Pricing of Waad Bil Mourabaha

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ABSTRACT---- Conventional contracts are limited in the point of view of ethics [5]. To meet the need of increasing financial market, we use engineering while respecting the principles of the Islamic Shariah¹, which contains the golden rules of ethics.

In this article we will model an alternative to the derivatives: especially the Call Option using the Waad. Waad is a promise of sale or purchase of goods, declared in a unilateral way. In spite of the unusual use of the Waad's contract because of the disagreement between some schools of Islamic law, this contract allows us to propose sophisticated and interesting contract: Waad Bil Mourabaha.

Mourabaha is a Kind of sale where the seller expressly mentions the cost he has incurred on goods for sale and sells it to another person by adding some profit or mark-up thereon which is known to the buyer. Waad Bil Mourabaha's contract provides the right to buy goods at a future date with a contract Mourabaha, but we can't sell this contract. Waad Bil Mourabaha can also be used to limit the risk; in

this paper we will study the possibility of Islamic hedging.

For the pricing of this contract, we will use the Black and Scholes model and the approach proposed by A.Jobst[6], according to S.Oumrana, R.Aboulaich[10] and S.Bouarfaoui[3] we will propose a new approach using a dynamic return rate of goods.

Keywords--- Islamic Finance, Shariah Compliance, Derivatives, Call Option, Black and Scholes Model, risks, hedging.

1 INTRODUCTION

According to Boubkeur $IJDIR^2$, the Islamic Finance is not only compatible with the Islamic ethics, but also with the ethics as a whole. For him, it is a mean to reconcile the financial world with the real economy.

Indeed, far from the excesses of more and more immoral financial world, the Islamic Finance prefers to focus on models of participative financing [5].

The value of the international derivatives is amounted to 693000^3 billion dollars, it means that ten times of the world's GDP(Gross domestic product) is invested without respecting the Shariah.

Islamic finance is based on five fundamental principles, we can say that a product is shariah compliant if it respects these five rules. The first rule is the prohibition of Haram: it means prohibition of everything which's baned by the Islamic religion (porc, alcohol, pornography..).

The second important rule in this study is the prohibition of Riba: the ban of the interest rate.

Riba is an increase of the capital without service or work. In other words, it is the money lent against remuneration. For example, the usurer can live without working, with the only pension of his money. The objective of the ban of Riba is to place relationships between people on the register of transparency, equity, and humanity. It is hence a question of refusing quite typical exploitation and of encouraging a fair trade.

The third rule to respect is the prohibition of Gharar, which's uncertainty or randomness.

The fourth rule is the obligation of Sharing profits and Risk, and the last rule is the asset backing: all the assets used in Islamic transactions must be tangible.⁴

¹ Islamic law

² Manager of the company IFAAS, Islamic Finance Advisory and Insurances Advisory

³ The international Bank for settlements 2013

⁴ Anything that has long-term physical existence: equipment, machinery, plant, property...

The majority of Islamic schools accept that Waad is Shariah compliant. In this work we will propose an innovative contract which can substitute the Call Option, assuming that the interest is null and using a rate of return. Finally, referring to the article S.Al-Suwailem[1] and [2] we will study the possibility to use the Waad Bil Mourabaha for Islamic hedging.

2 SHARIAH COMPLIANCE OF THE WAAD

According to the articles [11] and [9], the Call option can be seen as analogue to Khiyar Al-Shart, and the premium is considered as monetary compensation: Daman. This proposal was criticized by Islamic scholars, the Khiyar can change the details of a contract and not the contract itself.

For the Urboun, which is a particular alternative of the Call option, the premium is supposed to be a part of goods's price. In our research, we will model Waad Bil Mourabaha, an alternative to the Call Option, where the premium is totally independent from goods's price.

Using the articles [4] and [7], the opinion of the majority of Islamic schools, especially Hanafisme, Chafiisme and Hanbalisme is that the Waad made by a person to another one is religiously compulsory, however it is not legally compulsory. Malikites judge that the promise becomes a legal obligation (legally compulsory) if it causes damage in the part witch be promised.

