On Γ-CI-Algebras

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ABSTRACT— ***The purpose of this paper is to introduce the notion of a Γ-CI-algebras, we study Γ-ideals, Γ-subalgebras and upper sets in Γ-CI-algebras. Some characterizations of Γ-ideals and Γ-subalgebras are obtained. Moreover, we investigate relationships between Γ-ideals, Γ-subalgebras and upper sets in Γ-CI-algebras.***

Keywords— Γ-CI-algebra, Γ-ideal, Γ-subalgebra, upper set, CI-algebra.

1. INTRODUCTION

In what follows, let X denote an CI-algebra unless otherwise specified. By an CI-algebra we mean an algebra $(X;*,1)$ of type (2, 0) with a single binary operation "$*$" that satisfies the following identities: for any $x,\ y,\ z \in X$,

1. $x*x=1$,
2. $1*x=x$,
3. $x*(y*z)=y*(x*z)$.

In, 2006, H. S. Kim and Y. H. Kim defined a BE-algebra. Biao Long Meng, defined notion of CI–algebra as a generation of a BE–algebra [9]. BE-algebras and CI-algebras are studied in detail be some researchers [3, 7, 9, 11] and some fundamental properties of CI–algebra are discussed.

In this paper is to introduce the notion of a Γ-CI-algebras, we study Γ-ideals, Γ-subalgebras and upper sets in Γ-CI-algebras. Some characterizations of Γ-ideals and Γ-subalgebras are obtained. Moreover, we investigate relationships between Γ-ideals, Γ-subalgebras and upper sets in Γ-CI-algebras.

2. BASIC PROPERTIES

In this section is to introduce the notion of a Γ-CI-algebras.

Definition 2.1. Let X and Γ be any nonempty sets. If there exists a mapping $X\times\Gamma\times X \to X$ written as (x,γ,y) by $x\gamma y$, then $(\Gamma,X;1)$ is called a Γ-CI-algebra if

1. $x\gamma x = 1$ for all $x\in X$ and $\gamma\in\Gamma$,
2. $1\gamma x = x$ for all $x\in X$ and $\gamma\in\Gamma$,
3. $x\gamma(y\beta z) = y\gamma(x\beta z)$

for all $x,y,z\in X$ and $\gamma,\beta\in\Gamma$.

Example 2.2. Let $(\Gamma, X;1)$ be an arbitrary CI-algebra and Γ any nonempty set. Define a mapping $X\times\Gamma\times X \to X$, by $x\gamma y = xy$ for all $x, y \in X$ and $\gamma \in \Gamma$. It is easy to see that X is a Γ-CI-algebra. Indeed,

1. $x\gamma x = xx = 1$,
2. $1\gamma x = 1x = x$,
3. $x\gamma(y\beta z) = x(yz) = y(xz)$

for all $x, y, z \in X$ and $\gamma, \beta \in \Gamma$. Thus every CI-algebra implies a Γ-CI-algebra.

Example 2.3. Let $X = \{1, a, b\}$ in which "$\cdot$" is defined by

$\cdot$	1	a	b
1	1	b	a
a	a	1	b
b	b	a	1

Then X is not a CI-algebra. Let $\Gamma \neq \varnothing$. Define a mapping $X\times\Gamma\times X \to X$ by $x\gamma y = yx$ for all $x, y \in X$ and $\gamma \in \Gamma$. Then X is a Γ-CI-algebra. Indeed

1. $x\gamma x = (x\cdot x)\cdot(x\cdot x) = 1\cdot 1 = 1$,
2. $1\gamma x = x\cdot 1 = x$,
3. $x\gamma(y\beta z) = (y\beta z)\cdot x = (z\cdot y)\cdot x = (z\cdot x)\cdot y = (x\beta z)\cdot y = y\gamma(x\beta z)$

for all $x, y, z \in X$ and $\gamma, \beta \in \Gamma$. Therefore X is a Γ-CI-algebra.

3. Γ-CI-ALGEBRAS

In this section we first introduce the notion of Γ-CI-algebras and next study some of elementary properties.

Lemma 3.1. Let X be a Γ-CI-algebra. Then $x\gamma y = x\beta y$ for any $x, y \in X$ and $\gamma, \beta \in \Gamma$.

Proof. Let X be a Γ-CI-algebra X, $x, y \in X$ and let $\gamma, \beta \in \Gamma$. Then

$$x\gamma y = x\gamma(1\beta y) = 1\gamma(x\beta y) = x\beta y.$$

Hence $x\gamma y = x\beta y$.

