An Incentive Mechanism Game Theory Approach to Musharakah Contracts☆

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ABSTRACT--- ***Musharakah represents an Islamic practice of profit and loss sharing contracts. It is claimed to be a fair economic mode of investment as it entails the sharing, by the participants, of profits and risks. This mode of financing, however, suffers from asymmetric information in the form of adverse selection and moral hazards. In this research we focus on reducing moral hazards and investigate whether introducing an incentive mechanism can have a positive effect on participants' payoff in a profit and loss sharing contract. We use a two agents' game theoretical approach involving a financier and an entrepreneur. We found that an entrepreneur can be induced to participate with higher capital contribution as long as a specified minimum of incentive is put in place. We also found evidence that given the same incentive mechanism, the financier is indiffer- ent between profit and loss sharing ratios. Under the same incentive mechanism we can induce the agent to perform a higher effort contributing to a higher social value. We also found that, in case of the project success, the entrpreneur can also be an incentive giver ensuring a fairer distribution of profits. Due to its positive outcomes our model can definitely be commercialized.***

Keywords: Musharakah, sharing contracts, incentive mechanism, strictly dominant strategies, social value, moral hazard, Nash equilibrium.
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1. INTRODUCTION

Musharakah can be defined as a form of partnership between two or several parties in which they share capital and/or labor to form a business. Before commencing the partnership the parties must agree on the exact profit sharing ratio each one is entitled

☆Musharakah contract is a form of profit and loss sharing contracts

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to[1]. Losses, on the other hand, should not exceed each partners contribution[2]. To validate the musharakah contract, the participants must have the full capacity of entering into a contract with their free consent. A specified amount of investment is contributed by all parties with one or several partners acting as agents and managers. While management of the business is allowed to be undertaken by all partners, one or several managing partners may be appointed by mutual consent[3]. The non-managment parties (Silent partners) are entitled to a share of profits not allowed to exceed their share of investment[3]. The determined sharing ratio must be based on future expected profits. It cannot be determined as a lump sum or a percentage of the capital investment otherwise it becomes a guaranteed profit. Upon agreement of all partners on new terms, the profit-sharing ratio can be changed at a later stage. Partners can decide to retain profits in the venture for further investment. In terms of liability, the participants in a musharakah contract normally have unlimited liabilities.
The musharakah contract is terminated if [2]:

- The partnership was limited to a given time frame
- The purpose of the partnership has been achieved
- The continuity of the project is compromised by the withdrawal of one or several partners
- Any of the partners die before the end of the agreement
- In the case of premature termination, the business shall be liquidated and the settlement distributed pro-rata

Conventional and Islamic musharakah have different features. Islamic musharakah is based on ethical considerations governed by the Islamic jurisprudence (Shari'ah). For example musharakah is interest-free financing and prohibits the financing of illicit projects (gambling, casinos, wine and pornography). This restriction limits the number of investment opportunities of Islamic musharakah compared to their conventional counterparts.

Islamic musharakah participants do not receive fixed compensation like their conventional counterparts. The former may lose due to the profit and loss sharing principle. In addition, because the entrepreneur is most often not the provider of funds he may behave in opportunistic ways.
Our Paper proposes an extension of Nabi's model [4] which deals with the opportunistic behavior of the entrepreneur in the form of moral hazard. We extend his model using a new incentive mechanism game theoretical approach.
The paper is organized as follows:

[3]'THE CONCEPT OF MUSHARAKAH' presented at AlHuda CIBE Workshop at NIBAF-State Bank of Pakistan Islamabad by Dr.Muhammad Zubair Usmani, Sharia advisor, Muslim Commercial Bank Ltd, Jamia Darul Uloom Karachi

Section 2 starts with a literature review of asymmetric information in a financier-entrepreneur environment. Section 3 revisits the original model and presents our proposed model to reduce moral hazards. Section 4 presents the methodology. Section 5 represents the results. Section 6 discusses the outcomes and finally section 7 concludes with a summary and possible extensions.

2. LITERATURE REVIEW

In general a financier-entrpreneur relationship suffers from asymmetric information where the entrepreneur has an informational advantage about the project compared to the financier. To overcome this problem, the use of dissipative signals by the financier is of great importance. For example, collateral can be used by efficient agents as a signaling mechanism of their type. This signaling method is consistent with some research that claims that banks can use collateral in debt contracts to overcome information asymmetries, in particular arising from ex-ante adverse selection [5]. This is in line with the proposition of Karim [6] who proclaims that the submission of a warranty can resolve the adverse selection problem in a profit and loss contract. While collateral and warranties can be applied in a conventional system, it is, however, prohibited in musharakah under the Islamic jurisprudence (Shari'ah law). Yet, the recourse to a warranty is permissible if there is a proof of negligence or non-respect of the contract terms by the entrepreneur [4].

