

Interdependence of ISE 30 and Gold Spot, Natural Gas, Brent Oil: Copula-Garch Method

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ABSTRACT---- *This paper aims to examine the relationship between ISE 30 and gold spot, natural gas and Brent Oil with the COPULA-GARCH method. In the study, we use closing prices of ISE 30 and Gold Spot, Brent Oil and Natural Gas. The results show that there is a weak dependence between ISE 30 and Brent Oil, Gold Spot but there is more strong between ISE 30 and Natural Gas.*

Keywords---- ISE 30, COPULA-GARCH, Gold Spot, Brent Oil, Natural Gas

1. INTRODUCTION

Today, oil, an important energy source, directly or indirectly affects the economies of all countries in the world. Changes in oil prices affect the economy in various ways [1]. First, the rise in oil prices increases the cost of production which results decrease in productivity. Second, it disrupts the foreign trade balance of oil importing countries where oil importer countries transfer wealth towards oil exporter countries. As a result of the wealth transfer, the purchasing power of companies and households falls in the oil importer countries. This effect has different implications due to being an oil importer or exporter country. Third, due to the real balance effect [2], the rise in oil prices increases the demand for money. As the monetary authority fails to meet the increase in money demand, interest rates rise which results a reduction in the economic activity. As the monetary authority fails to meet the increase in money demand, an increase in interest rates will cause a fall in economic activity [3]. Fourth, the increase in oil prices will causes inflation which leads to the spiral of price-wage increases. Fifth, it has a negative impact on consumption, investment and stocks. Increasing oil prices leads to a decrease in disposable income and in consumption. Furthermore, investments decrease due to increased costs. Finally, the rise in oil prices reduces employment in the country [4-5] .

Among the emerging economies, Turkey has a significant energy consumption. In the process of economic growth and development, Turkey's hunger for energy has been increasing. Moreover, energy, as a significant input that facilitates economic growth and development, has a strategic importance. However, Turkey does not have the sufficient energy production which results the increase in costs in Turkey, in case of any increase in the price of energy, due to being dependent on external sources.

The main purpose of this study is to examine the dependence structures and/or tail dependence between oil price changes and stock market indices using the Copula-Garch method. In the majority of previous studies, researchers used traditional time series models such as vector autoregression (VAR) and cointegrated vector error-correction (VEC) models [6-7-8-9-10-11-12] . The dependence structure estimated via copulas is more robust since it separates the dependence structure from the choice of margins [13]. The Copula method provides two important advantages in multivariate analysis [14]. Copulas capture nonlinear dependencies and can provide the structure of the dependence and are invariant to increasing and continuous transformations [15]. First, this method allows researchers to separately model marginal distributions and dependencies and their related effects. Second, the copula method captures the dependence between variables completely. There exist a large number of copulas to capture a myriad of dependence structure, allowing fat-tailedness, asymmetry, tail dependence and so on. The dependence structures among financial markets are important for effective risk management and international asset allocation [16].

2. MATERIAL AND METHOD

2.1. Copula Functions

The copula function is presented to measure dependence of multivariate variables. Based on the famous Sklar's theorem [17], copulas allow to put in place the fruitful idea of splitting the specification of a multivariate model into two parts: the marginal distributions on one side, the dependence structure (copula) on the other part. Let X and Y be random variables with continuous distribution functions F_X and F_Y , which are uniformly distributed on the interval $[0,1]$. Then, there is a copula such that for all $x, y \in R$,

$$F_{XY}(X, Y) = C(F_X(X), F_Y(Y)) \quad (1)$$

The copula C for (X, Y) is the joint distribution function for the pair $F_X(X), F_Y(Y)$ provided F_X and F_Y continuous. The copula C for (X, Y) is the joint distribution function for the pair $F_X(X), F_Y(Y)$ provided F_X and F_Y continuous. The joint probability density of the variables X and Y is obtained from the copula density $(u, v) = \frac{\partial^2 C(u, v)}{\partial u \partial v}$, as follows:

$$f_{xy}(x, y) = c(u, v) f_x(x) f_y(y), \quad (2)$$

where $f_x(x)$ and $f_y(y)$ are the marginal densities of the random variables X and Y . According to Sklar (1959) an n-dimensional joint distribution can be decomposed into its n-univariate marginal distributions and an n-dimensional copula. In the extension of Sklar's theorem to continuous conditional distributions, Patton (2006) shows that the lower (left) and upper (right) tail dependence of two random variables is given for the copula as:

