# Some Basic Properties of Weakly Completely Primary Ideals in $\Gamma$ -Near Rings

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ABSTRACT— In this paper, we study completely primary and weakly completely primary ideals in  $\Gamma$ -near-rings. Some characterizations of completely primary and weakly completely primary ideals are obtained. Moreover, we investigate relationships completely primary and weakly completely primary ideals in  $\Gamma$ -near rings. Finally, we obtain necessary and sufficient conditions of a weakly completely primary ideal to be a completely primary ideals in  $\Gamma$ -near rings.

 $\textbf{Keywords} \hspace{-0.5cm} \quad \Gamma \text{-near-ring, completely primary, weakly completely primary, quasi completely primary.}$ 

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#### 1. INTRODUCTION

Throughout this paper, by a  $\Gamma$ -near-ring N we always mean a zero-symmetric near-ring with identity 1. For basic definitions in near-rings one may refer [20]. In 1970 W. L. M. Holcombe was introducing the notions of (0, 1, 2)-prime ideals of a near ring. In 1977 G. Pilz, was introducing the notion of prime ideals of a near ring. In 1988 N.J.Groenewald was introducing the notions of completely (semi) prime ideals of a near ring. In 1991 N.J.Groenewald was introducing the notions of 3-(semi) prime ideals of a near ring . In 2003 D. D. Anderson and E. Smith defined weakly prime ideals in commutative rings, an ideal P of a ring R is weakly prime if  $0 \neq ab \in P$  implies  $a \in P$  or  $b \in P$ .

The concept of  $\Gamma$ -near ring, a generalization of both the concepts near-ring and  $\Gamma$ -ring was introduced by Satyanarayana [21]. Later, several authors such as Booth and Booth, Groenewald [4, 5, 6] studied the ideal theory of  $\Gamma$ -near rings. Groenewald [12] introduced semi uniformly strongly prime near-rings.

In this paper we study completely primary and weakly completely primary ideals in  $\Gamma$ -near-rings. Some characterizations of completely primary and weakly completely primary ideals are obtained. Moreover, we investigate relationships completely primary and weakly completely primary ideals in  $\Gamma$ -near rings. Finally, we obtain necessary and sufficient conditions of a weakly completely primary ideal to be a completely primary ideals in  $\Gamma$ -near rings.

## 2. BASIC RESULTS

In this section we refer to [24, 25] for some elementary aspects and quote few theorem and lemmas which are essential to step up this study. For more details we refer to the papers in the references.

**Definition 2.1.** [25] All near-rings considered in this paper are left distributive. A Γ-near-ring is a triple  $(N, \Gamma, +)$ , where

- (i) (N,+) is a group (not necessarily abelian);
- (ii)  $\Gamma$  is a non-empty set of binary operations on N such that for each  $\gamma \in \Gamma$ ,  $(N, +, \gamma)$  is a right near -ring and;
  - (iii)  $(a\gamma b)\alpha c = a\gamma(b\alpha c)$ , for all  $a,b,c \in N$  and  $\gamma,\alpha \in \Gamma$ .

 $\Gamma$  -near rings generalize near-rings in the sense that every near-ring N is a  $\Gamma$  -near ring, with  $\Gamma = \{\cdot\}$ , where  $\cdot$  is the multiplication defined on N.

**Definition 2.2.** [25] Let N be a  $\Gamma$ -near ring, then a normal subgroup A of  $(N,\Gamma,+)$  is said to be

- (i) left ideal if  $m\gamma(n+a)-m\gamma n\in A$ , for all  $a\in A, \gamma\in \Gamma$ , and  $m,n\in N$ ;
- (ii) right ideal if  $a\gamma n \in A$ , for all  $a \in A, \gamma \in \Gamma$ , and  $n \in N$ ;
- (iii) ideal if it is both left and right ideal of N.

If A is an ideal of N, then it is denoted by  $A \triangleleft N$ . The ideal generated by  $a \in N$ , is denoted by  $a \triangleleft N$ .

**Lemma 2.3.** [25] Let A be a left ideal of a  $\Gamma$ -near ring N. Then  $(A:n)_{\gamma}$  is a left ideal of N, where  $(A:n)_{\gamma} = \{m \in N : m\gamma n \in A\}$ .

