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One Plan Suspension System with Bayesian Multiple Deferred State Sampling Plan

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ABSTRACT--- ***Acceptance sampling plans are practical tools for quality assurance applications involving product quality control. Acceptance sampling systems are advocated when small sample size are necessary or desirable towards costlier testing for product quality. When single plan is used with suspension rule, the system is called One-Plan (OP) Suspension System. In OP suspension system, a lot-by-lot sampling plan is used to decide whether individual lots are accepted or rejected.***

This paper provides a new procedure for Designing of One Plan Suspension System with Bayesian Multiple Deferred State (0,2) Sampling Plan as reference plan. Tables and procedures are also provided for the selection of parameter for the plan. Numerical illustrations are also provided for the shop- floor applications of these procedures.

Keywords---- Suspension system, sampling plan, stopping rule, Average probability of acceptance, Operating Ratio.

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1. INTRODUCTION

To improve the quality of any product and services, it is customary to modernize the quality practices and simultaneously reduce the cost for quality improvement. An efficient quality improvement program can be instrumental in increasing productivity at reduced cost. As a result of increasing customer quality requirements and development of new product technology, many existing quality assurance practices and techniques need to be modified. The need for statistical and analytical techniques in quality assurance is rapidly increasing owing to stiff competition in industry towards product quality management.

Acceptance sampling is a tool for consumer to reject bad lots as well as producer to quicken the process control. In a progressive atmosphere of production with increased chances of occurrence of non-conforming material, statistical process control optimizes the process capability and acceptance sampling acts logically to prevent passing out non-conforming units.

This paper is concerned with acceptance sampling plans when small samples are necessary or desirable, for example, when production quantities are small or when inspection is either costly or destructive. Under these conditions, an attribute plan with small sample size is not very effective, since the discrimination between good and bad quality is not sufficient. Nor does lot-by-lot inspection provide an incentive for the producer to turn out consistently good quality.

2. REVIEW OF LITERATURE

Calvin (1984) has provided procedures and tables for implementing Bayesian Sampling Plans. A set of tables presented by Oliver and Springer (1972) which are based on assumption of Beta prior distribution with specific posterior risk to achieve minimum sample size, which avoids the problem of estimating cost parameters. It is generally true that Bayesian Plan requires a smaller sample size than a conventional sampling plan with the same producer and consumer risks. Schafer (1967) discusses single sampling by attributes using three prior distributions for lot quality. Hald (1960) has given an extensive account of sampling plans based on discrete prior distributions of product quality. Case and Keats (1982) have provided a table for the classification of attributes sampling plan design methodologies.

Lilly Christina (1995) have studied the Selection of One Plan Suspension System with Repetitive Group Sampling (RGS) plan as reference plan indexed with Operating ratio and Average run length and also provided the

One Plan Suspension System with Chain Sampling Plan as reference plan indexed with Probability of lot acceptance and also studied Modified Suspension System.

Suresh and Indira (2013) have studied the Construction and Selection of One Plan Suspension System with Bayesian Chain Sampling Plan through Acceptable and Limiting quality levels. She has also studied Designing of One Plan Suspension System with Repetitive Deferred sampling Plan indexed through Probability of acceptance.

3. SELECTION OF SAMPLING PLAN

3.1 Bayesian Multiple Deferred Sate Sampling Plan BMDS (r, b)

The probability density function with Gamma prior distribution, having parameters s and t is,

$$w(p;s,t)=\begin{cases}=e^{-pt}p^{s-1}t^{s}/\Gamma(s), s>0, t>0, p>0\\ =0, otherwise\end{cases} \qquad 3.1.1$$

Let the defects per unit in the submitted lot p, is modeled through a Gamma distribution with parameters s and t. if these lots are subjected to a MDS (0,1) plan and with parameters n and m for a fixed value of p, then the proportion of lots expected to be accepted with operating characteristics function defined through MDS (0,1) plan is given in the equation

$$P(n,m/p)=e^{-np}+e^{-np(1+m)}np \qquad 3.1.2$$

The average probability of acceptance is given as

$$\overline{P}=\int_{0}^{\infty}P(n,m/p)w(p;s,t)dp \qquad 3.1.3$$

On simplification, the APA function for MDS (0,2) plan is

$$\overline{P}=\frac{s^{s}}{(s+n\mu)^{s}}+\frac{n\mu s^{s+1}}{(s+n\mu+mn\mu)^{s+1}}+\frac{(n\mu)^{2}s^{s+1}(s+1)}{2(s+n\mu+n\mu m)^{s+2}} \qquad 3.1.4$$

Where μ=s/t, it the mean value of the product quality p.