In this regard, guidance may be taken from the verdict issued by OIC Islamic Fiqh Academy in its Fifth Round, held at Kuwait from 10th to 15th December 1988.

The OIC(Organization of Islamic Conference) Islamic Fiqh Academy stated that a Waad (Promise) in commercial transactions is binding on the promisor with the following conditions:

- The Waad should be a one-sided, must have caused the promisee⁵ to incur a liability and should not be taken as the concluded sale.
- If the promisee is to purchase something, the actual sale must take place at the appointed time throug the exchange of ofier and acceptance.
- If the promisor⁶ backs out of his Waad, the court may force him to purchase the commodity, or pay actual damages to the seller, and those latter damages will include his actual monetary loss suffered by him.

In this study we consider that the Waad is a religiously and legally commitment that includes the payment of the Daman, which is a compensation of damages caused to the seller in case the Waad is cancelled.

3 WAAD BIL MOURABAHA

3.1 Definition

Waad Bil Mourabaha is an agreement between the seller and the buyer wishing to acquire goods at the future, with a given price.

Steps of occurring Waad Bil Mourabaha contract are as follows:

Step 1: The buyer who wishes to acquire goods later at t=T, makes a Waad to the seller upon which is going to buy goods. The price of goods (P) is fixed by mutual agreement at the moment t=0 and it is calculated using the modified Black and Scholes Model. The Waad implies a payment, the price of the Daman (guarantee). Hence, the seller makes commitment to sell goods at the moment t=T. Goods which are in the possession of the seller are Amana(trust, consignment), and if it is not available to the buyer at maturity it's considered as Tafrit(negligence).

In this paper we will consider the perfect case: goods are always available.

• <u>Step 2:</u> At maturity if the buyer decides to execute the Waad, the seller pay him back the premium, and they conclude the Mourabaha with the price fixed on the Waad. If the buyer decides not to execute theWaad, the seller keeps the premium as a compensation of the losses, it's considered as Taaddi(violation, transgression).

To assume the safety of this transaction, we might introduce a neutral third part. For example, an Islamic Bank that which will be responsible for the transition of the Daman between the buyer and the seller.

⁵ The person to whom a promise is made, the person to whom the Waad was done

⁶ the person who makes a promise, the person who makes the Waad

3.2 Comparison of Waad Bil Mourabaha and Call Option

Waad Bil Mourabaha and the European Call Option are close in terms of functioning because in both cases: the buyer can refuse at maturity to practice his right. However, there are important differences: The Call is an optional contract, in opposit of theWaad which is an obligation. The idea of the Option is to have the choice at maturity, and this choice costs the Call's premium, but the Waad is concluded to make goods available at a precise time. It's a kind of reservation, and the Daman(the premium) is paid if the Waad is not respected. The decision of the buyer of the Call Option is taken at maturity but the decision of the promisor is taken at the moment of making the Waad.

In Waad Bil Mourabaha, it is about a rate of return and not about a rate of interest. The Call Option concerns a tangible or intangible asset, while Waad Bil Mourabaha is absolutely concerned with the very precise goods and thees goods have to be Shariah compliant⁷. The only purpose of the Waad is to acquire goods, it can't neither be sold nor bought. Contrary to the option which can be used for speculation.

In the following sections we will use the Call option, improve it, to model Waad Bil Mourabaha.

4 MODELING WAAD BIL MOURABAHA

In his work[6], A.Jobst assumes that the Islamic rate of interest is constant and he uses the Black and Scholes model for pricing the Islamic derivatives.

In this work, we will adapt the Black and Scholes model toWaad Bil Mourabaha contract, assuming that the rate of interest is null and the rate of return R can be computed by the model. We refer to [10] for a similar work about Al Arboun, and to [3] where S. Bouarfaoui assumes that the rate of return is constant.

The hypothesis that R isn't constant makes the pricing more difficult.

