

Investigating the Course of Recovery in High Risk Suicide using Power Spectral Density

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ABSTRACT— *This pilot study attempts to address the question of whether there is a statistically significant change in patients' vocal characteristics as they progress from the initial recording session (labeled as high risk suicidal) to the second and third recording session that was collected a few days after receiving treatments. Interview speech and reading speech were collected from five male patients and eight female patients. Using the Power Spectral Density (PSD) feature, the normalized Euclidean distance from each vector to the separating hyperplane was measured in order to capture the progression or regression of patients after three treatment sessions. One-tailed paired sample t-test demonstrates statistically significant difference between the first and second session which represents improvement. In some cases, the first and third session also demonstrates statistically significant difference which shows that patients does not experiencing relapse.*

Keywords— Speech, suicide, depression, severity

1. INTRODUCTION

Speech is a rich source of information that contains hidden messages such as emotions, mental state and attitude of the speaker. Various studies have recognized the relationship between human speech and an individual's psychological state [2] - [17]. One manner in which this relationship may be applied is in the prediction and identification of a depressed patient with and without elevated risk of suicidal behavior. The primary method of psychiatric assessment requires extensive efforts by the clinician which involves gathering comprehensive information about the patient and answering a series of questions in the presence of a trained clinician. This comprehensive information is collected on the basis of organizing the risk factors for suicide. These factors are related to psychological milieu (life events, environmental factors and medical illnesses), presence of psychiatric disorders, biological factors, health records, family history and history of previous suicide attempts, if exists [1].

Diagnosis of patient with imminent suicidal risk is a complex task but it is crucial in order to select the appropriate treatment and safety management. Patients are usually given antianxiety, antipsychotic and sleep medications. Apart from that, they are segregated from seclusion and are encouraged to get involve in psychosocial interventions such as group therapy and socializing with other patients in order to reduce anxiety. Traditionally, treatment for high risk suicidal patient mainly focused on the psychiatric disorder that dominates the diagnosis. For example, high risk suicidal patient with major depression is treated for their depressive disorder and treatment for psychotic patients with the presence of suicidal behavior is directed at diminishing the expressions of the psychosis [18]. Psychological screening by means of human speech may be used as a secondary tool in helping clinicians to recognize psychiatric disorders and prevent misdiagnosis, particularly in a life threatening case.

The psychological state of a patient is expected to improve after receiving treatments. In certain cases, their condition improves rapidly after admission but in most cases, they often recover at a gradual pace. Development in the sense that the patient is no longer considered being acute high risk suicidal even though the condition still prolonged after the treatment. There are no laboratory tests or sophisticated diagnostic instruments that are available for psychiatrist to

confirm whether the given treatments and medications have a positive effect on the patients. In addition, current environment imposes difficult challenges on psychiatrist. Patients that are facing severe risk of suicidal are admitted into inpatient unit for a short length of stay where both treatment time and duration are made limited by insurance requirements [19]. Due to some limitations and requirements, discharged patients that have gone through short-term treatments may experience an illusion effect of the medication where they seem to look better but in reality, may still be at an imminent risk of committing suicide. Because of the complexity, erroneous improvement is merely impossible to distinguish from the actual improvement.

The primary goal of this study is to model the course of recovery and observe significant improvement in patients that were admitted for suicidal predisposition by a qualitative representation of changes in their vocal characteristic. Significant changes in the movement of a particular speech measure known as the Power Spectral Density (PSD) were observed as they progress from the first recording session (labeled as high risk suicidal) to the next recording sessions (condition made unknown to researchers) that were collected a few days after receiving treatments.

2. PREVIOUS WORK

A number of studies performed by [20]-[23] observed significant changes in the prominent features of speaking behavior and voice characteristics over the time course recovery from depression. These features comprise of speech rate, pause duration, vocal timing, fundamental frequency, energy and pitch variability. The analyses also revealed correlation between the changes in acoustic measurements and well-known clinical rating scores. Among the speech features that have been examined in the field of psychological disorders are the Power Spectral Density (PSD), Mel-Frequency Cepstral Coefficient (MFCC), fundamental frequency (F0), formant, glottal flow spectrum, transition parameters and interval probability density function [2]-[7]. It has been reported that the PSD appears to be a distinguishing vocal feature for discriminating between high risk suicidal and major depressed patients [3], [5]-[7].

An early research by Hardy [20] analyzed the significant changes that occur in patients' vocal characteristics before the onset of treatment and after the final evaluation made within 48 hours of discharge or change of medication in major depressed patients. Speech Pause Time (SPT) improved significantly between the two periods but not Phonation Time (PT) which was also demonstrated in [21].

