\beta z))\alpha z)}{2}$ we have $t_1 \in [0,1]$. It follows that $x, y \in \mu_{t_1}$ and $(x\gamma(y\beta z))\alpha z \notin \mu_{t_1}$. This is a contradiction, and therefore μ is a fuzzy Γ-ideal of X.

Lemma 3.6. Let X be a Γ-CI-algebra and let $x \in X$. If μ is a fuzzy Γ-ideal of X, then $\mu(1) \geq \mu(x)$.

Proof. By Definition 3.2, we get $\mu(1) = \mu(x\gamma x) \geq \mu(x)$ for all $x \in X, \gamma \in \Gamma$. Then $\mu(1) \geq \mu(x)$.

Lemma 3.7. Let X be a Γ-CI-algebra and $t \in [0,1], a, b \in X, \gamma, \beta \in \Gamma,\ a, b \in \mu_t$. If μ is a fuzzy Γ-ideal of X, $A(a\gamma, b\beta) \subseteq \mu_t$.

Proof. Assume that μ is a fuzzy Γ-ideal of X and let $a, b \in \mu_t$. By Definition 3.4, we have $\mu(a) \geq t$ and $\mu(b) \geq t$. To show that $A(a\gamma, b\beta) \subseteq \mu_t$. Let $x \in A(a\gamma, b\beta)$. By Definition 2.9, we have $x = 1$ or $a\gamma(b\beta x) = 1$.

Case 1. $x = 1$. By Lemma 3.6, we get $\mu(x) = \mu(1) \geq \mu(a) \geq t$ so that $x \in \mu_t$. Therefore $A(a\gamma, b\beta) \subseteq \mu_t$.

Case 2. $a\gamma(b\beta x) = 1$. Since $a\gamma(b\beta x) = 1$, we get

$$\begin{aligned} \mu(x) &= \mu(1\alpha x) \\ &= \mu((a\gamma(b\beta x))\alpha x) \\ &\geq \min\{\mu(a), \mu(b)\} \\ &\geq t. \end{aligned}$$

Therefore $x \in \mu_t$ and hence $A(a\gamma, b\beta) \subseteq \mu_t$.

Theorem 3.8. Let X be a Γ-CI-algebra and $t \in [0,1], a, b \in X, \gamma, \beta \in \Gamma$. Then μ is a fuzzy Γ-ideal of X if and only if μ satisfies the following assertion: if $a, b \in \mu_t$, then $A(a\gamma, b\beta) \subseteq \mu_t$.

Proof. $\Rightarrow$ Clearly, from Lemma 3.17.

$\Leftarrow$ Assume that μ is a fuzzy Γ-ideal of X and let $a, b \in \mu_t$. By Definition 2.9, we have $1 \in A(a\gamma, b\beta) \subseteq \mu_t$. Let $x, y, z \in X$ be such that $x\gamma(y\alpha z) \in \mu_t$ and $y \in \mu_t$. To show that $x\alpha z \in \mu_t$. Since

$$\begin{aligned} 1 &= (x\gamma(y\alpha z))\beta(x\gamma(y\alpha z)) \\ &= (x\gamma(y\alpha z))\beta(y\gamma(x\alpha z)), \end{aligned}$$

we get $(x\gamma(y\alpha z))\beta(y\gamma(x\alpha z))=1$. By Definition 2.9, we have $x\alpha z\in A((x\gamma(y\alpha z)),y\gamma)\subseteq\mu_t$. It follows from theorem 3.3 that μ_t is a Γ-ideal of X. Hence, by Theorem 3.5, μ is a fuzzy Γ-ideal of X.

Theorem 3.9. Let μ be a fuzzy Γ-ideal of a Γ-CI-algebra X and $t\in[0,1], a,b\in X, \gamma,\beta\in\Gamma$. If $\mu_t\neq\emptyset$, then $\mu_t=\bigcup_{a,b\in u_t} A(a\gamma,b\beta)$.