Low job protection can also be made similar to a high pledged collateral .i.e. a confident manager will demand a high reward in case of success but also signs for a low job protection in case of failure. This is consistent with previous research as in [7].Low job protection, however, can be seen as unfair to the entrepreneur since failure of the project can be due to factors beyond the entrepreneur's control. Demanding security by the financier, in the form of low job protection, can be seen as making the entrepreneur lose more than his contribution. This, however, contradicts the musharakah principle which calls for a fair sharing of profits and losses as mentioned earlier by Usmani [2].

Information sharing can be used to reduce information asymmetry. In fact, credit bureaus have been shown to increase efforts from borrowers [8].Information sharing is useful if borrower mobility is higher [9] and if asymmetric information problems are more important [10]. Empirical research has shown that, information sharing is correlated with higher access to credit [9], especially in developing countries with inefficient creditor rights [11], but lower lending to low-quality borrowers [12]. However, it is shown that due to information sharing, benefits from banking relationships are reduced resulting in a weaker banking competition [13].

One problem with Musharakah is misreporting. This happens when the agent announces losses while the project is making profits. To overcome this problem, Al

[4]Adoption of AAOIFI Shariah Standard No. 12 pertaining to Sharika (Musharaka) and Modern Corporations.Clause 3/1/4/1: "All partners of Sharika shall be deemed to be trustees in respect of Sharika assets; however, as trustees they shall be jointly and severally liable for misconduct, negligence or breach of contract."

Suwailm [14] argues that, there should be a higher due diligence from the part of the Islamic institutions as compared to conventional banks. In our paper, this is reflected in the expertise of the financier in the follow up of the project. The cost of such expertise is embeded in the opportunity cost of capital of the financier.
One argument suggests that the agency problem is based on an unfair distribution of returns if the project fails [15]. Taking into consideration the risks related to a project, the financial institution may demand a higher sharing ratio. This, according to Shaikh [15] however may result in less motivation of the entrepreneur and therefore a lower project returns. In our paper, we have developped an incentive scheme which gives the entrpreneur the choice beween a high or low ratio. This choice, interestingly, stabilises[5] the returns to both the entrepreneur and the financier.

Also, it is proposed that the entrepreneur's participation in the capital can reduce information asymmetries in a profit and loss contract [6].Consistent with this finding, and to induce the entrepreneur to exert high effort and therefore reduce moral hazard, a research suggested a minimum capital contribution by the entrepreneur given a minimum profit sharing ratio [4].

Our paper represents an extension to this model and differs by the usage of a game theoretical approach. We also found the minimum ratio, required by the financier to be restrictive in nature. This, therefore, gives less room for negotiation. Our incentive mechanism allows for increasing or reducing the minimum profit sharing ratio without affecting the payoffs of the financier.

In dealing with moral hazard, one research suggested that it can be solved under mudharabah[6] but cannot be solved under musharakah [16].This can be criticized in a sense that under mudharabah the financier provides the whole capital and therefore assumes all monetary risks. On the other hand under musharakah the capital is shared and intuitively the risk of losing capital is shared. This is, also, inconsistent with our findings and the findings of Nabi [4] which proposes that moral hazards can be solved subject to a contribution from the entrpreneur. i.e moral hazard is more likely to be solved under a musharakah contract than under a mudharabah contract. This is also inconsistent with the findings of Inness [17] who argues that sharing contract is not feasible in case of total external financing of the project.
Another research, proposed the usage of two profit sharing ratios instead of one to reflect the effort of the entrepreneur compared to the financier [18]. The model proposed, however, suffers from the non treatment of asymmetric information.

[5]Stability of returns does not mean a guranteed profit. rather we mean that the project yields its estimated profit when the entrepreneur excert a high effort. the project might still fail if conditions are beyond the entrepreneurs capabilities

[6]An Islamic term for a form of business in which the financier is the sole provider of capital(Rab'al Mal) and the entrepreneur is the provider of work and managment (Mudarib)

3. THE MODEL

Our model is an extension of the original model of Nabi's Work [4]. We have kept the same treatment under the symmetric case but differed in terms of the asymmetric case in which we applied a game theoretical approach. The model strives to reduce the moral hazard problem in a sharing contract between risk neutral financier and an entrepreneur. The later is willing to undertake a project which requires funding F. He is endowed with an initial fund f but requires an additional funding F-f . The success of the project depends on the effort of the entrepreneur. The project is estimated to result in a verifiable high output $\bar{\pi}$ with probability θ_e or low output value $\underline{\pi}$ with probability 1- θ_e.