**Lemma 2.4.** Let A be an ideal of  $(N, +, \cdot)$ . Then is a  $\Gamma$  -near-ring under the operations: For all  $a, b \in N$  (a+A)+(b+N)=(a+b)+A and (a+A)(b+N)=(ab)+A.

**Lemma 2.5.** Let A and B be ideals of  $(N, \Gamma, +)$ . Then  $(A+B)/A \approx B/(A \cap B)$ . Furthermore, if  $A \subseteq B$ , then  $(N/A)/(B/A) \approx N/B$ .

**Definition 2.6.** Let  $(N,\Gamma,+)$  be a  $\Gamma$ -near-ring and A be a subset of N. We write  $\sqrt{A} = \left\{ a \in N : a^k \in A \text{ for some positive integer } k \right\}.$ 

**Definition 2.7.** A ideal P of a  $\Gamma$ -near-ring N is called a completely primary ideal if for  $a,b \in N$  and  $\gamma \in \Gamma$  such that  $a\gamma b \in P$  implies that  $a^n \in P$  or  $b \in P$ , for some positive integer n.

**Definition 2.8.** A ideal P of a  $\Gamma$ -near-ring N is called a weakly completely primary ideal if for  $a,b \in N$  and  $\gamma \in \Gamma$  such that  $0 \neq a\gamma b \in P$  implies that  $a^n \in P$  or  $b \in P$ , for some positive integer n.

Clearly every completely primary ideal is weakly completely primary and  $\{0\}$  is always weakly completely primary ideal of N. The following example shows that a weakly completely primary ideal need not be a completely primary ideal in general.

**Example 2.9.** Let  $N = \{0, a, b, c, d, 1, 2, 3\}$  and  $\Gamma = \{0, 1\}$ . Define addition and multiplication in N as follows:

+	0	1	2	3	а	b	С	d
0	0	1	2	3	а	b	С	d
1	1	2	3	0	d	С	а	b
2	2	3	0	1	b	а	d	С
3	3	0	1	2	С	d	b	а
а	а	d	b	С	2	0	1	3
b	b	С	а	d	0	2	3	1
С	С	а	d	b	1	3	0	2
d	d	b	С	а	3	1	2	0

	0	1	2	2			-	•
•	Ü	1	2	3	а	b	С	d
0	0	0	0	0	0	0	0	0
1	0	1	2	3	а	b	С	d
2	0	2	0	2	2	2	0	0
3	0	3	2	1	b	а	С	d
а	0	а	2	b	а	b	С	d
b	0	b	2	а	b	а	С	d
С	0	С	0	С	0	0	0	0
$\overline{d}$	0	$\overline{d}$	0	d	2	2	0	0

Then  $(N, +, \cdot)$  is a  $\Gamma$ -near ring. Here  $\{0, c\}$  is a weakly completely primary ideal, but not a completely primary, since  $2 \cdot \gamma \cdot 2 = 0 \in \{0, c\}$ .

#### 3. MAIN RESULTS

We start with the following theorem that gives a relation between weakly completely primary and completely primary ideals in a  $\Gamma$ -near-ring. Our starting points is the following lemma:

**Lemma 3.1.** If N is a  $\Gamma$ -near-ring with identity, then  $a\gamma b = a\alpha b$  for all  $a, b \in N$  and  $\gamma, \alpha \in \Gamma$ .

**Proof.** Let N be a  $\Gamma$ -near-ring and e be the identity of N, and let  $a,b \in N, \gamma, \alpha \in \Gamma$  therefore we have

$$a\gamma b = a\gamma(e\alpha b)$$
  
=  $(a\gamma e)\alpha b$   
=  $a\alpha b$ .

Hence  $a\gamma b = a\alpha b$ .

**Lemma 3.2**. Let N be a  $\Gamma$ -near-ring, and let A be a left ideal of N. Then  $(A:\Gamma:B)$  is a left ideal in N, where  $(A:\Gamma:B) = \{n \in N : n\Gamma B \subseteq A\}$ .

