On the Classical Primary Radical Formula and Classical Primary Subsemimodules

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ABSTRACT— ***In this paper, we characterize the classical primary radical of subsemimodules and classical primary subsemimodules of semimodules over a commutative semirings. Furthermore we prove that if N_j is a classical primary subsemimodule of M_j, then N_j is to satisfy the classical primary radical formula in M_j if and only if $M_1 \times M_2 \times \ldots \times M_{j-1} \times N_j \times M_{j+1} \times \ldots \times M_n$ is to satisfy the classical primary radical formula in M.***

Keywords— classical primary subsemimodule, primary subsemimodule, classical primary radical, classical primary radical formula.

1. INTRODUCTION

Throughout this paper a semiring will be defined as follows: A semiring is a set R together with two binary operations called addition "$+$" and multiplication "$\cdot$" such that $(R,+)$ is a commutative semigroup and $(R,\cdot)$ is semigroup; connecting the two algebraic structures are the distrubutive laws : $a(b+c) = ab+ac$ and $(a+b)c = ac+bc$ for all $a,\ b,\ c \in R.$ A semimodule M over a semiring R is a commutative monoid M with additive identity 0, together with a function $R \times M \to M,$ defined by $(r,m) \mapsto rm$ such that:

1. $r(m+n) = rm+rn$
2. $(r+s)m = rm+sm$
3. $(rs)m = r(sm)$
4. $r0 = 0 = 0m$
5. $1m = m$

for all $m,n \in M$ and $r,s \in R.$ Clearly every ring is a semiring and hence every module over a ring R is a left semimodule over a semiring $R.$ A nonempty subset N of a R-semimodule M is called subsemimodule of M if N is closed under addition and closed under scalar multiplication.

J. Saffar Ardabili S. Motmaen and A. Yousefian Darani in (2011) defined a different class of subsemimodules and called it classical prime. A proper subsemimodule N of M is said to be classical prime when for $a,b \in R$ and $m \in M, abm \in N$ implies that $am \in N$ or $bm \in N.$

A proper subsemimodule N of M is said to be classical primary when for $a,b \in R$ and $m \in M, abm \in N$ implies that $am \in N$ or $b^n m \in N,$ for some positive integer $n.$ A classical primary radical of N in $M,$ denoted by $c.prad_M(N),$ is defined to be the intersection of all classical primary subsemimodules containing $N.$ Should there be no classical primary subsemimodule of M containing $N,$ then we put $c.prad_M(N) = M.$ In this note, we shall need the notion of the envelope of a submodule introduced by R. L. McCasland and M. E. Moore in [11]. For a submodule N of an R-module $M,$ the envelope of N in $M,$ denoted by $E_M(N),$ is defined to be the subset $\{rm : r \in R$ and $m \in M$ such that $r^k m \in N$ for some $k \in \mathbb{Z}^+\}$ of $M.$ Note that, in general, $E_M(N)$ is not an R-module. With the help of envelopes, the notion of the radical formula is defined as follows: a submodule N of an R-module M is said to satisfy the radical formula in $M,$ if $\langle E_M(N)\rangle = rad_M(N).$ Also, an R-module M is said to satisfy the radical formula, if every submodule of M satisfies the radical formula in $M.$ The radical formula has been studied extensively by various authors (see [8], [13] and [14]).

In this paper we introduce the concept of the radical formula and study some basic properties of this class of subsemimodules. Moreover, we prove that if N_j is a classical primary subsemimodule of $M_j,$ then N_j is to satisfy the classical primary radical formula in M_j if and only if $M_1 \times M_2 \times \ldots \times M_{j-1} \times N_j \times M_{j+1} \times \ldots \times M_n$ is to satisfy the classical primary radical formula in $M.$