4.1 Assumptions

4.1.1 Black and Scholes Model assumptions

- Assumptions on the asset using [12]: The option is European: it can only be exercised at expiration T. Random walk: the instantaneous log returns of the stock price is an infinitesimal random walk with drift, it is a geometric Brownian motion, and we will assume constant drift and volatility. The stock does not pay a dividend. Each little is perfectly divisible.
- Assumptions on the market: There is no arbitrage opportunity⁸. The market operates continuously. The above transactions do not incur any fees or costs⁹.

4.1.2 Shariah conformity assumptions

The underlying asset must be Shariah compliant. The underlying asset must be tangible, identifiable and owned by the owner of the asset at the contract's settlement. The rate of interest must be equal to zero.

4.2 Black and Scholes Model

In their work [10], and for the pricing of Al Arboun the autors supposed that the rate of return is constant. In this paper, we propose to compute the price of goods's purchase and the variable rate of return. We will use the following datas: S_0 the good's sale price, P the good's purchase price using Mourabaha, T the time remaining before expiration (maturity), V the contract's premium (Daman), R the rate of return and σ the volatility. Waad Bil Mourabaha's Payoff is:

$$Max\left(S-P;0\right) \tag{1}$$

The purchase price of the good is equal to the sale price of the which we subtract the Daman:

$$V = S - P \tag{2}$$

⁷ follows the principles of the Islamic sharia

⁸ There is no way to make a riskless profit

⁹ frictionless market

The price of the underlying asset is:

$$dS = Sdt + Sdw \tag{3}$$

This asset's price follows a Geometric Brownian motion with constant drift and volatility, and dW is simple Brownian. In the case of Waad Bil Mourabaha, the initial condition when t=T is given by:

$$V = Max (S - P; 0)$$
(4)

In the conventional European option, the premium of the contract is expressed by:

$$V(t,S) = S0 e^{-\mu 1t} N(d1) - P e^{-\mu 2t} N(d2)$$
(5)

 μ_1 is the interest's rate, μ_2 is the return's Rate.

The distances to default d1 and d2, using [8] can be represented by:

$$d2 = d1 - \sigma \sqrt{t} \tag{6}$$

N(.) the cumulative standard normal distribution function.

We will adapt the Black and Scholes Model to the previous principles and assumptions. In the following we consider that:

$$\mu 1 = 0; \ \mu 2 = R$$
 (7)

Then we obtain the new equation:

$$V(t,S) = S0 N(d1) - P e^{-Rt} N(d2)$$
 (8)

We notice that in equation (8), we have two Unknown parameters V and R. We introduce another expression using the actualization of the rate of return.

We remind the general formulas of actualization:

$$P = (S0 - V) (1 + R)^{t}$$
(9)

where P and (S0 - V) are the future value and the present value of goods respectively. (9)

Using (8) and (9) the rate of return is given by:

$$R = \sqrt[t]{\frac{P}{S0 - V} - 1} \tag{10}$$

Our purpose is to compute R and V using the equations (8) and (10).

4.3 Resolution

In the following we will compute P = P(S; t) the price of purchase of goods and R = R(S; t) the rate of return. Using (2) and (8) we have:

$$V(t,S) = S0 - P = S0 N(d1) - P e^{-Rt} N(d2)$$
(11)

$$S0 - S0 N(d1) = P - P e^{-Rt} N(d2)$$
 (12)

We obtain the expression of the price P:

$$P = \frac{S0(1 - N(d1))}{(1 - e^{-Rt}N(d2))}$$
(13)

The expressions of distances to default are given by:

(14)

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$$d1 = \frac{\left[(\ln P/S0) + (-R - 1/2\sigma^{2})t\right]}{\sigma\sqrt{t}}$$
$$d2 = \frac{\left[(\ln P/S0) + (-R + 1/2\sigma^{2})t\right]}{\sigma\sqrt{t}}$$
(15)

Using (10) and (13) we obtain the expression of return's rate R:

$$R = \sqrt[1/t]{\frac{P}{S0(1 - N(d1)) + P e^{-Rt}N(d2)}} - 1$$
 (16)