**Proof.** Let N be a  $\Gamma$ -near-ring, and let A be a left ideal of N. Suppose that  $n \in N$  and  $m, n \in (A : \Gamma : B)$ . Then  $m\Gamma B \subseteq A$  and  $n\Gamma B \subseteq A$  so that

$$(n-m)\Gamma B = n\Gamma B - m\Gamma B \subseteq A.$$

Therefore  $n-m \in (A:\Gamma:B)$ . For  $a \in (A:\Gamma:B)$  and  $n \in N$ ,

$$(n+a-n)\Gamma B = n\Gamma B + a\Gamma B - n\Gamma B$$

$$\subseteq n\Gamma B + A - n\Gamma B$$

$$\subset A$$

since A is a left ideal of N. Therefore,  $n+a-n\in (A:\Gamma:B)$ . Thus  $(A:\Gamma:B)$  is a normal subgroup of N. Let  $m,n\in N,a\in (A:\Gamma:B)$  and  $\beta$   $\gamma\in \Gamma$ . Then

$$(m\gamma(n-a)-m\gamma n)\Gamma B = (m\gamma(n-a))\Gamma B - (m\gamma n)\Gamma B$$

$$= m\gamma((n-a)\Gamma B) - (m\gamma n)\Gamma B$$

$$= m\gamma(n\Gamma B - a\Gamma B) - (m\gamma n)\Gamma B$$

$$= m\gamma(n\Gamma B - a\Gamma B) - (m\gamma n)\Gamma B$$

$$\subseteq A.$$

Thus  $m\gamma(n-a)-m\gamma n\in (A:\Gamma:B)$ . Hence  $(A:\Gamma:B)$  is a left ideal in N.

**Theorem 3.3**. Let N be a  $\Gamma$ -near-ring, and let A be an ideal of N. If A is a weakly quasi completely primary (quasi completely primary) ideal of N, then  $(A:\Gamma:B)$  is a weakly quasi completely primary (quasi completely primary) ideal in N, where  $B \not\subset A$ .

**Proof.** Let N be a  $\Gamma$ -near-ring, and let A be a weakly completely quasi primary ideal of N. Suppose that  $0 \neq m\gamma n \in (A:\Gamma:B)$  and  $m^k \notin (A:\Gamma:B)$ , for all positive integer k. Then

$$0 \neq m\gamma(n\Gamma B) = (m\gamma n)\Gamma B \subseteq A.$$

By Definition of weakly quasi completely primary ideal, we get  $m^k \in A$  or  $n\Gamma B \subseteq A$  for some positive integer k so that  $n \in (A : \Gamma : B)$ . Hence  $(A : \Gamma : B)$  is a weakly quasi completely primary ideal in N.

Corollary 3.4. Let N be a  $\Gamma$ -near-ring, and let A be a weakly quasi completely primary (quasi completely primary) ideal of N. Then  $(A:m)_{\gamma}$  is a weekly quasi completely primary (quasi completely primary) ideal in N, where  $m \in N-A$ .

**Proof.** This follows from Theorem 3.3.

**Theorem 3.5.** Let N be a  $\Gamma$ -near-ring, and let P be an ideal of N. If P is a weakly completely primary ideal that is not completely primary. Then  $\sqrt{P} = \sqrt{0}$ .

**Proof.** Let N be a  $\Gamma$ -near-ring with identity. First, we prove that  $P^2 = 0$ . Suppose that  $P^2 \neq 0$  we show that P is weakly completely primary. Let  $a\gamma b \in P$ , where  $a, b \in N, \gamma \in \Gamma$ . If  $a\gamma b \neq 0$ , then either

$$a \in \sqrt{P}$$
 or  $b \in P$ 

since P is weakly completely primary ideal. So suppose that  $a\gamma b = 0$ . If  $P\gamma b \neq 0$ , then there is an element p' of P such that  $p'\gamma b \neq 0$ , so that

$$0 \neq p'\gamma b = (p'+a)\gamma b \in P,$$

and hence P weakly completely primary ideal gives either  $p'+a\in \sqrt{P}$  or  $b\in P$ . As  $p'+a\in \sqrt{P}$  and  $p'\in P\subseteq \sqrt{P}$  we have either  $a\in \sqrt{P}$  or  $b\in P$ . So we can assume that  $P\gamma b=0$ . Similarly, we can assume that  $P\gamma a=0$ . Since  $P^2\neq 0$ , there exist  $c,d\in P$  such that  $c\gamma d\neq 0$ . Then

$$(a + c)\gamma(b + d) \in P$$
,

so either  $p+c\in P$  or  $q+d\in \sqrt{P}$ , and hence either  $p\in P$  or  $q\in \sqrt{P}$ . Thus P is completely primary ideal. Clearly,  $\sqrt{0}\subseteq \sqrt{P}$ . As  $P^2=0$ , we get  $\sqrt{P}\subseteq \sqrt{0}$ , hence  $\sqrt{P}=\sqrt{0}$ , as required.