3.2 One Plan Suspension system

Cone and Dodge (1963) have first shown that the effectiveness of a small sample lot-by-lot sampling system can be greatly improved by using cumulative results as a basis for suspending inspection. Suspending inspection required the producer to correct what is wrong and submit satisfactory written evidence of action taken before inspection is resumed. The small sample is due to small quantity of production or costly or destructive nature of sample. Usually small sample size is not very effective since the discrimination between good and bad quality is not sufficient. Hence Cone and Dodge (1963) used the cumulative results principle to suspend inspection.

Troxell (1972) has applied the suspension principle to acceptance sampling system. A suspension rule is a procedure used to decide when to suspend inspection of a production process, where product is submitted for inspection in lots. The decision to suspend is based on the observed sequence of lot acceptance and rejections.

A suspension rule, which is designated (j,k), 2≤ j ≤ k is a rule of suspending inspection based on finding j lot rejections in k or less lots. Given j and k at least j lots must be inspected before a decision is possible upon the beginning of a new process or from the time of the last suspension. A suspension system is a combination of suspension rule and a single lot-by-lot sampling plan or pair plans. When a single plan is used with a suspension rule it is called One Plan (OP) Suspension System and when two plans, Tightened and Normal are used, it is called Two Plan (TP) Suspension System.

3.3 Conditions for application

- Production is reasonably steady. So that results on current and proceeding lots are broadly indicative of a continuous process.
- Samples are taken from lot substantially in the order of production so that observed variations in quality of product reflect process performance.
- Inspection is performed close to the production source so that inspection information can be made available promptly.
- Inspection is by attributes, with quality measured in terms of fraction defective p.
- A single sample of size n or double or multiple samples of equal size n is taken from each sampled lot.

3.4 Operating Characteristic Curve

A different type of OC curve which has features not common to type B OC curve (1959) has been used here to study the suspension system. Since ARL for the rule (j,k) is some function of incoming quality p, this correspondence allows an operating characteristic to be plotted in the following way . In a large number of lots N, the number of lots for which the process is judged conforming, that is the number of lots for which suspension does not occur, is given approximately by N(1-1/ARL). Therefore (1-1/ARL) is interpreted as the average fraction of lots for which the process is acceptable, or the probability of accepting the process. This value is denoted as P_A,

$$P_A(j,k) = 1 - 1/ARL(j,k) \qquad 3.2.1$$

and hence $$P_A(2,k) = \frac{1 + P_a - P_a^{\ k}}{2 - P_a^{\ k-1}} \qquad 3.2.2$$

$$P_A(2,\infty) = \frac{1 + P_a}{2} \qquad 3.2.3$$

The OC Curve is a graph of P_A as a function of fraction defective.

3.5 Operating Ratio

A usual measure of discrimination of a sampling plan is the operating ratio (OR), defined as the ratio of the two values of fraction defective for which the probability of acceptance of lots is 0.10 and 0.95 respectively or OR= $p_{0.10} / p_{0.95}$. In order to assess the ability of the rules in anyone class to discriminate between good and bad quality, an index called the Operating Ratio is often used. The operating ratio was first proposed by Peach (1947) for measuring quantitatively the relative discrimination power of sampling plans. The operating ratio for a suspension system is defined as follows.

Choose α and β, where for the particular class (j, k), α and β are restricted such that $1\text{-}1/j \leq \beta < \alpha \leq 1$. α and β are probabilities of not suspending inspection (P_A) corresponding to ARL's of 1/1-α and 1/1-β, hence the restriction. It is necessary to find the fractions defective p_α and p_β which yield α and β, for any rule (j, k) and sampling plan (n,c). The OR is defined as the ratio of the two fractions defective for which the probability of not suspending is β and α, that is OR= p_β / p_α.

In the desire to refer to a suspension system by a numerical value of the OR, a subjective choice of α and β is made. The choice of α is 0.98, corresponding to an ARL of 50. The fraction defective value, which gives this answer, is denoted as $p_{0.98}$. That is, OR = $p_{0.80} / p_{0.98}$ for different values of α and β that is for different values of ARL it is possible to define and calculate OR. But two values of ARL, 50 and 5 are proposed as standards which used to define the OR of a sampling plan.