The opportunity cost of the entrepreneur is at how much he values his effort. In our case, C_h and C_l are the corresponding disutility for the high effort and low effort respectively. The opportunity cost of the financier is ρ i.e. The expected output generated by the project should at least be equal to (1+ ρ) F. The expected output under the high effort and low effort case are given respectively as:

$$E^{h} = \theta_h \bar{\pi} + (1 - \theta_h)\underline{\pi} \tag{1}$$

$$E^{l} = \theta_l \bar{\pi} + (1 - \theta_l)\underline{\pi} \tag{2}$$

A relationship between the expected output under each level of the entrepreneur's effort and the required project fund F is presumed to follow the following assumption:
Assumption1: We assume expected output is superior to the fund F only in the case of success.

$$E^{h} > F \tag{3}$$

$$E^{l} < F \tag{4}$$

Also, an increase in the level of effort by the entrepreneur is assumed to take place only and only if there is an extra benefit from undertaking higher effort. Hence, the following assumption is considred:

Assumption2: Under self financing, and taking the difference between (3.1) and (3.2), the entrepreneur will perform a high effort only if the additional benefit exceeds the additional cost:

$$(\theta_h - \theta_l)(\bar{\pi} - \underline{\pi}) \geq C_h - C_l \tag{5}$$

We consider a one period contract. The entrepreneur and the financier agree on a partnership contract (x; F, α, β=x) whereby the entrepreneur commits to undertake a high effort and invest f= (1- x) F. A profit sharing ratios α and a loss sharing β are assigned to the financier in case of success and failure of the project respectively[4].
We can then modify "assumption 2" to get "assumption 3" by including partial financing of the entrpreneur:

Assumption3: Under partial financing, the entrepreneur will perform a high effort only if the additional shared benefit exceeds his additional cost:

$$(\theta_h - \theta_l)((1-\alpha)\bar{\pi} - (1-\beta)\underline{\pi}) \geq C_h - C_l \tag{6}$$

We should note the Islamic particularity of this contract that each participant's losses can't exceed their personal contribution in the project. This, however, is not a condition under the conventional system where one party can demand security against project losses.

To respect the Islamic particularity of the contract, we should also add to the original model that the financier is not a "sleeping partner" but rather contributes to the project throught consultancy, reputation and expertise. the cost of these contributions is reflected in the opportunity cost of capital of the financier.

3.1 The symmetric case

Under this case the financier observes the effort of the entrepreneur who can only undertake a high effort. The financier sets the profit sharing ratio α_0 in such a way that it gets its opportunity cost (break-even). So, the expected wealth of the financier $E(W^f)$ is:

$$E(W^f) = \theta_h \alpha_0 \bar{\pi} + (1-\theta_h)\beta\underline{\pi} = (1+\rho)xF \tag{7}$$

The sharing ratio is then:

$$\alpha_0 = \frac{x}{\theta_h \pi}[(1+\rho)F - (1-\theta_h)\underline{\pi}] \tag{8}$$

The entrepreneur has an incentive to perform higher effort if his contribution in the business exceeds a threshold $(1-\underline{x}) = (1-\beta_0)$such that:

$$\underline{x} = \theta_h \frac{(E_\pi^h - E_\pi^l) - (C_h - C_l)}{((1+\rho)F - \underline{\pi})(\theta_h - \theta_l)} \tag{9}$$

applying α_0 and β_0 we can have a special case of "assumption 3" where the entrepreneur is indifferent between the high and low effort:

$$(\theta_h - \theta_l)((1-\alpha_0)\bar{\pi} - (1-\beta_0)\underline{\pi}) = C_h - C_l \tag{10}$$

i.e under partial financing the entrepreneur is assumed to excert a high effort if his marginal payoff is equal or more than his marginal cost. We turn now to the assymetric case:

3.2. The asymmetric case

The original model of Nabi [4] provides us a minimum entrepreneurial capital requirement for no effort deviation and its corresponding profit sharing ratio. Can we find a better combination between the profit sharing ratio and the entrepreneur's capital contribution and get better payoffs for the participants in the sharing contract? To do so, we propose truncating the minimum profit sharing ratio α_0 using a given allowance γ and the financier minimal capital contribution β_0 using an allowance λ such that:

$$\beta_n = \beta_0(1 + \lambda_n) \tag{11}$$

$$\alpha_n = \alpha_0(1 + \gamma_n) \tag{12}$$

Where the subscript $n \in \{h,0,l\}$ stand for higher ,minimal, and lower values of β and α and

$\lambda_n, \gamma_n \in \{-1,1\}$ and so λ_l , $\gamma_l < 0$ while $\lambda_h, \gamma_h > 0$ and $\lambda_0, \gamma_0 = 0$

To insure that the agent does not deviate from his commitments we propose introducing an incentive $\Gamma(\alpha_n, \beta_n)$ that makes his payoff under high effort $U_E \mid h$ exceeds his payoff $U_E \mid l$ under the low effort. We can have then:

$U_E \mid h \geq U_E \mid l$
if
$\theta_h(1-\alpha_n)\bar{\pi} + (1-\theta_h)(1-\beta_n)\underline{\pi} + \theta_h\Gamma(\alpha_n,\beta_n) - C_h \geq \theta_l(1-\alpha_n)\bar{\pi} + (1-\theta_l)(1-\beta_n)\underline{\pi} + \theta_l\Gamma(\alpha_n,\beta_n) - C_l$

Plugging the allowances and making use of (10) we get the following assumption:

Assumption 4:

For no deviation the incentive $\Gamma(\alpha_n,\beta_n)$ must at least be greater or equal than a certain minimum threshold $\underline{\Gamma}(\alpha_n,\beta_n) = \gamma_n\alpha_0\bar{\pi} - \lambda_n\beta_0\underline{\pi}$ such that

$$\Gamma(\alpha_n, \beta_n) \geq \underline{\Gamma}(\alpha_n, \beta_n) \tag{13}$$

Now that we set up the minimum incentive and given assumption (3) the entrepreneur will exert a higher effort and therefore we can set up the game such that the entrepreneur will perform the high effort action. Since the loss ratio to the financier is the same as his capital contribution ratio the problem of the financier can be formulated as:

Maximize $E(W^f) = \theta_h\alpha_n\bar{\pi} + (1-\theta_h)\beta_n\underline{\pi} - \theta_h\Gamma(\alpha_n,\beta_n)$
Subject to constraints:

$\theta_h\alpha_n\bar{\pi} + (1-\theta_h)\beta_n\underline{\pi} - \theta_h\Gamma(\alpha_n,\beta_n) \geq (1+\rho)\beta_n F$ (PC_F)
$\theta_h(1-\alpha_n)\bar{\pi} + (1-\theta_h)(1-\beta_n)\underline{\pi} + \theta_h\Gamma(\alpha_n,\beta_n) > C_h$ (PC_E)
$\Gamma(\alpha_n, \beta_n) \geq \underline{\Gamma}(\alpha_n, \beta_n)$ (IC_E)
$0 \leq \beta_n \leq 1$ (FC)
$0 \leq \alpha_n \leq 1$ (FC)

Where (PC_F), (PC_E) stands for the participation constraints of the financier and the entrepreneur respectively while (IC_E) stands for the incentive constraints of the entrepreneur. The last two constraints are the feasibility constraints where the contribution of the financier and his profit sharing ratio need to be between 0 and 100% .

4. METHODOLOGY

We follow a game theoretical approach to decide on the best strategies of each agent. Before doing so we need to decide on the available strategies of each of them.

4.1 Strategies of the game's participants

In this theoretical game, the financier has three strategies regarding the sharing ratio $(\alpha_h; \alpha_0; \alpha_l)$. The Entrepreneur, on the other hand, has the following strategies regarding his participation in capital$(f_l; f_0; f_h)$. Here $(\alpha_h; \alpha_0; \alpha_l)$ represents a high, minimal, low profit sharing requirment of the financier respectively and $(f_l; f_0; f_h)$ represents a low, minimal, high capital contribution of the entrepreneur respectively. We should not that this also represent an important variation of the original model of Nabi [4]. In fact the entrpreneur's wealth f is now ranging from a minimal f_l to a maximal value f_h. The intuition is that we would like to see if it is socially optimal to pull out a maximum fund from the entrepreneur for investment purposes.

We present our case in the strategic game form below.

Table 1: Payoffs to the financier and entrepreneur in a normal form game

		Entrepreneur		
		f_l	f_0	f_h
Financier	α_h	$U_F(\alpha_h, f_l); U_E(f_l, \alpha_h)$	$U_F(\alpha_h, f_0); U_E(f_0, \alpha_h)$	$U_F(\alpha_h, f_h); U_E(f_h, \alpha_h)$
	α_0	$U_F(\alpha_0, f_l); U_E(f_l, \alpha_0)$	$U_F(\alpha_0, f_0); U_E(f_0, \alpha_0)$	$U_F(\alpha_0, f_h); U_E(f_h, \alpha_0)$
	α_l	$U_F(\alpha_l, f_l); U_E(f_l, \alpha_l)$	$U_F(\alpha_l, f_0); U_E(f_0, \alpha_l)$	$U_F(\alpha_l, f_h); U_E(f_h, \alpha_l)$

Each combination represents, respectively, the payoff to the financier and to the entrepreneur given their relevant actions:

4.2 An incentive mechanism under each combination of players' strategies

As shown in table 4 , we can map the incentives under each combination of the financier-entrepreneur strategies.