In a longitudinal study performed by Ellgring [22], correlation between voice parameters and the Voice Analogue Scale for Subjective Well-being (VAS) was computed using standardized interviews for depression. Analyses were chosen from samples obtained within five days of admission and after 50 days of treatment. Only for female subjects, significant difference were observed between depressed and recovered phases for the acoustic measures of mean fundamental frequency (MF0), speech rate (SR) and mean pause duration (MPD). They were also found to be significantly correlated with the VAS.

Another investigation performed by Stasen [23] was aimed at evaluating the relationship between psychopathology and speech characteristics in hospitalized depressive patients throughout the first 2 weeks of antidepressant treatment. They reported that over time, a single parameter only correlates with a certain aspects of individual courses of affective disorders, thus going against the simple approach which applies in general. Later on, Cannizzaro [24] attempted to replicate the results reported by Stasen [23] with an exception of using recordings that were made in less than ideal conditions and using relevant voice acoustical metrics from samples of spontaneous speech (interview). Three acoustic measures of speaking rate, percent pause time and pitch variation were extracted from five male and two female recordings. The results demonstrated the ability of speech features to objectively identify severity of depression. However, the recordings were independent and performed at a particular point in time. The Pearson product-moment correlation analysis revealed a significant negative correlation between speaking rate and the HAMD. Even though HAMD scores demonstrated large negative correlation with pitch variation and moderate significance with percent pause time, neither achieved statistically significant, possibly due to the small sample size.

Two separate studies were performed by Mundt [25, 26] using different depression severity measures and different methods of assessment for 35 depressed and non-depressed patients. In the initial study, pitch variability across the second harmonic and vocal acoustics relating to pauses and vocalization in speech were significantly correlated with the Interactive Voice Response (IVR) HAMD scores when measured using Pearson's correlation. Also reported, total pause time during automatic speech where patients were asked to read from a standardized passage, counting from 1 to 20 and pronouncing vowels for 5s reveal stronger correlation, whereas vocalization-pause-ratio reveals better correlation during free speech. In the second study, logistic regression analyses were conducted on acoustics measures of speech in order to classify between 105 patients who responded to treatment and those who did not. Among the seven acoustic measures that were found to be significantly correlated with depression severity measures in the previous study, six of them were also found to be significant in this study. Results across both studies were consistent thus provide strong evidence for the value of vocal acoustic features as an indicator of depression severity.

3. DATABASE

All recording sessions were conducted at the Vanderbilt Emergency Department and Psychiatric Hospital. The first recording session were taken from new case patients. Patients under the influence of alcohol, toxicity or experiencing respiratory problems such as shortness of breath were excluded. All recordings were made in a standard, empty psychiatric interview room without the benefit of soundproof or acoustically ideal environment, mimicking the real-world clinical environment. Group assignment was made according to assessment made by experienced clinicians using the Beck Depression Inventory (BDI-II), MINI International Neuropsychiatric Interview, Pierce Suicide Intent Scale (SIS) and Hamilton Depression Rating Scale (HAMD). A number of patients attended three recording sessions, some had two recordings and the rest only had one session. The second and third recordings were collected a few days after receiving treatments. Entry criteria restricted inclusion to patients who were labeled as high risk suicidal during their first recording session. The state of the patient during the next recording sessions were made blind to the acoustic engineering researchers and categorized as *others*.

In this case, *other* may or may not indicate that the patient is no longer considered high risk. All speech samples were digitized using a 32-bit analog to digital converter at 44.1 kHz sampling rate for both databases. Audio acquisitions were made using a portable high-quality field recorder, a TASCAM DR-1, with a frequency response of 40Hz to 20kHz, Samsung Q40 laptop with Intel Core i5 2.4GHz 4G memory and Windows 7.

Two types of speech samples that were collected from male and female patients are called the interview speech and the reading speech. For the interview speech, patients engaged in an interview with a clinician answering a series of questions such as feelings of guilt, thoughts of suicide, interest level, presence of anxiety and somatic complaints. For the reading speech, patients were asked to read from a standardized “rainbow passage” which contains every sound in the English language and is considered to be phonetically balanced with the ratios of assorted phonemes similar to the ones in normal speech [27]. Table 4.1 displays the information on the number of patients and recordings that were used in this analysis. However, for this study, only patients with three recording sessions were used.