By fixing the time, t = T, the analytical solution of Waad Bil Mourabaha using the adapted Black and Scholes Model is given by:

$$P = \frac{S0(1 - N(d1))}{(1 - e^{-RT}N(d2))}$$
(17)

$$d1 = \frac{\left[(\ln P/S0) + (-R - 1/2\sigma^2)T\right]}{\sigma\sqrt{T}}$$
(18)

$$d2 = \frac{\left[(\ln P/S0) + (-R + 1/2\sigma^2)T\right]}{\sigma\sqrt{T}}$$
(19)

$$R = \sqrt[1/T]{\frac{P}{S0(1 - N(d1)) + P e^{-RT}N(d2)}} - 1$$
 (20)

P, R, d1 and d2 depends on the maturity T, the price of purchase of goods P, and the rate of return R. In the next part we will solve the system using the fixed point method.

5 FIXED POINT METHOD AND IMPLEMENTATION

In order to compute numerically the solution of the differential equation of Black and Schole's, we will use the fixed point method.

5.1 Implementation

The algorithm is given by: **Definition of the parameters:** T, S₀, σ and ε **Initialization:** P₀, R₀ **Calculation:** P₁ with the initial condition: P₁ = f(P₀) R₁ with the initial condition: R₁ = f(R₀) $P(n + 1) = f(Pn) = \frac{S0(1 - N(d1(Pn, Rn)))}{(1 - e^{-RnT}N(d2(Pn, Rn)))}$ R(n + 1) = f(Rn) = $\frac{^{1/T}}{\sqrt{\frac{Pn}{S0(1-N(d1(Pn,Rn)))+Pn e^{-RnT}N(d2(Pn,Rn))}}} - 1$ While | P(n+1) - P(n) | > ε and | R(n+1) - R(n) | > ε do: P(n + 1) = f(P(n)) R(n + 1) = g(R(n)) End While P(n+1) R(n+1)

5.2 Numerical results

In Table1, we present the numerical results of Waad Bil Mourabaha contract and the European Call Option. These numerical results are given using MATLAB.

The result of Waad Bil Mourabaha is calculated using the equations (17), (18), (19) and (20). The result of the European Call Option is calculated using the equation (5) with: $\mu_1 = R = 0.0072$; $\mu_1 = 0.07$.

The rate of return of the Call Option is constant.

The rate of return of theWaad Bil Mourabaha is dynamic and it's obtained by the proposed algorithm. To obtain the results: T = 5; $\sigma = 0.3$.

S ₀	V(Waad Bil Mourabaha)	V(Call Option)	
1000	15,1863	18,7893	
500	7,5931	9,3946	
100	1,5186	1,8789	
50	1,7594	0,9395	
10	0,1519	0,1879	

Table1: Comparison between Waad Bil Mourabaha and the European Call Option

6 PROFIT AND LOSS OF WAAD BIL MOURABAHA

6.1 Table of Profit and Loss

The price of the underlying asset at maturity St can take infinity of values, it can be presented in the Figure 1:



Using Figure1, we present the profit and loss of Waad Bil Mourabaha in the Table2:

Cases	Value of Underlying Asset	PL of the Buyer	PL of the Seller
Case1	St < P - V	-V	$+\mathbf{V}$
Case2	$P - V \le St < P$	St -P < 0	P -St ≥0
Case3	$P < St \le P + V$	St -P≥0 P	P-St < 0
Case4	St > P + V	St - P -V≥ 0	P-St + V < 0

Table2: Profit and loss of Waad Bil Mourabaha

Islamic religion does not encourage the transactions with very high profit, by the fact that this engenders a big loss to the other side.

The idea is to minimise the loss, that why when St > P + V Min Bab Al Ihssan(out of charity), the buyer allows the seller to keep the Daman to not cause him a big loss(case4).

The money which is exchanged during Waad Bil Mourabaha is:

- case 1: -V + V = 0
- case 2: St P + P St = 0
- case 3: St P + P St = 0
- case 4: St -P -V + P-St + V = 0

We notice that the money which is earned by one side is the same one which is lost by the other side: it's a zero sum game.