Proof. By Theorem 3.8, we get $A(a\gamma,b\beta)\subseteq\mu_t$ for all $a,b\in\mu_t$. Then $\bigcup_{a,b\in u_t} A(a\gamma,b\beta)\subseteq\mu_t$. Now, let $y\in\mu_t$. By Definition 2.9, we get $y\in A(y\gamma,y\alpha)\subseteq\bigcup_{a,b\in u_t} A(a\gamma,b\beta)$. Then $\mu_t\subseteq\bigcup_{a,b\in u_t} A(a\gamma,b\beta)$ and hence $\mu_t=\bigcup_{a,b\in u_t} A(a\gamma,b\beta)$.

Definition 3.10. Let X be a Γ-CI-algebra. A fuzzy set μ in X is called an anti fuzzy Γ-ideal of X if

1. $\mu(x\gamma y)\leq\mu(y)$, for all $x, y\in X$ and $\gamma\in\Gamma$.
2. $\mu((x\gamma(y\alpha z))\beta z)\leq\max\{\mu(x), \mu(y)\}$, for all $x, y, z\in X$ and $\gamma,\alpha,\beta\in\Gamma$.

Definition 3.11. Let μ be a fuzzy set of Γ-CI-algebra X. For a fixed $t\in[0,1]$, the set $\mu^t=\{x\in X \mid \mu(x)\leq t\}$ is called the lower level subset of μ.

Remark. Clearly, $\mu^t\cup\mu_t=X$ for all $t\in[0,1]$ if $t_1\leq t_2$ then $\mu^{t_1}\subseteq\mu^{t_2}$.

Theorem 3.12. Let X be a Γ-CI-algebra and $u_t\neq\emptyset$. If u^t is a Γ-ideal of X, then μ is an anti fuzzy Γ-ideal of X.

Proof. Assume that u^t is a Γ-ideal of X. If $\mu(x\gamma y)>\mu(y)$ for some $x,y\in X,\gamma\in\Gamma$, then

$$\mu(x\gamma y)>\frac{\mu(x\gamma y)+\mu(y)}{2}>\mu(y)$$

by taking $t_0=\frac{\mu(x\gamma y)+\mu(y)}{2}$. Thus $x\gamma y\notin\mu^{t_0}$ and $y\in\mu^{t_0}$ which is a contradiction. X. Let $x,y,z\in X$ and $\gamma,\alpha,\beta\in\Gamma$ such that $\mu((x\gamma(y\beta z))\alpha z)>\max\{\mu(x), \mu(y)\}$. Then

$$\mu((x\gamma(y\beta z))\alpha z)>\frac{\min\{\mu(x), \mu(y)\}+\mu((x\gamma(y\beta z))\alpha z)}{2}>\min\{\mu(x), \mu(y)\}.$$

Taking $t_1=\frac{\min\{\mu(x), \mu(y)\}+\mu((x\gamma(y\beta z))\alpha z)}{2}$ we have $t_1\in[0,1]$. It follows that $x,y\in\mu^{t_1}$ and $(x\gamma(y\beta z))\alpha z\notin\mu^{t_1}$. This is a contradiction, and therefore μ is an anti fuzzy Γ-ideal of X.

Theorem 3.13. Let X be a Γ-CI-algebra and $u^t \neq \varnothing$. If μ is an anti fuzzy Γ-ideal of X, then u^t is a Γ-ideal of X.

Proof. Assume that μ is an anti fuzzy Γ-ideal of X. Let $t \in [0,1], x \in X$ and $y \in u^t$. Then $\mu(y) \leq t$ and so $\mu(x\gamma y) \leq \mu(y) \leq t$, for all $x, y \in X$ and $\gamma \in \Gamma$. By Definition 3.11, we have $x\gamma y \in u^t$. To show that $(a\gamma(b\beta x))\alpha x \in u^t$ for all $x \in X, \gamma, \alpha, \beta \in \Gamma$ and $a, b \in u^t$. Let $x \in X, \gamma, \alpha, \beta \in \Gamma$ and $a, b \in u^t$. Then $\mu(a) \leq t$ and $\mu(b) \leq t$. Since $\mu((a\gamma(b\beta x))\alpha x) \leq \max\{\mu(a), \mu(b)\} \leq t$, we get $(a\gamma(b\beta x))\alpha x \in u^t$. Hence u^t is a Γ-ideal of X.