Table 1: Number of patients and recordings

Database B	Male Interview	Male Reading	Female Interview	Female Reading
Total number of patients	8	8	14	13
Total number of recordings	19	18	34	32
Total number of 20 second segments	473	49	479	78
• Number of patients with three recordings	5	4	8	7
• Number of patients with two recordings	1	2	4	5
• Number of patients with one recording	2	2	2	1
Number of recordings labeled as high risk (HR)	8	7	13	12

4. METHODOLOGY

4.1 Distance Measurement from the Separating Hyperplane

The decision hypersurface in the l -dimensional space is a hyperplane that is represented by

$$g_{12}(x) = w^T x + w_o \quad (1)$$

where,

$$w = \Sigma^{-1}(\mu_1 - \mu_2) \quad (2)$$

$$w_o = \frac{1}{2}(\mu_2^T \Sigma^{-1} \mu_2 - \mu_2^T \Sigma^{-1} \mu_1) \quad (3)$$

w is known as the weight vector and w_o as the threshold. On one side of the hyperplane, $g_{12}(x) > 0(+)$ for vectors that are classified as class 1 takes the positive values and $g_{12}(x) < 0(-)$ for class 2 takes the negative values [23]. A geometry illustration of the decision hyperplane is shown in figure 3. The normalize Euclidean distance from each vector to the decision hyperplane can be represented by,

$$G_{norm} = \frac{g_{12}(x)}{\sqrt{w_1^2 + w_2^2}} \quad (4)$$

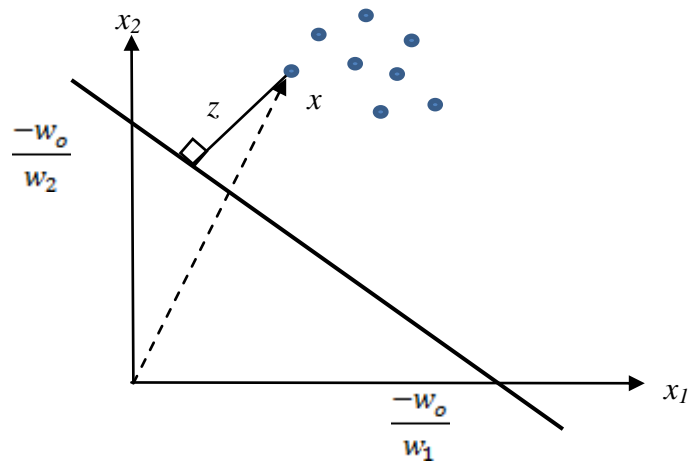


Figure 3: Geometry of the decision hyperplane[8]

4.2 Feature Extraction

The analysis was performed separately for both male and female patients using their interview and reading speech samples. Only the voiced speech segments were collected using the voice/unvoiced/silence detection [4] and the collected voiced signals were then split into 20-second segments. Energy distribution for four equal 500Hz bands (band1:0-500Hz, band2:501-1000Hz, band3:1001-1500Hz, band4:1501-2000Hz) were extracted from each 20 second segments as demonstrated in [7, 8]. Band4 was removed from the analysis because it contains information that is linear dependent on the other three spectral energy bands. Patients will have varying numbers of 20-second segments depending on the length of the recordings. Finally, we measured the distance for each vector of the 20-second segment to the separating hyperplane and represent the measured distance as GNorm.

4.3 Analysis of Significant Measures

Two-sample statistical analysis was performed separately on the estimated GNorm distance to identify the statistical significance between HR(session 1)-‘others’(session 2), HR(session 1)-‘others’(session 3) and ‘others’(session 2)-‘others’(session 3). The analysis was done on all combinations of 0-1500Hz (4 bands) for both interview and reading samples. Each patient had three recording sessions and each recording session have varying number of 20 second segments. All patients were identified to be in the high risk suicidal during session 1. After receiving treatment, medication and being hospitalized for a period of time, patients continued for their second session and then had their last session before getting discharged. In order to keep the same number of observations across sessions for every patient, the mean of the estimated GNorm for each session was calculated. The significance of the group means and standard deviation were obtained using a paired two sample for means t-test analysis with one tailed p-value due to the expectation of one mean to be larger than the other.

5. RESULTS AND DISCUSSION

5.1 Results for Statistical Analysis and Significant Differences

Figure 1 displays an example plot of using 4PSD band 2 with respect to 4PSD band 3 obtained from one of the female patient’s interview. The normalized Euclidean distance (GNorm) is calculated based on the distance of each vector to the separating hyperplane with one side being more positive (high severity) and the other side being more negative (low severity). By observation, there is a general progression of moving in the negative direction along the normal line as the sessions progressed which equates to getting better.

For each combination of 4PSD bands, we calculated the mean normalized Euclidean distance of patients in four categories obtained from sessions 1 to 3 as shown in Figure 2. Similarly, the general trend that was observed in Figure 1 can also be seen here in Figure 2. The mean values demonstrate a general trend of progression in the negative direction during the second session and small progression or regression towards the hyperplane was observed occurring from the second session to the third session.