Table 2: A maping of the suggested incentive mechanism under each combiantion of the financier-entrepreneur strategies

		Entrepreneur		
Strategy		f_l	f_0	f_h
Financier	α_h	$\gamma_h\alpha_0\bar{\pi} - \lambda_h\beta_0\underline{\pi}$	$\gamma_h\alpha_0\bar{\pi} > 0$	$\gamma_h\alpha_0\bar{\pi} - \lambda_l\beta_0\underline{\pi} > 0$
	α_0	$-\lambda_h\beta_0\underline{\pi} < 0$	0	$-\lambda_l\beta_0\underline{\pi} > 0$
	α_l	$\gamma_l\alpha_0\bar{\pi} - \lambda_h\beta_0\underline{\pi} < 0$	$\gamma_l\alpha_0\bar{\pi} < 0$	$\gamma_l\alpha_0\bar{\pi} - \lambda_l\beta_0\underline{\pi}$

This table shows some interseting implications. The incentive can be either positive or negative , except for the combination $(\alpha_0; f_0)$[7]. This means that the entrpreneur can act as an incentive giver or taker. This, can be intuitive since the entrepreneur is now a provider of funds and of work. We should also stress that this incentive, from both sides, happens only when gains occure. This ensures a fairer distribution of profit. The incentive is not given in the case of failure of the project as this can be thought of as a security against project failure. This also make the incentive giver lose more than his capital contribution. The last points are not acceptable under Islamic jurisprudence as we mentioned by Usmani [2].

Next step is to look if there exists any strictely dominant strategy to each participant in the game. Then we construct a final reduced form game.

5. RESULTS

5.1 Dominant strategies for the entrpreneur

In our game ,for the entrepreneur, for the strategy f_h to strictely dominate f_0 and f_l we need to have the following two conditions:

$$U_E(f_h, \alpha_n) > U_E(f_0, \alpha_n) \tag{14}$$

and

$$U_E(f_h, \alpha_n) > U_E(f_l, \alpha_n) \tag{15}$$

We start by comparing the payoffs to the entrepreneur under the high and minimal capital contribution strategy given any strategy of the financier. We have the following:

$$U_E(f_h, \alpha_n) = \theta_h(1-\alpha_n)\bar{\pi} + (1-\theta_h)(1-\beta_l)\underline{\pi} + \theta_h\Gamma(\alpha_n, \beta_l) - C_h$$

and

$$U_E(f_0, \alpha_n) = \theta_h(1-\alpha_n)\bar{\pi} + (1-\theta_h)(1-\beta_0)\underline{\pi} + \theta_h\Gamma(\alpha_n, \beta_0) - C_h + f_h - f_0$$

We should note that in the second equation we added the difference $f_h - f_0$. This represents the saving that the entrepreneur is making when deciding to invest less i.e investing f_h rather than f_0 taking the difference and Making use of (11) and (12) we have

$$U_E(f_h, \alpha_n) - U_E(f_0, \alpha_n) = (\beta_0 - \beta_l)(F - \underline{\pi}) > 0, since, \beta_l < \beta_0, and, \underline{\pi} < F \tag{16}$$

So strategy f_h strictely dominates f_0.

Now does strategy f_h strictely dominates f_l ? We need to prove that:

[7] we did not mention the sign of the combination $(a_h; f_l) and (a_l; f_h)$ as this depend on how high or low the profit sharing ratio and the entreprenerial contribution are negotiated

$$U_E(f_h, \alpha_n) > U_E(f_l, \alpha_n)$$

To do so, we need to compare the payoff to the entrepreneur under the high and low capital contribution strategy given any strategy of the financier. We have the following:

$$U_E(f_h, \alpha_n) = \theta_h(1-\alpha_n)\bar{\pi} + (1-\theta_h)(1-\beta_l)\underline{\pi} + \theta_h\Gamma(\alpha_n, \beta_l) - C_h$$

and

$$U_E(f_l, \alpha_n) = \theta_h(1-\alpha_n)\bar{\pi} + (1-\theta_h)(1-\beta_h)\underline{\pi} + \theta_h\Gamma(\alpha_n, \beta_l) - C_h + f_h - f_l$$

Like before, investing f_l means saving $f_h - f_l$ Making use of (11) and (12) and taking the difference we have:

$$U_E(f_h, \alpha_n) - U_E(f_l, \alpha_n) = (F + \underline{\pi})(\beta_h - \beta_l) > 0, since, (\beta_h > \beta_l) \qquad (17)$$

So from (18) and (19) the strategy f_h of the entrepreneur strictly dominates the strategies f_0 and f_l. This proves that, under a proper incentive scheme, it is always better for the entrepreneur to contribute more with a personal capital in the project.