2. PRELIMINARIES

Let $R = \prod_{i=1}^{n} R_i,$ where each R_i is a commutative semiring with identity. Then an ideal $I = \prod_{i=1}^{n} I_i$ of P is primary if and only if I_i is equal to the corresponding semiring R_i and the other is primary. Moreover, any R-semimodule M can be uniquely decomposed into a direct product of semimodules, i.e. $M = \prod_{i=1}^{n} M_i,$ where

$$M_i = (0,0,0,\ldots,0,1,0,\ldots0)M$$

is an R_i-semimodule with action $(r_1, r_2,\ldots,r_n)(m_1,m_2,\ldots,m_n) = (r_1m_1, r_2m_2,\ldots,r_nm_n),$ where $r_i \in R_i$ and $m_i \in M_i.$

Proposition 2.1. Let $N = N_1 \times N_2$ be a subsemimodule of $M.$ Then $\langle E_M(N)\rangle = \langle E_{M_1}(N_1)\rangle \times \langle E_{M_2}(N_2)\rangle.$

Proof. Let $x = \sum_{i=1}^{k}(r_i,s_i)(m_i,n_i) \in \langle E_M(N)\rangle$ where $(r_i,s_i)^{k_i}(m_i,n_i) \in N,$ for some $k_i \in \mathbb{Z}^+$ if and only if

$$u = \sum_{i=1}^{k} r_i m_i \in \langle E_{M_1}(N_1)\rangle, \text{ with } r_i^{k_i} m_i \in N_1$$

and

$$v = \sum_{i=1}^{k} s_i n_i \in \langle E_{M_2}(N_2)\rangle, \text{ with } s_i^{k_i} n_i \in N_2.$$

Then $x = (u,v) \in \langle E_M(N)\rangle$ if and only if $u \in \langle E_{M_1}(N_1)\rangle$ and $v \in \langle E_{M_2}(N_2)\rangle$ as required.

Corollary 2.2. Let $N=\prod_{i=1}^{n} N_i$ be a subsemimodule of M. Then $\langle E_M(N)\rangle = \prod_{i=1}^{n}\langle E_{M_i}(N_i)\rangle$.

Proof. This follows from Proposition 2.1

Proposition 2.3. If N is a classical prime subsemimodule of M, then $\langle E_M(N)\rangle = N$.

Proof. Clearly, $N\subseteq\langle E_M(N)\rangle$. To show that $\langle E_M(N)\rangle\subseteq N$. Let $x=\sum_{i=1}^{k} r_i m_i\in\langle E_M(N)\rangle$, where $r_i^{k_i}m_i\in N$ for some $k_i\in\mathbb{Z}^+$. Since N is a classical prime subsemimodule of M, we have $r_i m_i\in N$. Then $x=\sum_{i=1}^{k} r_i m_i\in N$ so that $\langle E_M(N)\rangle\subseteq N$. Hence $\langle E_M(N)\rangle = N$.

3. CLASSICAL PRIMARY SUBSEMIMODULES

In this section, we give some characterizations for classical primary subsemimodules of R-semimodule M.

Lemma 3.1. Let $M=M_1\times M_2$, where M_i is an R_i-semimodule. A subsemimodule $N_1\times M_2$ is a classical primary subsemimodule of M if and only if N_1 is a classical primary subsemimodule of M_1.

Proof. Suppose that $N_1\times M_2$ is a classical primary subsemimodule of R-semimodule M. We will show that N_1 is a classical primary subsemimodule of M_1. Clearly, N_1 is a proper subsemimodule of R_1-semimodule M_1. To show that classical primary subsemimodule properties of N_1 hold, $m\in M_1$ and $a,b\in R_1$ such that $abm\in N_1$. Then

$$(a,1)(b,1)(m,n)=(abm,n)\in N_1\times M_2.$$

Since $N_1\times M_2$ is a classical primary subsemimodule of R-semimodule M, it follows that