Proposition 3.2. Any Γ-CI-algebra X satisfies for any $x, y \in X$ and $\gamma, \beta, \alpha \in \Gamma$, $y\gamma\left[(y\alpha x)\beta\alpha\right] = 1$

Proof. Let X be a Γ-CI-algebra X, $x, y \in X$ and let $\gamma, \beta, \alpha \in \Gamma$. Then

$$\begin{aligned} y\gamma\left[(y\alpha x)\beta\alpha\right] &= (y\alpha x)\gamma(y\beta\alpha) \\ &= (y\beta x)\gamma(y\beta\alpha) \\ &= 1. \end{aligned}$$

Hence $y\gamma\left[(y\alpha x)\beta\alpha\right] = 1$.

Proposition 3.3. Any Γ-CI-algebra X satisfies for any $x, y \in X$ and $\gamma, \beta \in \Gamma$, $(x\gamma 1)\beta(y\gamma 1) = (x\gamma y)\beta 1$.

Proof. Let X be a Γ-CI-algebra X, $x, y \in X$ and let $\gamma, \beta, \alpha \in \Gamma$. Then

$$\begin{aligned} (x\gamma 1)\beta(y\gamma 1) &= (x\gamma 1)\beta\left(y\gamma\left[(x\gamma y)\alpha(x\gamma y)\right]\right) \\ &= (x\gamma 1)\beta\left((x\gamma y)\gamma\left[y\alpha(x\gamma y)\right]\right) \\ &= (x\gamma 1)\beta\left((x\gamma y)\gamma\left[x\alpha(y\gamma y)\right]\right) \\ &= (x\gamma 1)\beta\left((x\gamma y)\gamma\left[x\alpha 1\right]\right) \\ &= (x\gamma y)\beta\left((x\gamma 1)\gamma(x\gamma 1)\right) \\ &= (x\gamma y)\beta 1. \end{aligned}$$

Hence $(x\gamma 1)\beta(y\gamma 1) = (x\gamma y)\beta 1$.

Proposition 3.4. Let X be a Γ-CI-algebra. If $x\gamma(x\beta y) = x\beta y$ for any $x, y \in X$ and $\gamma, \beta \in \Gamma$, then $x\gamma 1 = 1$.

Proof. Let X be a Γ-CI-algebra X, $x \in X$ and let $\gamma \in \Gamma$. Then

$$\begin{aligned} x\gamma 1 &= x\gamma(x\gamma x) \\ &= x\gamma x \\ &= 1. \end{aligned}$$

Hence $x\gamma 1 = 1$.

Proposition 3.4. Let X be a Γ-CI-algebra. If $x\gamma(y\beta x) = x$ for any $x, y \in X$ and $\gamma, \beta \in \Gamma$, then $x\gamma 1 = 1$.

Proof. Let X be a Γ-CI-algebra X, $x \in X$ and let $\gamma \in \Gamma$. Then

$$\begin{aligned} x\gamma 1 &= 1\gamma(x\gamma 1) \\ &= 1. \end{aligned}$$

Hence $x\gamma 1 = 1$.

4. Γ-IDEALS IN Γ-CI-ALGEBRAS

In this section we first introduce the notion of Γ-ideal in Γ-CI-algebras and next study some of elementary properties.

Definition 4.1. Let X be a Γ-CI-algebra and A a nonempty subset of X. A is said to be a Γ-ideal of X if it satisfies: $x \in X$ and $\gamma, \beta, \alpha \in \Gamma$

1. $X\Gamma A \subseteq A$,
2. $a, b \in A$ imply that $(a\gamma(b\beta x))\alpha x \in A$.

Definition 4.2. Let X be a Γ-CI-algebra and A a nonempty subset of X. A is said to be a Γ-subalgebra of X if $AA \subseteq A$.

For any Γ-CI-algebra X, $\{1\}$ and X are trivial ideals (resp. Γ-subalgebras) of X. Obviously every ideal in a Γ-CI-algebra is a Γ-subalgebra.

Lemma 4.3. Let X be a Γ-CI-algebra. Then

(1) Every Γ-ideal of X contains 1,

(2) If A is a Γ-ideal of X, then $(a\gamma x)\beta x \in A$ for all $a \in A, x \in X$ and $\gamma, \beta \in A$.

Proof. Let X be a Γ-CI-algebra X and let A be a Γ-ideal of X.

(1) Since A is a Γ-ideal of X, we have $1 = a\alpha a \in A\Gamma A \subseteq X\Gamma A \subseteq A$.

(2) Let $a \in A$ and let $x \in X$. Then $(a\gamma x)\beta x = (a\gamma(1\alpha x))\beta x \in A$.

Definition 4.4. Let X be a Γ-CI-algebra and $a \in X, \gamma \in \Gamma$. Define $A(a\gamma)$ by

$$A(a\gamma) = \{1\} \cup \{x \in X : a\gamma x = 1\}.$$

Then we call $A(a\gamma)$ the initial section of the element a.