Waad Bil Mourabaha contract allows the limitation of risks, but don't contribute directly to the creation of wealth.

6.2 Example

A Company will need raw material in three years, the current price of this quantity is S0 = 50000DH, the manager concludes a Waad to buy this quantity in three years.

- Compute the parameters P, R and V :

The price of purchase will be lower than the current price, because the farmer will make a reduction to the buyer who is going to buy his goods before they are available. The buyer will pay the farmer the Daman which is equal to the amount of the obtained reduction.

The price of purchase of goods is: P = 53057DHThe Daman is: V = 1681.9DHThe rate of return is: R = 0.0317It would be useful to calculate P + V = 54738DH and P - V = 51375DH.

- Give the profit and loss values knowing(St = 45000DH) It's Case1, the buyer will not honor his Waad and will choose to buy goods directly from the market. To compensate the loss will keep the Daman. The buyer loses: V = - 1681.9DH The seller wins: V = +1681.9DH

- Give the profit and loss values knowing(St = 52000DH): It's Case2, the buyer will pay P to conclude the Mourabaha even if he loses, his loss will be less than the the value of the Daman.

The buyer loses: St - P = 52000-53057 = -1057DH < VThe seller is going to win: P - St = +1057DH

- Give the profit and loss values knowing(St = 54000DH): It's Case3, the buyer will pay P to conclude the Mourabaha and the buyer will pay him back the Daman V. The buyer wins: St - P = +943 The seller loses: P- St = - 943

- Give the profit and loss values knowing (St = 60000DH): It's Case4, the buyer will pay P to conclude the Mourabaha and the seller will keep Daman Min Bab Al Ihssan. The buyer wins: St - P - V = 60000 - 53057 - 1681.9 = +5261.1DH The seller loses: P - St + V = -5261.1DH

7 STABILITY DATA ANALYSIS

7.1 Stability of Time



Figure2: Variation of the price according to the time





time T (maturity) considering $S_0 = 100$ and $\sigma = 0.3$.

We can say that by using Waad Bil Mourabaha's modeling it is interesting to choose the maturity less than seven years. However, beyond the fifth year it becomes dificult to predict the value of goods.

7.2 Stability of volatility





Figure5: Variation of the rate a of return according to the volatility

8 WAAD BIL MOURABAHA HEDGING

In the Call Option, the buyer is free not to perform the contract. In return, he must pay a premium to the seller. The risk is known and limited to the premium, while the gain is potentially unlimited. He has the right to abandon the option if the price at the maturity is more favorable than the option's market price, or exercise the option if the option price is favorable relative to market, or sell the option if there is a profit to be made (Speculation which's impossible in Waad Bil Mourabaha). However, the seller of the call option is obligated to sell, if the buyer decides to exercise its right. The gain of the seller is limited but losses are unknown.

According to the article^[2], the risk can be defined as possibility of loss, and it's clear from an Islamic perspective that risk as such is not desirable. It's very important to differentiate the legitimate risk from the undesirable risk.

The Islam does not encourage highly risked transactions, it's a form of gambling. The risk is legitimate when it's tolerable, inevitable, insignificant and unintentional.

Waad Bil Mourabaha can be used for hedging, because it can cover the risk the buyer and the seller at the same time. Table3 presents profit and loss's intervals of Waad Bil Mourabaha and the Call Option considering t=T.

Note: WBM = Waad Bil Mourabaha.