$$(am,n)=(a,1)(m,n)\in N_1\times M_2$$

or

$$(b^n m,n)=(b,1)^n(m,n)\in N_1\times M_2,$$

for some positive integer n. That is, $am\in N_1$ or $b^n m\in N_1$. Therefore N_1 is a classical primary subsemimodule of R_1-semimodule M_1. Conversely, suppose that N_1 is a classical primary subsemimodule of R_1-semimodule M_1. We will show that $N_1\times M_2$ is a classical primary subsemimodule of R-semimodule M. Clearly, $N_1\times M_2$ is a proper subsemimodule of R-semimodule M. To show that classical primary subsemimodule properties of $N_1\times M_2$ hold, let $(m,n)\in M$ and $(a_1,a_2),(b_1,b_2)\in R$ such that

$$(a_1b_1m,a_2b_2n)=(a_1,a_2)(b_1,b_2)(m,n)\in N_1\times M_2.$$

Since N_1 is a classical primary subsemimodule of R_1-semimodule M_1 and $a_1b_1m\in N_1$, we have $a_1m\in N_1$ or $b_1^n n\in N_1$, for some positive integer n. Therefore

$$(a_1,a_2)(m,n)=(a_1m,a_2n)\in N_1\times M_2$$

or

$$(b_1,b_2)^n(m,n)=(b_1^n m,b_2^n n)\in N_1\times M_2.$$

Hence $N_1\times M_2$ is a classical primary subsemimodule of R-semimodule M.

Corollary 3.2. Let $M = M_1 \times M_2$, where M_i is an R_i-semimodule. A subsemimodule $M_1 \times N_2$ is a classical primary subsemimodule of R-semimodule M if and only if N_2 is a classical primary subsemimodule of R_2-semimodule M_2.

Proof. This follows from Lemma 3.1.

Corollary 3.3. Let $M = \prod_{i=1}^{n} M_i$, where M_i is an R_i-semimodule. A subsemimodule

$$M_1 \times M_2 \times \ldots \times M_{j-1} \times N_j \times M_{j+1} \times \ldots \times M_n$$

is a classical primary subsemimodule of R-semimodule M if and only if N_j is a classical primary subsemimodule of R_j-semimodule M_j.

Proof. This follows from Lemma 3.1 and Corollary 3.2.

Lemma 3.4. Let $M = M_1 \times M_2$, where M_i is an R_i-semimodule. If $N_1 \times \{n\}$ is a classical primary subsemimodule of M, then N_1 is a classical primary subsemimodule of M_1.

Proof. Suppose that $N_1 \times \{n\}$ is a classical primary subsemimodule of R-semimodule M. We will show that N_1 is a classical primary subsemimodule of M_1. Clearly, N_1 is a proper subsemimodule of R_1-semimodule M_1. To show that classical primary subsemimodule properties of N_1 hold, let $m \in M_1$ and $a, b \in R_1$ such that $abm \in N_1$. Then

$$(a,1)(b,1)(m,n) = (abm,n) \in N_1 \times \{n\}.$$

Since $N_1 \times M_2$ is a classical primary subsemimodule of R-semimodule M, it follows that

$$(am,n) = (a,1)(m,n) \in N_1 \times \{n\}.$$

or

$$(b^n m,n) = (b,1)^n (m,n) \in N_1 \times \{n\}.$$

for some positive integer n. That is, $am \in N_1$ or $b^n m \in N_1$. Therefore N_1 is a classical primary subsemimodule of R_1-semimodule M_1.

Corollary 3.5. Let $M = M_1 \times M_2$, where M_i is an R_i-semimodule. If $\{n\} \times N_2$ is a classical primary subsemimodule of R-semimodule M, then N_2 is a classical primary subsemimodule of R_2-semimodule M_2.

Proof. This follows from Lemma 3.4.

Corollary 3.6. Let $M = \prod_{i=1}^{n} M_i$, where M_i is an R_i-senimodule. If $\{m_1\} \times \{m_2\} \times \ldots \times N_j \times \ldots \times \{m_n\}$ is a classical primary subsemimodule of R-semimodule M, then N_j is a classical primary subsemimodule of R_j-semimodule M_j.