Definition 4.5. Let X be a Γ-CI-algebra and $a, b \in X, \gamma, \beta \in \Gamma$. Define $A(a\gamma, b\beta)$ by

$$A(a\gamma, b\beta) = \{1\} \cup \{x \in X : a\gamma(b\beta x) = 1\}.$$

We call $A(a\gamma, b\beta)$ an upper set of a and b.

Remark. By Definition 4.5, we have $A(a\gamma, b\beta) = A(b\gamma, a\beta)$ for all $a, b \in X$ and $\gamma, \beta \in \Gamma$.

Proposition 4.6. Let X be a Γ-CI-algebra, $x, y \in X$ and $\gamma, \beta \in \Gamma$. Then

(1) $1 \in A(a\gamma)$, $1 \in A(a\gamma, b\beta)$.

(2) $a \in A(a\gamma)$.

(3) If $y\beta 1 = 1$, then $A(a\gamma) \subseteq A(a\gamma, b\beta)$.

(4) If $y\beta 1 \neq 1$, then $A(a\gamma) - \{1\} \subseteq X - A(a\gamma, b\beta)$.

Proof. Let X be a Γ-CI-algebra X, $a, b \in X$ and let $\gamma, \beta \in \Gamma$.

(1) By Definition 4.4 and Definition 4.5, we have $1 \in A(a\gamma)$ and $1 \in A(a\gamma, b\beta)$.

(2) Since $a\gamma a = 1$, we have $a \in A(a\gamma)$.

(3) Let $x \in A(a\gamma)$. Then $x \in \{x \in X : a\gamma x = 1\}$ or $x = \{1\}$.

Case 1: $x = 1$. It is easy to see that $x \in A(a\gamma, b\beta)$.

Case 2: $x \in \{x \in X : a\gamma x = 1\}$. It is easy to see that $a\gamma x = 1$. Then

$$\begin{aligned} a\gamma(b\beta x) &= b\gamma(a\beta x) \\ &= b\gamma(a\gamma x) \\ &= b\gamma 1 \\ &= 1. \end{aligned}$$

Hence $x \in A(a\gamma, b\beta)$.

(4) Suppose that $y\beta 1 \neq 1$. Let $x \in A(a\gamma) - \{1\}$. Then $a\gamma x = 1$ and $x \neq 1$. We have

$$\begin{aligned} a\gamma(b\beta x) &= b\gamma(a\beta x) \\ &= b\gamma(a\gamma x) \\ &= b\gamma 1 \\ &\neq 1. \end{aligned}$$

Thus $x \notin A(a\gamma, b\beta)$ and hence $A(a\gamma) - \{1\} \subseteq X - A(a\gamma, b\beta)$.

5. CONCLUSION

Many new classes of Γ-CI-algebras have been discovered recently. All this has attracted researchers of the field to investigate these newly discovered classes in detail. This current article investigates we study Γ-ideals, Γ-subalgebras and upper sets in Γ-CI-algebras. Some characterizations of Γ-ideals and Γ-subalgebras are obtained. Moreover, we investigate relationships between Γ-ideals, Γ-subalgebras and upper sets in Γ-CI-algebras.

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7. REFERENCES

[1] Ahn S.S, Kim Y.H., So K.S., "Fuzzy BE-algebras. Journal of applied mathematics and informatics", vol. 29, 1049-1057, 2011.

[2] Ahn S.S., So Y.H., "On ideals and upper sets in BE-algebras", Sci. Math. Jpn. Online e-2008, vol. 2, 279-285, 2008.

[3] Borumand Saeid A., Rezaei A.,"Quotient CI–algebras", Bulletin of the Transilvania University of Bra¸sov, vol. 5, no. 54, 15-22, 2012.

[4] Hu Q.P., Li X., "On BCH-algebras", Math.Seminar Notes, vol. 11, 313-320, 1983.

[5] Hu Q. P., Li X., "On proper BCH-algebras", Math Japonica, vol. 30, 659-661, 1985.

[6] Iseki K., Tanaka S., "An introduction to theory of BCK-algebras", Math Japonica, vol. 23, 1-20, 1978.

[7] Kim K.H., "A Note on CI-algebras", International Mathematical Forum, vol. 6, no. 1, 1-5, 2011.

[8] Kim H.S., Kim Y.H., "On BE-Algebras", Sci. Math. Jpn., vol. 66, no. 1, 1299-1302, 2006.

[9] Meng B.L., "CI–algebra", Sci. Math. Jpn. vol. 71, no. 2, 695-701, 2010.

[10] Neggers J., Ahn S.S., Kim H.S., "On q-algebras", Int. J. Math. Math. Sci., vol. 27, no. 12, 749-757, 2001.

[11] Piekart B., Andrzej Walendziak A., "On filters and upper sets in CI-algebras", Algebra and Discrete Mathematics, vol. 11, no. 1, 109-115, 2011.