$$\lambda_l = \lim_{u \rightarrow 0} P(F_X(x) \leq u | F_Y(x) \leq u) = \lim_{u \rightarrow 0} C(u, u)/u \quad (3)$$

$$\lambda_u = \lim_{u \rightarrow 1} P(F_X(x) > u | F_Y(x) > u) = \lim_{u \rightarrow 1} 1 - 2u - C(u, u)/1 - u \quad (4)$$

where λ_l and $\lambda_u \in [0, 1]$.

2.2. Copula Models

We introduce several copula models in this section; Gumbel copula, Clayton copula, Frank copula Gaussian copula, Student t copula and Joe copula.

Gumbel Copula: This Archimedean copula is defined with the help of generator function $\phi(t) = (-\ln t)^\theta, \theta \geq 1$;

$$C_\theta(u, v) = \exp\left(-[(-\ln u)^\theta + (-\ln v)^\theta]^{1/\theta}\right) \quad (5)$$

where θ is the copula parameter restricted to $[1, \infty)$. This copula is asymmetric, with more weight in the right tail. Beside this, it is extreme value copula [17].

Clayton Copula: This Archimedean copula is defined with the help of generator function $\phi(t) = \frac{t^{-\theta} - 1}{\theta}$,

$$C_\theta(u, v) = (u^{-\theta} + v^{-\theta} - 1). \quad (6)$$

where θ is the copula parameter restricted to $(0, \infty)$. This copula is also asymmetric, but with more weight in the left tail [17].

Frank Copula: This Archimedean copula is defined with the help of generator function; $\phi(t) = -\ln \frac{e^{-\theta t} - 1}{e^{-\theta} - 1}$;

$$C_{\theta}(u, v) = -\frac{1}{\theta} \ln \left(1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{(e^{-\theta} - 1)} \right) \quad (7)$$

where θ is the copula parameter restricted to $[-1, 1]$.

Joe Copula: This Archimedean copula is defined with the help of generator function; $\phi(t) = -\ln[1 - (1-t)^{\theta}]$

$$C_{\theta}(u, v) = 1 - [(1-u)^{\theta} + (1-v)^{\theta} - (1-u)^{\theta}(1-v)^{\theta}]^{1/\theta} \quad (8)$$

where θ is the copula parameter restricted to $[1, \infty)$ [17].

Gaussian copula: The copula function;

$$C(u, v; \rho) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(\frac{2\rho rs - r^2 - s^2}{2(1-\rho^2)}\right) dr ds \quad (9)$$

where $u = F_{Y_1}(y_1)$, $v = F_{Y_2}(y_2)$ is the inverse of the standard normal distribution and ρ is the general correlation coefficient.

Student t copula: This copula allows for joint fat tails and an increased probability of joint extreme events compared with the Gaussian copula. This copula can be written as;

$$C_{\rho, \nu}(u, v) = \int_{-\infty}^{t_V^{-1}(u)} \int_{-\infty}^{t_V^{-1}(v)} \frac{1}{2\pi(1-\rho^2)^{1/2}} \left\{ 1 + \frac{x^2 - 2\rho xy + y^2}{\nu(1-\rho^2)} \right\}^{-(\nu+2)/2} ds dt \quad (10)$$

where ρ, ν parameters of the t copula.

The BB1 Copula (Clayton-Gumbel) copula is given by

$$C(u, v) = 1 - (u_1^{-\theta} + u_2^{-\theta} - u_1^{-\theta} u_2^{-\theta})^{1/\theta} \quad (11)$$

with $\theta \in [1, \infty)$.

The BB6 Copula (Joe-Gumbel) copula is

$$C(u_1, u_2, \theta, \delta) = 1 - (1 - \exp\{ - [(-\log(1 - u_1^{-\theta}))^{\delta} + (-\log(1 - u_2^{-\theta}))^{\delta}]^{\frac{1}{\delta}} \})^{\frac{1}{\theta}} \quad (12)$$

with $\theta \in [1, \infty) \cap \delta \in [1, \infty)$.