4. SELECTION PROCEDURE

In this section, a new procedure has been proposed by Multiple Deferred State Sampling plan as reference plan in One-Plan Suspension System. Hence the plan is named One Plan Suspension System with Multiple Deferred State Sampling plan as reference plan. The plan is designed with Average Probability of Acceptance and Operating Ratio. The construction and evaluation of the plan is studied.

4.1 Construction and Evaluation of the Plan:

The probability of Acceptance for One Plan Suspension System plan as defined in the equation given below

$$P_A(2,k) = \frac{1+P_a - P_a{}^k}{2-P_a{}^{k-1}} \qquad 4.1.1$$

Here $\overline{P}$ is the probability of acceptance for Bayesian Multiple Deferred State Sampling Plan. The Probability of acceptance for Bayesian Multiple Deferred State (0,2) Sampling Plan as reference plan in the equation as,

$$\overline{P} = \frac{s^s}{(s+n\mu)^s} + \frac{n\mu s^{s+1}}{(s+n\mu+mn\mu)^{s+1}} + \frac{(n\mu)^2 s^{s+1}(s+1)}{2(s+n\mu+n\mu m)^{s+2}} \qquad 4.1.2$$

Now, the average probability of acceptance (APA) for One Plan Suspension System with Bayesian Multiple Deferred State Sampling Plan as reference plan is defined as,

$$P_A(2,k) = \frac{1+\left(\left(s^s/s+n\mu\right)+\left(n\mu s^{s+1}/(s+n\mu+mn\mu)^{s+1}\right)+\left((n\mu)^2 s^{s+1}(s+1)\right)/2(s+n\mu+n\mu m)^{s+2}\right) - \left(\left(s^s/s+n\mu\right)+\left(n\mu s^{s+1}/(s+n\mu+mn\mu)^{s+1}\right)+\left((n\mu)^2 s^{s+1}(s+1)\right)/2(s+n\mu+n\mu m)^{s+2}\right)^k}{2-\left(\left(s^s/s+n\mu\right)+\left(n\mu s^{s+1}/(s+n\mu+mn\mu)^{s+1}\right)+\left((n\mu)^2 s^{s+1}(s+1)\right)/2(s+n\mu+n\mu m)^{s+2}\right)^{k-1}}$$

4 .1.3

5. CONSTRUCTION OF TABLES

For any Suspension rule (j, k) ARL is a function of P_a, the probability of lot acceptance. The expression for P_a is given in equations (4.1.3) respectively. Using the equation (4.1.3) different ARL values such as (2,1) it is equal to 2 by the method of successive approximation and it given a name in k, and also given values of s and m it is a parameters for Multiple Repetitive Deferred State Sampling Plan Using these values the table1.1 was obtained.

6. EXAMPLE

If the sample size is set at n_k =10, and it is desires to accept material with proportion defective p_k =0.048 (or defects per units) 90 percent of the time so that L(p_k) =0.90 to constructed the plan, scan the column headed L(p) =0.90, find out the value in Table 5.1.1equal to (just less than) n_k p_k =(10)*0.048=0.48. Using Table 1.1 this turns out to be 482872. This is Opposite to k=18, s=5 and m=1 which gives the parameters for the sampling Plan. Further it reveals that higher probability of acceptance with One Plan Suspension System is established at good incoming quality. Table 1.1 the Probability of acceptance values is tabulated which are found out using the equation (4.1.3) and also find out operating ratio in Table 1.2.

7. CONCLUSION

Acceptance sampling is the technique, which deals with the procedures in which decision to accept or reject lots or process based on their examination of past history or knowledge of samples. The present work mainly relates to the Designing of performance One Plan Suspension System with Multiple Deferred State Sampling Plan using Gamma-Poisson distribution. These types of plans are very much essential for the engineers to accept or reject the lots having at least one defective in the lot. The emphasis in the present work is that the selection of a sampling system with this procedure is more advantages for both producer and consumer with less inspection cost. Tables are provided which are tailor made, handy and ready-made uses to the industrial shop-floor conditions.