Corollary 3.6. Let N be a  $\Gamma$ -near-ring, and let P an ideal of N. If  $\sqrt{P} \neq \sqrt{0}$ , then P is completely primary if and only if P is weakly completely primary.

**Proof.** This follows from Theorem 3.5.

**Lemma 3.7.** Let N be a  $\Gamma$ -near-ring with identity, and let P be a proper ideal of N. If P is a weakly completely primary ideal of N, then  $(P:\Gamma:N\Gamma a)=P\cup(0:\Gamma:N\Gamma a)$ , where  $a\in N-\sqrt{P}$ .

**Proof.** Let N be a  $\Gamma$ -near-ring with identity, and let P be a weakly completely primary ideal of N. Clearly,

$$P \cup (0:\Gamma:N\Gamma a) \subseteq (P:\Gamma:N\Gamma a).$$

For the other inclusion, suppose that  $m \in (P : \Gamma : N\Gamma a)$ , so that

$$m\Gamma(N\Gamma a)\subseteq P$$
.

If  $0 \neq m\Gamma(N\Gamma a)$ , then  $N\Gamma a \subseteq P$  since P is weakly completely primary. If  $0 = m\Gamma(N\Gamma a)$ , then  $m \in (0:\Gamma:N\Gamma a)$  so we have the equality.

Corollary 3.8. Let N be a  $\Gamma$ -near-ring with identity, and let P be a proper ideal of N. If P is a weakly completely primary ideal of N, then  $(P:\Gamma:a) = P \cup (0:\Gamma:a)$ , where  $a \in N - \sqrt{P}$ .

**Proof.** This follows from Lemma 3.7.

**Corollary 3.9.** Let N be a  $\Gamma$ -near-ring with identity, and let P be a proper ideal of N. If  $(P:\Gamma:N\Gamma a)=P\cup (0:\Gamma:N\Gamma a)$ , then  $(P:\Gamma:N\Gamma a)=P$  or  $(P:\Gamma:N\Gamma a)=(0:\Gamma:N\Gamma a)$ , where  $a\in N-\sqrt{P}$ .

**Proof.** This follows from Lemma 3.7.

**Theorem 3.10.** Let N be a  $\Gamma$ -near-ring with identity, and let P be a proper ideal of N. If  $(P:\Gamma:n)=P$  or  $(P:\Gamma:n)=(0:\Gamma:n)$ , then P is a weakly completely primary ideal of N, where  $n\in N-\sqrt{P}$ .

**Proof.** Let N be a  $\Gamma$ -near-ring with identity, and let P be a proper ideal of N. Suppose that Let  $0 \neq m\gamma n \in P$ , where  $m \in N - \sqrt{P}$ ,  $\gamma \in \Gamma$ . Then

$$m \in (P:\Gamma:n) = P \cup (0:\Gamma:n)$$

by Corollary 3.9 hence  $m \in P$  since  $m\gamma n \neq 0$ , as required.

**Lemma 3.11**. Let  $N = N_1 \times N_2$ , where each  $N_i$  is a  $\Gamma$ -near-ring with identity. Then the following hold:

(i) If 
$$A$$
 is an ideal of  $N_1$ , then  $\sqrt{A \times N_2} = \sqrt{A} \times N_2$ .

(ii) If 
$$A$$
 is an ideal of  $N_2$ , then  $\sqrt{N_1 \times A} = N_1 \times \sqrt{A}$ .

**Proof.** The proof is straightforward.

**Theorem 3.12.** Let  $N=N_1\times N_2$ , where each  $N_i$  is a  $\Gamma$ -near-ring with identity. If P is a weakly completely primary (completely primary) ideal of  $N_1$ , then  $P\times N_2$  is a weakly completely primary (completely primary) ideal of N.

**Proof.** Suppose that  $N=N_1\times N_2$ , where each  $N_i$  is a  $\Gamma$ -near-ring with identity and P is a weakly completely primary ideal of  $N_1$ . Let

$$0 \neq (a,b)\gamma(c,d) = (a\gamma c,b\gamma d) \in P \times N_2$$

where  $(a,b),(c,d) \in N, \gamma \in \Gamma$  so either  $a \in \sqrt{P}$  or  $c \in P$  since P is weakly completely primary. It follows that either

$$(a,b) \in \sqrt{P} \times N_2 = \sqrt{P \times N_2}$$
 or  $(c,d) \in P \times N_2$ .