Proof. This follows from Lemma 3.4 and Corollary 3.5.

4. RADICAL OF CLASSICAL PRIMARY SUBSEMIMODULES

A subsemimodule N of an R-semimodule M is said to satisfy the classical primary radical formula in M, if $\langle E_M(N)\rangle$ = $c.prad_M(N)$.

Lemma 4.1. Let $M = M_1 \times M_2$, where M_i is an R_i-semimodule. If W is a classical primary subsemimodule of R-semimodule M and $P = \{x \in M_1 : (x,y) \in W\}$, then $P = M_1$ or P is a classical primary subsemimodule of R_1-semimodule M_1.

Proof. Suppose that $P \neq M_1$. We will show that P is a classical primary subsemimodule of R_1-semimodule M_1. It is clear that, P is a proper subsemimodule of R_1-semimodule M_1. To show that classical primary subsemimodule properties of P, let $a,b \in R_1$ and $m \in M_1$ such that $abm \in P$. Then $(a,1)(b,1)(m,y) = (abm,y) \in W$. Since W is a classical primary subsemimodule of M, we have

$$(am,1) = (a,1)(m,y) \in W$$

or

$$(b^n m, y) = (b,1)^n (m,y) \in W,$$

for some positive integer n. It follows that $am \in P$ or $b^n m \in P$. Therefore P is a classical primary subsemimodule of M_1.

Corollary 4.2. Let $M = M_1 \times M_2$, where M_i is an R_i-semimodule. If W is a classical primary subsemimodule of R-semimodule M and $P = \{x \in M_2 : (0,x) \in W\}$, then $P = M_2$ or P is a classical primary subsemimodule of R_2-semimodule M_2.

Proof. This follows from Lemma 4.1.

Corollary 4.3. Let $M = \prod_{i=1}^{n} M_i$, where M_i is an R_i-semimodule. If W is a classical primary subsemimodule of R-semimodule M and $P = \{x \in M_j : (m_1, m_2, \ldots, x, m_{j+1}, \ldots, m_n) \in W\}$, then $P = M_j$ or P is a classical primary subsemimodule of R_j-semimodule M_j.

Proof. This follows from Lemma 4.1 and Corollary 4.2.

Lemma 4.4. Let $M = M_1 \times M_2$, where M_i is an R_i-semimodule and let N be a subsemimodule of R_1-semimodule M_1. Then $m \in c.prad_{M_1}(N)$ if and only if $(m,y) \in c.prad_{M_1}(N \times \{y\})$.

Proof. Suppose that $M = M_1 \times M_2$, where M_i is an R_i-semimodule. Let N be a subsemimodule of R_1-semimodule M_1 and let $m \in c.prad_{M_1}(N)$.

If there is no classical primary subsemimodule of M containing $N \times \{y\}$, then $c.prad_M(N \times \{y\}) = M$. Therefore $(m,y) \in c.prad_M(N \times \{y\})$.

If there is classical primary subsemimodule of M containing $N\times\{y\}$, then there exists a classical primary subsemimodule W with $N\times\{y\}\subseteq W$. By Lemma 4.1 and $P=\{x\in M_1:(x,y)\in W\}$, we have $P=M_1$ or P is a classical primary subsemimodule of R_1-semimodule M_1.

Case 1: $P=M_1$. Since $m\in c.prad_{M_1}(N)$, we have $m\in P$. Then $(m,y)\in W$. Therefore if W is a classical primary subsemimodule of M containing $N\times\{y\}$, then $(m,y)\in W$.