Cases	St	Buyer WBM	Buyer Call	Seller WBM	Seller Call
Case1	St < P - V	[-V,-V]	[-V,-V]	[V,V]	[V,V]
Case2	$P - V \le St < P$	[-V,0[[-2V,-V[[0,V[[V,2V[
Case3	$P < St \le P + V$]0,V]]-V,0]]-V,0]]0,V]
Case4	St > P + V]0;+∞[]0;+∞[]-∞; 0[]-∞; 0[

Table3: Profit and loss's intervals of Waad Bil Mourabaha and Call Option

When the price is lower than P - V, the behavior of the Waad Bil Mourabaha and the Call option is the same: the transaction is canceled and the Daman (respectively the premium) is paid. (Case1)

- According to Table3, the buyer and the seller can have different positions depending on the choisen contract:
 - For the buyer
 - If he chooses the Waad Bil Mourabaha contract and the price falls between P-V and P: $(P - V \le St < P) \rightarrow (-V \le St - P < 0)$ He loses an amount in the interval [-V,0[(case2). If the price rise between P and P+V: $(P < St \le P + V) \rightarrow (0 < St \le V)$ He wins an amount in the interval]0,V](case3). And if the price is higher than P+V his gain is unlimited (case4). If he chooses the Call and the price falls between P - V and P: $(P - V \le St \le P) \rightarrow (-2V \le St - P - V \le -V)$

He loses an amount in the interval [-2V,-V[(case2)

If the price rises between P and P+V: $(P < St \le P + V) \rightarrow (-V < St \le 0)$ he loses an amount in the interval]-V,0](case3). And if the price is higher than P+V his gain is unlimited (case4).

• For the seller

If he chooses the Waad Bil Mourabaha contract and the price falls between P - V and P: (P - V ≤ St < P) → (0 ≤ P - St < V) He wins an amount in the interval [0,V[(case2). If the price rises between P and P+V: (P < St ≤ P + V) → (-V < P - St ≤ 0) He loses an amount in the interval]-V,0](case3) And if the price is higher than P+V his losses are unlimited (case4)
If he chooses the Call and the price falls between P - V and P: (P - V ≤ St < P) → (0_≤ - St < V) He wins an amount in the interval [V,2V[(case2). If the price rise between P and P+V: (P < St ≤ P + V) → (-V < P - St ≤ 0) He loses an amount in the interval]0,V](case3). And if the price is higher than P+V his losses are unlimited (case4).

In case1, case2 and case4, the positions of gain and loss for the seller and the buyer are the same in Waad Bil Mourabaha contract and Call option(no matter if it'sWaad Bil Mourabaha or Call option, if the price rises the buyer wins). But the difference is in the amount which is earned or lost, especially in case2 we notice an amplification of gain and loss interval.

These transactions are zero sum game: Waad Bil Mourabaha contract try to avoid the excessive gain in one side, because it limits the loss in the other side.

In case 3, ($P < St \le P + V$), if we compare the position of the buyer in Waad Bil Mourabaha contract and the Call option, we notice that under the same price's shape there is a difference in the profit and loss: the buyer will win if he chooses the first contract and loses if he chooses the second.

In the four cases, the buyer inWaad Bil Mourabaha contract is sure to win in two cases, and only in one case in the Call option.

In the Call option, the buyer can have a very big profit and the seller unlimited loss. In contrast, Waad Bil Murabaha contract allows less profits than option, but it minimizes the loss.

In case4 it's difficult to limit the risk, that's why to improve the hedging it would be interesting to use the exotic options.

9 CONCLUSION

In Waad Bil Mourabaha contract the notion of gain and loss should be discussed, because both parts are winning (winwin).

In case of increase of goods's price, the buyer obtains his goods with a low price via the Mourabaha, and the seller is sure of the sale of its goods.

In case of reduction of goods's price, the buyer will have paid the premium of the contract and will buy goods with a low price but directly from the market without using the Mourabaha, the seller will have kept its goods besides premium of the contract to reward the loss which can possibly occur because of the canceling of the Waad.

The Waad Bil Mourabaha can hedge the risk of the actuations in the prices on the market, and minimize the loss.

It would be also interesting to test Waad Bil Mourabaha in other transactions as Muzara'ah or Istisna'ah.

To improve the Waad Bil Mourabaha model we can try to use the exotic options, to model the loss of the buyer in cases where the goods are not available, or to test the variation of the parameters by replacing the normal distribution in the model by a non-normal distribution for example α -stable.

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