8. REFERENCES

1. Calvin, T.W. (1984): How and When to perform Bayesian Acceptance Sampling. Vol.7, American Society for Quality Control, Milwaukee, Wisconsin.
2. Case, K.E and Keats, J.B. (1982): On the Selection of a prior Distribution in Bayesian Acceptance Sampling, Journal of Quality Technology, Vol. 14(1), 10-18.
3. Cone, H.F and Dodge, H.G. (1963): A Cumulative Results Plans for small sample inspection, Sandia Corporation Reprint, SCR-678, Albuquerque, NM, USA.Z
4. Hald, A. (1960): The Compound Hypergeometric Distribution and a system of Single sampling Inspection Plans based on Prior Distributions and Costs, Technometrics, Vol.2, pp. 275-340.
5. Lilly Christina, A. (1995): Construction to the study of Design and Analysis of Suspension System and Some other Sampling Plans, Ph.D. Thesis, Bharathiar University, Coimbatore, Tamil Nadu.
6. Oliver, L.R and Springer, M.D. (1972): A General Set of Bayesian Attribute Acceptance Plans, American Institute of Industrial Engineers, Norcross, G.A.
7. Peach, P. (1947): An Introduction to Industrial Statistics and Quality Control, 2nd Edition, Edwards and Broughton Company, Raleigh, North Carolina.
8. Schafer, R.E. (1967): Bayes Single Sampling Plans by Attributes Based on the Posterior Risks, Navel Research Logistics Quarterly, Vol.14. No. 1, March, pp.81-88.
9. Suresh, K.K and Indira, K. (2013): Construction and Selection of One Plan Suspension System with Repetitive Deferred Sampling Plan, Journal of Indian Association for Productivity, Quality and Reliability (IAPQR), Vol.38, No.1,pp. 67-81.
10. Suresh, K.K and Indira, K. (2013): Construction and Selection of One Plan Suspension System with Bayesian Chain Sampling Plan, Mathematical Sciences International Research Journal, Vol.2, Iss.2, pp.196-201.
11. Troxell, J.R (1972): An investigation of Suspension system for small sample inspection, Ph.D. Thesis, Rutgers University, New Jersey.
12. Troxell, J.R (1980): Suspension System for small sample inspection, Technometrics, Vol.22, No.14, pp. 517-533.
13. Vaerst.R (1981): A Procedures to Construct Multiple Deferred State Sampling Plans, Methods of Operation Research, Vol.37, pp.477-485.