Case 2: $P\neq M_1$. Since $P\neq M_1$, we have P is a classical primary subsemimodule of R_1-semimodule M_1. Let $x\in N$. Then $(x,y)\in N\times\{y\}$ so that $x\in P$. It follows that $N\subseteq P$. We have

$$\begin{aligned} c.rad_{M_1}(N) &\subseteq c.rad_{M_1}(P) \\ &= P \end{aligned}$$

so that $m\in P$. Therefore if W is a classical primary subsemimodule of M containing $N\times\{y\}$, then $(m,y)\in W$ and hence $(m,y)\in c.prad_{M_1}(N\times\{y\})$.

Corollary 4.5. Let $M=M_1\times M_2$, where M_i is an R_i-semimodule and let N be a subsemimodule of R_2-semimodule M_2. Then $m\in c.prad_{M_2}(N)$ if and only if $(x,m)\in c.prad_M(\{x\}\times N)$.

Proof. This follows from Lemma 4.4.

Corollary 4.6 Let $M=\prod_{i=1}^{n}M_i$, where M_i is an R_i-semimodule and let N be a subsemimodule of R_j-semimodule M_j. Then $m\in c.prad_{M_j}(N)$ if and only if

$$(x_1,\ldots,m,x_{j+1},\ldots,x_n)\in c.prad_M(\{x_1\}\times\{x_2\}\times\ldots\times N\times\{x_{j+1}\}\times\ldots\times\{x_n\}).$$

Proof. This follows from Lemma 4.4 and Corollary 4.5.

Lemma 4.7. Let $M=M_1\times M_2$, where M_i is an R_i-semimodule. If N_i be a subsemimodule of R_i-semimodule M_i, then $c.prad_{M_1}(N_1)\times c.prad_{M_2}(N_2)\subseteq c.prad_M(N_1\times N_2)$.

Proof. Suppose that $M=M_1\times M_2$, where M_i is an R_i-semimodule. Let N_1 be a subsemimodule of R_1-semimodule M_1. We will show that $c.prad_{M_1}(N_1)\times .cprad_{M_2}(N_2)\subseteq c.prad_M(N_1\times N_2)$. Let

$$(x,y)\in c.prad_{M_1}(N_1)\times c.prad_{M_2}(M_2).$$

Then $x\in c.prad_{M_1}(N_1)$ and $y\in c.prad_{M_2}(N_1)$. By Lemma 4.1 and Lemma 4.4, we have

$$(x,0)\in c.prad_M(N_1\times\{0\})\subseteq c.prad_M(N_1\times N_2)$$

and

$$(0,y)\in c.prad_M(\{0\}\times N_2)\subseteq c.prad_M(N_1\times N_2).$$

Then $(x,y)=(x,0)+(0,y)\in c.prad_M(N_1\times N_2)$ and hence

$$c.prad_{M_1}(N_1)\times c.prad_{M_2}(N_2)\subseteq c.prad_M(N_1\times N_2).$$

Corollary 4.8. Let $M=\prod_{i=1}^{n}M_i$, where M_i is an R_i-semimodule. If N_i be a subsemimodule of R_i-semimodule M_i, then $\prod_{i=1}^{n}c.prad_{M_i}(N_i)\subseteq c.prad_M(\prod_{i=1}^{n}N_i)$.

Proof. This follows from Lemma 4.7.

Theorem 4.9. Let $M=M_1\times M_2$, where M_i is an R_i-semimodule. If N is a subsemimodule of R_1-semimodule M_1, then $c.prad_{M_1}(N_1)\times c.prad_{M_2}(M_2)=c.prad_M(N_1\times M_2)$.