Lemma 3.14. Let X be a Γ-CI-algebra and let $x \in X$. If μ is a ant fuzzy Γ-ideal of X, then $\mu(1) \leq \mu(x)$.

Proof. By Definition 3.10, we get $\mu(1) = \mu(x\gamma x) \leq \mu(x)$, for all $x \in X, \gamma \in \Gamma$. Then $\mu(1) \leq \mu(x)$.

Lemma 3.15. Let X be a Γ-CI-algebra and $t \in [0,1], a, b \in X, \gamma, \beta \in \Gamma$, $a, b \in u^t$. If μ is an anti fuzzy Γ-ideal of X, $A(a\gamma, b\beta) \subseteq \mu^t$.

Proof. Assume that μ is an anti fuzzy Γ-ideal of X and let $a, b \in \mu^t$. By Definition 3.11, we have $\mu(a) \leq t$ and $\mu(b) \leq t$. To show that $A(a\gamma, b\beta) \subseteq \mu^t$. Let $x \in A(a\gamma, b\beta)$. By Definition 2.9, we have $x = 1$ or $a\gamma(b\beta x) = 1$.

Case 1. $x = 1$. By Lemma 3.14, we get $\mu(x) = \mu(1) \leq \mu(a) \leq t$ so that $x \in u^t$. Therefore $A(a\gamma, b\beta) \subseteq u^t$.

Case 2. $a\gamma(b\beta x) = 1$. Since $a\gamma(b\beta x) = 1$, we get

$$\begin{aligned} \mu(x) &= \mu(1\alpha x) \\ &= \mu((a\gamma(b\beta x))\alpha x) \\ &\leq \max\{\mu(a), \mu(b)\} \\ &\leq t. \end{aligned}$$

Therefore $x \in u^t$ and hence $A(a\gamma, b\beta) \subseteq u^t$.

Theorem 3.16. Let X be a Γ-CI-algebra and $t \in [0,1], a, b \in X, \gamma, \beta \in \Gamma$. Then μ is an anti fuzzy Γ-ideal of X if and only if μ satisfies the following assertion: if $a, b \in u^t$, then $A(a\gamma, b\beta) \subseteq u^t$.

Proof. $\Rightarrow$ Clearly, from Lemma 3.15.

$\Leftarrow$ Assume that μ is an anti fuzzy Γ-ideal of X and let $a, b \in u^t$. By Definition 2.9, we have $1 \in A(a\gamma, b\beta) \subseteq u^t$. Let $x, y, z \in X$ be such that $x\gamma(y\alpha z) \in u^t$ and $y \in u^t$. To show that $x\alpha z \in \mu_t$. Since

$$\begin{aligned} 1 &= (x\gamma(y\alpha z))\beta(x\gamma(y\alpha z)) \\ &= (x\gamma(y\alpha z))\beta(y\gamma(x\alpha z)), \end{aligned}$$

we get $(x\gamma(y\alpha z))\beta(y\gamma(x\alpha z)) = 1$. By Definition 2.9, we have $x\alpha z \in A((x\gamma(y\alpha z)), y\gamma) \subseteq u^t$. It follows from theorem 3.3 that u^t is a Γ-ideal of X. Hence, by Theorem 3.12, u is an anti fuzzy Γ-ideal of X.

Theorem 3.17. Let μ be an anti fuzzy Γ-ideal of a Γ-CI-algebra X and $t \in [0,1], a, b \in X, \gamma, \beta \in \Gamma$. If $u^t \neq \varnothing$, then $u^t = \bigcup_{a,b \in u^t} A(a\gamma, b\beta)$.

Proof. By Theorem 3.16, we get $A(a\gamma, b\beta) \subseteq u^t$ for all $a, b \in u^t$. Then $\bigcup_{a,b \in u^t} A(a\gamma, b\beta) \subseteq u^t$. Now, let $y \in u^t$. By Definition 2.9, we get $y \in A(y\gamma, y\alpha) \subseteq \bigcup_{a,b \in u^t} A(a\gamma, b\beta)$. Then $u^t \subseteq \bigcup_{a,b \in u^t} A(a\gamma, b\beta)$ and hence $u^t = \bigcup_{a,b \in u^t} A(a\gamma, b\beta)$.

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