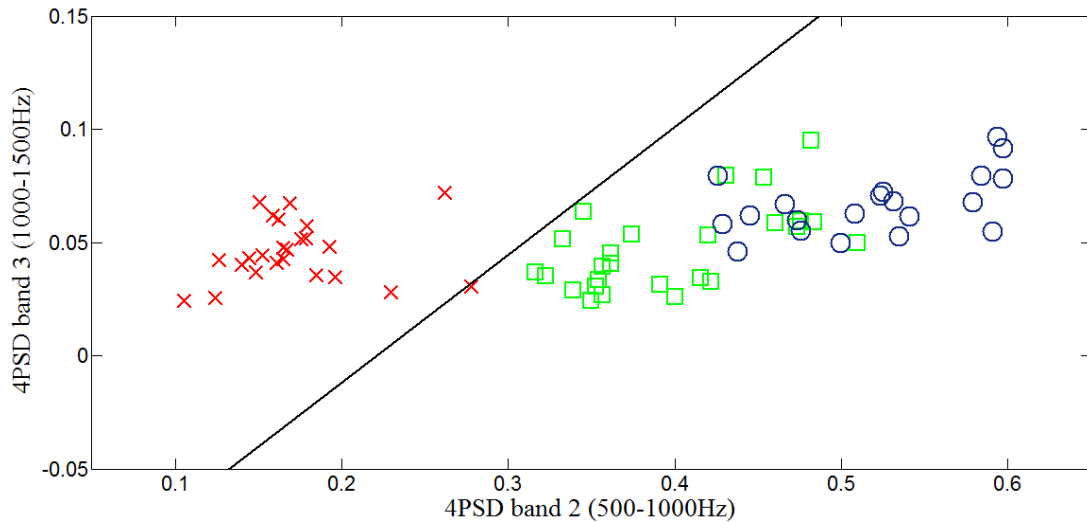


Figure 1: Example plot of 20 second segment vectors from one of the female patient’s interview using 4PSD band 2 with respect to 4PSD band 3 for session1 (‘Red_x’), session2 (‘Green_box’) and session3 (‘Blue_circle’).

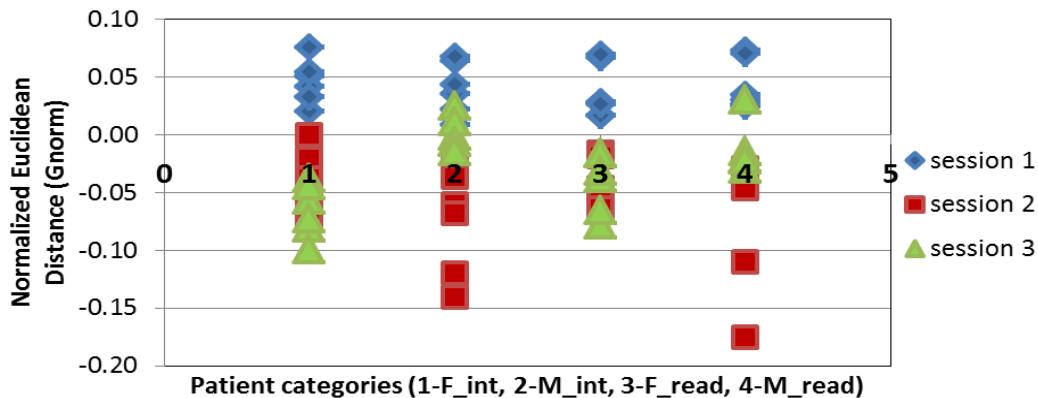


Figure 2: Plot of the mean normalized Euclidean distance to the separating hyperplane during session 1, 2, and 3 for each patient based on the female interview (1), male interview (2), female reading (3) and male reading (4) categories.

A one tailed paired sample t-test was performed on the measurements because of the same population but at different times and conditions with the expected mean of one group to be larger than the other. Results are demonstrated in Table 2. According to [28], an appropriate number of samples per estimated feature are of the 5:1 ratio in order for the results to be generalizable. Based on the number of patients in Table 1, a maximum of two features are considered adequate for male interview, one feature for male reading, three features for female interview and two features for female reading.

Mean difference between session 1 and session 2 (s1-s2) except for male interview using band 3 suggest that 95% to 99% confidence they are indeed distinctive and thus, providing evidence that there is positive changes in patient’s condition after receiving treatment, medication or being hospitalized. Except for male interview, mean difference between the first and third session for the other three categories suggest a 95% to 99% confidence that they are indeed separable, thus indicating that patients were not experiencing relapse. For all four categories of samples, there were no significant differences found between second and third session indicating that patient’s condition were not noticeably improving or deteriorating.