Proof. Suppose that $M=M_1\times M_2$, where M_i is an R_i-semimodule. Let N be a subsemimodule of R_1-semimodule M_1. By Lemma 4.7, we have $c.prad_{M_1}(N)\times c.prad_{M_2}(M_2)\subseteq c.prad_M(N\times M_2)$. We will show that $c.prad_M(N_1\times M_2)\subseteq c.prad_{M_1}(N_1)\times c.prad_{M_2}(M_2)$. If there is no classical primary subsemimodule of M containing N, then $c.prad_{M_1}(N)=M_1$. Then

$$c.prad_M(N_1\times M_2)\subseteq c.prad_{M_1}(N_1)\times c.prad_{M_2}(M_2).$$

If there is classical primary subsemimodule of M containing N, then there exists W is a classical primary subsemimodule of M_1 containing N. Then $W\times M_2$ is a classical primary subsemimodule of M, containing $N\times M_2$. Let P be a classical primary subsemimodule of M containing $N\times M_2$. Then

$$\begin{aligned} N\times M_2 &\subseteq c.prad_{M_1}(N)\times M_2 \\ &= c.prad_{M_1}(N)\times c.prad_{M_2}(M_2). \end{aligned}$$

Therefore $c.prad_M(N_1\times M_2)\subseteq c.prad_{M_1}(N_1)\times c.prad_{M_2}(M_2)$ and hence

$$c.prad_M(N_1\times M_2)=c.prad_{M_1}(N_1)\times c.prad_{M_2}(M_2).$$

Corollary 4.10. Let $M=M_1\times M_2$, where M_i is an R_i-semimodule. If N is a subsemimodule of R_2-semimodule M_2, then $c.prad_M(M_2\times N)=c.prad_{M_1}(M_2)\times c.prad_{M_2}(N)$.

Proof. This follows from Lemma 4.9.

Corollary 4.11. Let $M=\prod_{i=1}^{n}M_i$, where M_i is an R_i-semimodule. If N_i be a subsemimodule of R_i-semimodule M_i, then $\prod_{i=1}^{n}c.prad_{M_i}(N_i)=c.prad_M(\prod_{i=1}^{n}N_i)$.

Proof. This follows from Lemma 4.9 and Corollary 4.10.

Theorem 4.12. Let $M=M_1\times M_2$, where M_i is an R_i-semimodule. If N_1 is a classical primary subsemimodule of M_1, then N_1 is to satisfy the classical primary radical formula in M_1 if and only if $N_1\times M_2$ is to satisfy the classical primary radical formula in M.

Proof. Suppose that N_1 is a classical primary subsemimodule of M_1 and N_1 is to satisfy the classical primary radical formula in M_1. We will show that $N_1\times M_2$ is to satisfy the classical primary radical formula in M. Since N_1 is a classical primary subsemimodule of M_1, it follows that

$$\begin{aligned} c.prad_M(N_1\times M_2) &= c.prad_{M_1}(N_1)\times c.prad_{M_2}(M_2) \\ &= \langle E_{M_1}(N_1)\rangle \times M_2 \\ &= \langle E_M(N_1\times M_2)\rangle. \end{aligned}$$

Therefore $N_1\times M_2$ is to satisfy the classical primary radical formula in M. Conversely, suppose that N_1 is a classical primary subsemimodule of M_1 and $N_1\times M_2$ is to satisfy the classical primary radical formula in M. We will show that N_1 is to satisfy the classical primary radical formula in M_1. Since $N_1\times M_2$ is a classical primary subsemimodule of M, it follows that

$$\begin{aligned} \langle E_{M_1}(N_1)\rangle \times M_2 &= \langle E_M(N_1\times M_2)\rangle \\ &= c.prad_{M_1}(N_1)\times c.prad_{M_2}(M_2). \end{aligned}$$

Then $c.prad_{M_1}(N_1)=\langle E_{M_1}(N_1)\rangle$ and hence N_1 is to satisfy the classical primary radical formula in M_1.

Corollary 4.13. Let $M=\prod_{i=1}^{n} M_i$, where M_i is an R_i-semimodule. If N_j is a classical primary subsemimodule of M_j, then N_j is to satisfy the classical primary radical formula in M_j if and only if

$$M_1\times M_2\times\ldots\times M_{j-1}\times N_j\times M_{j+1}\times\ldots\times M_n$$

is to satisfy the classical primary radical formula in M.

Proof. This follows from Theorem 4.12.

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