Table 1.2- Parametric Values for OPSSS with BMDS (0, 2) plan

s	m	k	Probability of acceptance						
			0.99	**0.98**	**0.95**	**0.90**	**0.80**	**0.75**	**0.50**
1	1	2	0.324148	0.437581	0.682595	1.06794	2.025109	2.735441	43.9782
	2	3	0.197774	0.260836	0.408174	0.639311	1.206958	1.62147	43.87012
	3	4	0.14872	0.194751	0.31	0.496106	0.964432	1.316984	43.40152
	4	5	0.120258	0.161706	0.262105	0.422512	0.849894	1.186475	43.12546
	5	6	0.104021	0.138359	0.230812	0.378204	0.789914	1.119806	42.5621
	6	7	0.0911	0.125535	0.208085	0.347863	0.752437	1.082393	40.2546
	7	8	0.08126	0.115027	0.193995	0.328262	0.728653	1.059961	38.94561
	8	9	0.073777	0.103878	0.182373	0.314671	0.712963	1.045644	38.4251
3	1	10	0.204033	0.261004	0.391708	0.584597	1.009527	1.28917	24.34162
	2	11	0.141165	0.184927	0.286757	0.435649	0.782645	1.013628	23.30939
	3	12	0.116614	0.152849	0.239196	0.370788	0.691972	0.910101	23.49658
	4	13	0.100556	0.131804	0.21147	0.334445	0.644601	0.85601	23.70897
	5	14	0.087976	0.120116	0.190896	0.31	0.61681	0.827379	23.85695
	6	15	0.08294	0.11	0.178705	0.295562	0.59931	0.810152	23.95458
	7	16	0.077005	0.102611	0.168983	0.282328	0.587752	0.804214	24.01937
	8	17	0.072152	0.096191	0.161099	0.273175	0.579843	0.79735	24.06322
5	1	18	0.185474	0.239758	0.371349	0.565705	0.979116	1.228726	15.02521
	2	19	0.128031	0.171568	0.271836	0.42171	0.755428	0.962353	14.69195
	3	20	0.11	0.143324	0.227798	0.362491	0.662584	0.85884	12.54119
	4	21	0.092501	0.124936	0.202085	0.324757	0.619123	0.811001	12.64123
	5	22	0.081201	0.113467	0.081201	0.302682	0.59184	0.78371	12.74374
	6	23	0.078109	0.104431	0.172105	0.288217	0.574802	0.767881	12.78898
	7	24	0.07277	0.097251	0.162965	0.275049	0.563712	0.758339	12.81665
	8	25	0.068378	0.091466	0.155597	0.266289	0.556277	0.752424	12.83403
7 7	1	26	0.169457	0.230577	0.365243	0.561562	0.968704	1.20446	12.1863
	2	27	0.123799	0.165919	0.267279	0.419775	0.744987	0.937187	10.40001
	3	28	0.102884	0.136052	0.224098	0.358036	0.653863	0.839407	10.56902
	4	29	0.087911	0.121085	0.198919	0.323039	0.608543	0.791924	10.67267
	5	30	0.081394	0.11	0.180995	0.300607	0.581275	0.765302	10.73045
	6	31	0.074954	0.101081	0.169589	0.285822	0.56434	0.750088	10.76332
	7	32	0.069883	0.094264	0.16064	0.27288	0.553423	0.741115	10.78258
	8	33	0.065722	0.088805	0.153446	0.264066	0.546208	0.735707	10.79418
9	1	34	0.164709	0.225459	0.363382	0.560524	0.959612	1.187299	9.502126
	2	35	0.120579	0.162704	0.265817	0.419655	0.736404	0.926281	9.455778
	3	36	0.099381	0.133833	0.222835	0.357727	0.648537	0.828579	9.615326
	4	37	0.084977	0.118659	0.197806	0.322699	0.60264	0.78134	9.691523
	5	38	0.075091	0.11	0.179764	0.29972	0.57536	0.755095	9.729212
	6	39	0.072705	0.099019	0.168484	0.284595	0.558481	0.74025	9.763569
	7	40	0.067798	0.092424	0.159636	0.273997	0.547672	0.73163	9.792509
	8	41	0.063785	0.087158	0.152525	0.262988	0.540597	0.726539	9.805088

Table 1.2 Values of Operating Ratios for OPSS with BMDS (0,2) plan

s	m	k	0.98	0.80	OR
1	1	2	0.437581	2.025109	4.62796
	2	3	0.260836	1.206958	4.627266
	3	4	0.194751	0.964432	4.952137
	4	5	0.161706	0.849894	5.255784
	5	6	0.138359	0.789914	5.709161
	6	7	0.125535	0.752437	5.993866
	7	8	0.115027	0.728653	6.334638
	8	9	0.103878	0.712963	6.863439
3	1	10	0.261004	1.009527	3.867865
	2	11	0.184927	0.782645	4.232193
	3	12	0.152849	0.691972	4.527155
	4	13	0.131804	0.644601	4.89059
	5	14	0.120116	0.61681	5.135138
	6	15	0.11	0.59931	5.448277
	7	16	0.102611	0.587752	5.727989
	8	17	0.096191	0.579843	6.028054
5	1	18	0.239758	0.979116	4.083767
	2	19	0.171568	0.755428	4.403075
	3	20	0.143324	0.662584	4.622964
	4	21	0.124936	0.619123	4.955512
	5	22	0.113467	0.59184	5.215981
	6	23	0.104431	0.574802	5.504135
	7	24	0.097251	0.563712	5.796432
	8	25	0.091466	0.556277	6.081775
7	1	26	0.230577	0.968704	4.201224
	2	27	0.165919	0.744987	4.490069
	3	28	0.136052	0.653863	4.805994
	4	29	0.121085	0.608543	5.025767
	5	30	0.11	0.581275	5.284314
	6	31	0.101081	0.56434	5.583068
	7	32	0.094264	0.553423	5.871019
	8	33	0.088805	0.546208	6.150625
9	1	34	0.225459	0.959612	4.25626
	2	35	0.162704	0.736404	4.526043
	3	36	0.133833	0.648537	4.845859
	4	37	0.118659	0.60264	5.078763
	5	38	0.11	0.57536	5.230548
	6	39	0.099019	0.558481	5.64013
	7	40	0.092424	0.547672	5.925638
	8	41	0.087158	0.540597	6.202495