According to the assessment made by psychiatrist, patients were labeled as high risk during the first session and the other two sessions were kept unknown to the researcher. With the type of data in hand, the aim was not so much in making a clear decision between pairwise group of HR-DEP, DEP-REM or HR-REM. Instead, the hyperplane was used as a 1-D scale, measuring progression versus regression. By observation, patients seem to look better after each session and as shown by the results in this study, their condition does seem to correlate in the measured Euclidean distance. In addition, the significance of improvement and deterioration in their condition were represented in the statistical t-test analysis.

Table 2: Calculated one-tailed paired sample t-test and significance p-value for measuring the mean difference between each pairwise session for female interview (FI), male interview (MI), female reading (FR) and male reading (MR) using all possible combinations of 4 PSD bands. Boxes highlighted in yellow demonstrate p-values that are statistically significant with less than 0.01 and in blue with a significant difference of less than 0.05

Significance (p)	session number											
	s1-s2				s1-s3				s2-s3			
	FI	MI	FR	MR	FI	MI	FR	MR	FI	MI	FR	MR
4 PSD band 1	0.0073	0.0033	0.0328	0.0023	0.0551	0.2086	0.0145	0.0153	0.5902	0.9741	0.4068	0.9983
4 PSD band 2	0.0025	0.0036	0.0155	0.0398	0.0138	0.1099	0.0101	0.0166	0.5422	0.9718	0.5815	0.7783
4 PSD band 3	0.0188	0.0536	0.0106	0.0485	0.0155	0.5940	0.0187	0.0871	0.0815	0.9171	0.7187	0.5951
4 PSD band 1:2	0.0044	0.0082	0.0079	0.0193	0.0365	0.1281	0.0038	0.0498	0.7370	0.9042	0.4240	0.5951
4 PSD band 1:3	0.0031	0.0164	0.0228	0.0291	0.0070	0.1362	0.0339	0.0007	0.1792	0.8489	0.7216	0.8279
4 PSD band 2:3	0.0010	0.0073	0.0020	0.0141	0.0015	0.1164	0.0022	0.0142	0.2518	0.9268	0.5122	0.8291

Smaller bands combination can be explored for the statistical analysis besides the 4 PSD band combinations (i.e. using 6 PSD band combinations which divide into 333Hz bands and 8 PSD band combinations which divide into 250Hz bands). But since we were able to demonstrate a significant difference using only combinations from 4 PSD bands, we did not proceed with the analysis on smaller band combinations. Plus, the core analysis was studying the progression or regression along the normal line between pairwise sessions. Choosing other combination of bands only change the threshold of where zero is but it does not change the direction of the ‘movement’.

A linear separation was chosen as the hyperplane because it gives a unique normal line. The direction orthogonal to the hyperplane can be used to establish a direction indicating whether patients are getting better or worse as they go through the normal line. As illustrated in the results, they are strikingly moving in the same direction (in the negative direction) during the second session with the exception of a few cases.

Even though most patients did not display a significant improvement when going from the second to third session, there is at least a significant of improvement from the first session (HR) to the other two sessions with the exception of a few cases. The fact that the second and third session did not demonstrate a significant difference from each other shows that more or less, there is only minor progression or regression along the normal line. Importantly, this provides evidence that their condition does not return back to being high risk even after the third session.

The hyperplane was chosen in a way that it maximizes the number of HR vectors on one side and the rest on the other. By doing this, it guarantees that the second and third session will have a smaller distance than the first session. This is simply due to the orientation of the hyperplane. But it does not guarantee that the third session will be more negative than the second session and the possibility of the existence of outlier in either direction was neglected. The fact that they are indeed separable in most cases definitely shows that overtime with treatment, their distances to the hyperplane do tend to move in one direction.

6 CONCLUSION

The statistical analysis performed demonstrated that PSD features exhibit distinctive characteristics that could measure the progression of the psychological severity after the patient has been hospitalized and received treatments. Results demonstrate interesting trends where the features are separable and in most cases, progressing in one direction instead of being clustered up in a cloud which will then destroy the argument. Importantly, this method is unobtrusive and does not introduce any risk factors because it only uses the patient’s recorded speech. Considering the difficulty to obtain the high risk suicidal speech database, more effort could be put in to collecting more data on suicidal predisposition speech. Larger studies of vocal characteristics must be done in order to strongly demonstrate the correlation between psychological severity and vocal measurements.

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