5.2 Dominant strategy for the financier

As we noted earlier, the financier has three strategies regarding the sharing ratio $(\alpha_h; \alpha_0; \alpha_l)$

Which strategy, if any, dominates the others? To do this, we compare first the payoff of the strategy α_h with the strategy α_0.

For the strategy α_h to dominate α_0 we need to have the payoff to the financier using α_h to be superior to his payoff using α_0 no matter is the strategy of the entrepreneur i.e we need to have:

$$U_F(\alpha_h, f_n) > U_F(\alpha_0, f_n)$$

we have the following payoffs to the financier under each strategy:

$$U_F(\alpha_h, f_n) = \theta_h\alpha_h\bar{\pi} + (1-\theta_h)\beta_n\underline{\pi} - \theta_h\Gamma(\alpha_h, \beta_n) - (1+\rho)x_nF$$

and

$$U_F(\alpha_0, f_n) = \theta_h\alpha_0\bar{\pi} + (1-\theta_h)\beta_n\underline{\pi} - \theta_h\Gamma(\alpha_0, \beta_n) - (1+\rho)x_nF$$

taking the difference we have:

$$U_F(\alpha_h, f_n) - U_F(\alpha_0, f_n) = 0 \qquad (18)$$

So given a specified minimum incentive scheme the financier is indifferent between strategy α_h and strategy α_0.

Now we compare the payoffs of the strategies α_l with α_0.

For the strategy α_0 to dominate α_l we need to have the payoff to the financier using α_0 to be superior to his payoff using α_l no matter is the strategy of the entrepreneur i.e. we need to have:

$$U_F(\alpha_0, f_n) > U_F(\alpha_l, f_n)$$

We have the following payoffs to the financier under each strategy

$$U_F(\alpha_l, \boldsymbol{f_n}) = \theta_h \alpha_l \bar{\pi} + (1-\theta_h)\beta_n \underline{\pi} - \theta_h \Gamma(\alpha_l, \beta_n) - (1+\rho)x_n F$$

and

$$U_F(\alpha_0, \boldsymbol{f_n}) = \theta_h \alpha_0 \bar{\pi} + (1-\theta_h)\beta_n \underline{\pi} - \theta_h \Gamma(\alpha_0, \beta_n) - (1+\rho)x_n F$$

taking the difference we have:

$$U_F(\alpha_0, f_n) - U_F(\alpha_l, f_n) = 0 \qquad (19)$$

So from (20) and (21), we conclude that the financier is indifferent between α_0, α_l and α_h. This means that, using a suitable incentive the financier can increase or lower his sharing ratio without affecting his payoff and at the same time can induce the entrepreneur to contribute with more capital.

5.3 Reduced form game

Since the entrepreneur has one single strictly dominant strategy, f_h, our game has the following reduced form:

Table 3: A financier-entrepreneur reduced form game

		Entrepreneur
	Stratrgy	f_h
Financier	α_h	$U_F(\alpha_h, f_h); U_E(f_h, \alpha_h)$
	α_0	$U_F(\alpha_0, f_h); U_E(f_h, \alpha_0)$
	α_l	$U_F(\alpha_l, f_h); U_E(f_h, \alpha_l)$

We should note that each sharing ratio should be coupled with its relevant incentive mechanism as detailed below:

Table 4: The financier sharing ratio and its relevant entrepreneur's incentive mechanism

Sharing ratio	Relevant incentive
α_h	$\gamma_h \alpha_0 - \lambda_l \beta_0$
α_0	$-\lambda_l \beta_0$
α_l	$\gamma_l \alpha_0 - \lambda_l \beta_0$

6. DISCUSSION

6.1 Payoff stability and nash equilibrium

We can show that whatever the strategy of the financier in terms of sharing ratio and incentive mechanism each player gets the same payoff as long as the entrepreneur selects its strictly dominant strategy. So we need to prove that:

$$\begin{aligned}(U_F(\alpha_h, f_h);\, U_E(f_h, \alpha_h)) &= (U_F(\alpha_0, f_h);\, U_E(f_h, \alpha_0)) \\ &= (U_F(\alpha_l, f_h);\, U_E(f_h, \alpha_l))\end{aligned}$$

For the financier:
We have shown from (20) and (21) that given a specified minimum incentive, the financier is indifferent between $\alpha_h, \alpha_l, and, \alpha_0$.Therefore, the financier gets the same payoff under all three alternatives.