By Definition of weakly completely primary ideal, we have  $P \times N_2$  is a weakly completely primary ideal of N.

Corollary 3.13. Let  $N=N_1\times N_2$ , where each  $N_i$  is a  $\Gamma$ -near-ring with identity. If P is a weakly completely primary (completely primary) ideal of  $N_2$ , then  $N_1\times P$  is a weakly completely primary (completely primary) ideal of N.

**Proof.** This follows from Lemma 3.12.

Corollary 3.14. Let  $N = \prod_{i=1}^n N_i$ , where each  $N_i$  is a  $\Gamma$ -near-ring with identity. If P is a weakly completely primary (completely primary) ideal of  $N_j$ , then  $N_1 \times N_2 \times \ldots \times P_j \times N_{j+1} \times \ldots \times N_n$  is a weakly completely primary (completely primary) ideal of N.

**Proof.** This follows from Theorem 3.12 and Corollary 3.13.

**Theorem 3.15.** Let  $N = N_1 \times N_2$ , where each  $N_i$  is a  $\Gamma$ -near-ring with identity. If P is a weakly completely primary ideal of N, then either P = 0 or P is completely primary.

**Proof.** Let  $N=N_1\times N_2$ , where each  $N_i$  is a  $\Gamma$ -near-ring with identity and let  $P=P_1\times N_2$  be a weakly completely primary ideal of N. We can assume that  $P\neq 0$ . So there is an element (a,b) of P with  $(a,b)\neq (0,0)$ . Then  $(0,0)\neq (a,1)\gamma(1,b)\in P$ ,

wehere  $\gamma \in \Gamma$ , gives either

$$(a,1) \in \sqrt{P}$$
 or  $(1,b) \in \sqrt{P} = \sqrt{P_1} \times N_2$ .

If  $(a,1) \in P$ , then  $P = P_1 \times N_2$ . We show that  $P_1$  is completely primary hence P is weakly completely primary by Theorem 3.12. Let  $c\gamma d \in P_1$ , where  $c, d \in N_1$ . Then

$$(0,0) \neq (c,1)\gamma(d,1) = (c\gamma d,1) \in P$$
,

so either  $(c,1) \in P$  or  $(d,1) \in \sqrt{P} = \sqrt{P_1} \times N_2$  and hence either  $c \in P_1$  or  $d \in P_1$ . By a similar argument,  $N_1 \times P_2$  is completely primary.

**Proposition 3.16.** Let  $A \subseteq P$  be proper ideals of a  $\Gamma$ -near-ring N. Then the following hold:

- (i) If P is weakly completely primary (completely primary), then P/A is weakly completely primary (completely primary).
- (ii) If A and P/A are weakly completely primary (completely primary), then P is weakly completely primary).

**Proof.** (i) Let  $0 \neq (a + A)\gamma(b + A) = a\gamma b + A \in P/A$ , where  $a, b \in N, \gamma \in \Gamma$  so  $ab \in P$ . If  $a\gamma b = 0 \in A$ , then  $(a+A)\gamma(b+A) = 0$ , a contradiction. So if P is weakly completely primary, then either  $a \in P$  or  $b \in \sqrt{P}$ , hence either  $a + A \in P/A$  or  $b + A \in \sqrt{P/A}$ , as required.

(ii) Let  $0 \neq a\gamma b \in P$ , where  $a,b \in N$ , so  $(a+A)\gamma(b+A) \in P/A$ . For  $a\gamma b \in A$ , if A is weakly completely primary, then either  $a \in A \subseteq P$  or  $b \in A \subseteq P \subseteq \sqrt{P}$ . So we may assume that  $a\gamma b \notin A$ . Then either  $a+A \in P/A$  or  $b+A \in \sqrt{P/A}$ . It follows that either  $a \in P$  or  $b \in \sqrt{P}$  as needed.

**Theorem 3.17.** Let P and Q be weakly completely primary ideals of a  $\Gamma$ -near-ring N that are not completely primary. Then P+Q is a weakly completely primary ideal of N.

**Proof.** Since  $(P+Q)/Q \approx Q/(P \cap Q)$ , we get that (P+Q)/Q is weakly completely primary by Proposition 3.16 (i). Now the assertion follows from Proposition 3.16 (ii).

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