$$U_F(\alpha_h, f_h) = U_F(\alpha_0, f_h) = U_F(\alpha_l, f_h) \tag{20}$$

For the entrepreneur:
We know from (18) and (19) that the strictly dominant strategy for the entrepreneur is f_h. Given this information, what are the payoffs to the entrepreneur from the strategies of the financier α_l, α_0,α_h?To do this we need to measure the payoff to the entrepreneur under each strategy of the financier. We have then:

$$U_E(f_h, \alpha_l) = \theta_h(1-\alpha_l)\bar{\pi} + (1-\theta_h)(1-\beta_l)\underline{\pi} + \theta_h\Gamma(\alpha_l, \beta_l) - C_h$$

$$U_E(f_h, \alpha_0) = \theta_h(1-\alpha_0)\bar{\pi} + (1-\theta_h)(1-\beta_l)\underline{\pi} + \theta_h\Gamma(\alpha_0, \beta_l) - C_h$$

$$U_E(f_h, \alpha_h) = \theta_h(1-\alpha_h)\bar{\pi} + (1-\theta_h)(1-\beta_l)\underline{\pi} + \theta_h\Gamma(\alpha_h, \beta_l) - C_h$$

In order to decide on the best outcome, we subtract the payoffs from each others. We also make use of the relevant incentive at each strategy. After a simple mathematical manipulation we have:

$$U_E(f_h, \alpha_l) - U_E(f_h, \alpha_0) = 0$$

and

$$U_E(f_h, \alpha_h) - U_E(f_h, \alpha_0) = 0$$

This proves that under a specified incentive the entrepreneur gets the same payoff no matter is the strategy of the financier

$$U_E(f_h, \alpha_l) = U_E(f_h, \alpha_0) = U_E(f_h, \alpha_h) \tag{21}$$

So from (24) and (25) when the entrepreneur chooses the higher capital contribution strategy,f_h, the payoff to both participants is stabilized across the three stratgies of the financier. We have therefore three Nash equilibriums.

6.2 Higher social value

We have managed to show that the entrepreneur has a strictly dominant strategy f_h, i.e. this strategy allows the entrepreneur the biggest possible monetary payoff regardless of the strategy of the financier.
Does this strictly dominant strategy of the entrepreneur, however, allow the financier to get the maximum payoff compared to the rest of the strategies of the entrepreneur? To answer this question, we compare the payoff to the financier under each strategy of the entrepreneur (f_h ,f_0 ,f_l).

Lemma 1 under a specified incentive mechanism, the financier gets the highest possible payoff when the entrepreneur chooses a higher capital contribution strategy f_h.

Proof See the appendix

Table 5: Representation of the highest payoff to the financier given the strategies of the entrepreneur

Strategy of the Financier	Payoffs to be compared	highest payoff
A_h	$U_F(\alpha_h; f_h); U_F(\alpha_h; f_0); U_F(\alpha_h; f_l)$	$U_F(\alpha_h; f_h)$
α_0	$U_F(\alpha_0; f_h); U_F(\alpha_0; f_0); U_F(\alpha_0; f_l)$	$U_F(\alpha_0; f_h)$
A_l	$U_F(\alpha_l; f_h); U_F(\alpha_l; f_0); U_F(\alpha_l; f_l)$	$U_F(\alpha_l; f_h)$

We notice that the highest payoff to the financier happens when the entrepreneur chooses a higher contribution strategy f_h. We should also remember from (18) and (19) that strategy f_h of the agent dominates the strategies f_l and f_0. Also we have shown from (20) and (21) that given a specified minimum incentive, the financier is indifferent between $\alpha_0, \alpha_h, \alpha_l$.

We have also shown from (25)that the entrepreneur gets the same payoff when he chooses a higher capital contribution strategy. Without an incentive, the entrepreneur might be induced to exert a low effort strategy resulting in a gain to him but a loss or at least lower profit to the financier. A win to a party at the expense of a loss to another party, referred to as a zero sum game in the literature, is not an acceptable form of an Islamic musharakah contracts [14]. Therefore, we can deduce that the entrepreneur's strategy f_h matched with a specific incentive mechanism to any strategy of the financier, can result in the highest social value SV^h:

$$\begin{aligned} SV^h &= U_F(\alpha_h, f_h) + U_E(f_h, \alpha_h) \\ &= U_F(\alpha_0, f_h) + U_E(f_h, \alpha_0) \\ &= U_F(\alpha_l, f_h) + U_E(f_h, \alpha_l) \end{aligned}$$

This result further emphasizes some of the main objectives of the Islamic jurispru-

dence (M'qasid al shari'ah)[8] by encouraging:

- more investing to saving as this stimulates the economic activity of a society.
- profit making through undertaking more risk via higher investment
- high entrepreneurial effort and perfection of work (referred to as Al Itkane)

7. CONCLUSION

In this research we have tried to reduce the moral hazard problem using a specified incentive mechanism. We have started from a bottom line reference that specifies a minimum capital contribution of the entrepreneur for no deviation purpose. This is coupled with a minimum break even profit sharing ratio for the financier. We proceeded by truncating the minimum sharing ratio and the entrepreneur's capital using some designated allowances. We have shown that higher contribution from the entrepreneur can dominant his minimum and lower capital contribution strategies. In this case it is shown that, under a suitable incentive mechanism, the entrepreneur can be induced to contribute more with capital. It is also shown that the entrepreneur gets the same payoff under his dominant strategy no matter is the strategy of the financier. The incentive mechanism we have provided resulted in the highest possible social value. A possible extension of this proposed model is to test its validity in a repeated game over multiple periods.

8. APPENDIX A. PROOF OF LEMMA 1

We need to proove that no matter is the startegy of the financier α_n, the highest payoff to the financier happens when the entrepreneur chooses the high capital contribution option f_h.
First, we compare the payoff to the financier under f_h and f_0:

$$U_F(\alpha_n, f_h) = \theta_h \alpha_n \bar{\pi} + (1-\theta_h)\beta_l \underline{\pi} - \theta_h \Gamma(\alpha_n, \beta_l) - (1+\rho)(F - f_h)$$

and

$$U_F(\alpha_n, f_0) = \theta_h \alpha_n \bar{\pi} + (1-\theta_h)\beta_0 \underline{\pi} - \theta_h \Gamma(\alpha_n, \beta_0) - (1+\rho)(F - f_0)$$

knowing that

$$\beta_l = \frac{F - f_h}{F}, \text{ and, } \beta_0 = \frac{F - f_0}{F}$$

taking the difference we have:

$$U_F(\alpha_n, f_h) - U_F(\alpha_n, f_0) = \underline{\pi}(\beta_l - \beta_0) - (1-\theta_h) - (1+\rho)(f_0 - f_h) + \underline{\pi}\beta_0 \lambda_l$$

[8]Dr Mohamed Karrat, Professor of Islamic Jursprudence in Al Qarawiyyin University(Morocco), Associate partner at Al Maali Consulting (Islamic finance consulting firm ,Morocco), member of the 'International Islamic Centre for Reconciliation and Arbitrage (IICRA)'

simplifying we get:

$$U_F(\alpha_n, f_h) - U_F(\alpha_n, f_0) = \beta_0\lambda_l[\theta_h\underline{\pi} + (1-\theta_h)\underline{\pi} - F(1+\rho)]$$

which is simply:

$$U_F(\alpha_n, f_h) - U_F(\alpha_n, f_h) = \beta_0\lambda_l(\underline{\pi} - F(1+\rho))$$

which is positive because

$$\lambda_l < 0, and, \underline{\pi} < F(1+\rho)$$

therefore

$$U_F(\alpha_n, f_h) > U_F(\alpha_n, f_0) \quad (A.1)$$

So it is benificial to the financier that the entrepreneur chooses a high capital contribution than if the entrepreneur chosses the minimal capital contribution. Second, we compare the payoff to the financier under f_h and f_l. We have the following payoffs to the financier under each strategy:

$$U_F(\alpha_n, \boldsymbol{f_h}) = \theta_h\alpha_n\bar{\pi} + (1-\theta_h)\beta_l\underline{\pi} - \theta_h\Gamma(\alpha_n, \beta_l) - (1+\rho)(F - \boldsymbol{f_h})$$

and

$$U_F(\alpha_n, \boldsymbol{f_l}) = \theta_h\alpha_{\boldsymbol{f}}\bar{\pi} + (1-\theta_h)\beta_l\underline{\pi} - \theta_h\Gamma(\alpha_n, \beta_l) - (1+\rho)(F - \boldsymbol{f_l})$$

knowing that

$$\beta_l = \frac{F-f_l}{F}, and, \beta_h = \frac{F-f_l}{F}$$

taking the difference we have:

$$U_F(\alpha_n, f_h) - U_F(\alpha_n, f_l) = \underline{\pi}(\beta_l - \beta_0) - (1-\theta_h) - (1+\rho)(f_l - f_h) + \underline{\pi}\beta_0(\lambda_l - \lambda_h)$$

simplifying we get

$$U_F(\alpha_n, f_h) - U_F(\alpha_n, f_l) = \beta_0(\lambda_l - \lambda_h)[\theta_h\underline{\pi} + (1-\theta_h)\underline{\pi} - F(1+\rho)]$$

which is simply

$$U_F(\alpha_n, f_h) - U_F(\alpha_n, f_l) = \beta_0(\lambda_l - \lambda_h)(\underline{\pi} - F(1+\rho))$$

the last term is positive since

$$\lambda_l < \lambda_h, and, \underline{\pi} < F(1+\rho)$$

therfore

$$U_F(\alpha_n, f_h) > U_F(\alpha_n, f_l) \quad (A.2)$$

So, it is benificial to the financier that the entrepreneur chooses a high capital contribution than if the entrepreneur chosses the lower capital contribution.

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