The BB7 (Joe-Clayton) copula is given by

$$C(u_1, u_2, \theta, \delta) = 1 - (1 - [(1 - u_1^{-\theta})^{-\delta} + (1 - [(1 - u_2^{-\theta})^{-\delta} - 1]^{\frac{1}{\delta}})]^{\frac{1}{\theta}} \quad (13)$$

with $\theta \in [1, \infty) \cap \delta \in [0, \infty)$.

The BB8 (Frank-Joe) copula is

$$C(u_1, u_2, \theta, \delta) = \frac{1}{\delta} (1 - [1 - \frac{1}{1-(1-\delta)^{\theta}} (1 - (1 - \delta u_1)^{\theta})(1 - (1 - \delta u_2)^{\theta})]^{\frac{1}{\theta}}) \quad (14)$$

with $\theta \in [1, \infty) \cap \delta \in (0, 1]$.

2.3. Marginal Modelling

In order to build the model for bivariate distribution with the copula, primarily the marginal distribution for the series must be formed. For this, there are models that it has been commonly accepted financial time series returns. ARCH and GARCH model, proposed Engle (1986) and Bollerslev (1986), which have been widely applied Financial series. There are a few GARCH model. In this paper, we combine ARMA (m,n) and ARCH(p,q), GARCH (1,1), EGARCH nad GJR-GARCH (p,q) model to daily financial returns. This models;

$$r_t = \lambda_0 + \sum_{j=1}^m \lambda_j r_{t-j} + \varepsilon_t - \sum_{i=1}^n \theta_i \varepsilon_{t-i} \tag{15}$$

$$r_t = w_0 + \sum_{i=1}^q \beta_i u_{t-1}^2 \tag{16}$$

$$r_t = w_0 + \sum_{i=1}^q \alpha_i u_{t-1}^2 + \sum_{j=1}^p \beta_j u_{t-1}^2 \tag{17}$$

$$\log(r_t) = w_0 + \sum_{i=1}^q \alpha_i \frac{|u_{t-i}|}{\sqrt{r_{t-i}}} + \sum_{i=1}^q \gamma_i \frac{u_{t-i}}{\sqrt{r_{t-i}}} + \sum_{j=1}^p \beta_j \log(u_{t-j}) \tag{18}$$

$$r_t = w_0 + \sum_{i=1}^p \beta_i r_{t-i} + \sum_{j=1}^q \alpha_j u_{t-j}^2 + \sum_{i=1}^q \gamma_j u_{t-j}^2 I_{t-j} \tag{19}$$

where m,n,p, q are positive integers , $u_t = \eta_t \sqrt{h_t}$, $\eta_t \sim f(0,1)$, respectively λ_j , θ_i parameters of (AR) and (MA), $w_0, \beta_i, \alpha_j, \gamma_j$ are ARCH(p,q), GARCH (1,1), EGARCH nad GJR-GARCH (p,q) model parameters.

3. DATA

XU030 data was downloaded from www.investing.com as daily prices between 02.01.2003 – 13.04.2017. Table 1 submit the stochastic properties of financial series over the period (2003-2017) and Table 2 submit the stochastic properties of the daily returns of the XU030, Gold Spot, Brent Oil and Natural Gas over the period (2003-2017). This table indicate the positive average return series for Gold Spot and Brent Oil, the negative average return series for XU030 and Natural Gas. Among all the series, Natural Gas is the most volatile as indicated by the standard deviation. The positive values for skewness are extensive for all the series with the exception of the Natural Gas. The kurtosis value is greater than three, indicating the presence of a fat tailed distribution for all the returns series. In addition, the Jarque-Bera test rejects the null of Gaussian distribution for all the series. By applying the two-unit root tests (ADF and PP), the stationary test (KPSS), we find that all return series are stationary. Finally, the Autoregressive Conditional Heteroscedasticity –Lagrange Multiplier (ARCH-LM) test indicates that there exist strong ARCH effects in all the four financial return series.

Table 1. Summary Statistics

	XU030	Gold Spot	Brent Oil	Natural gas
Mean	70502,58	1010,029	71,68646	5060,293
Median	71042,27	1125,250	65,26000	4405,000
Maksimum	147935,8	1898,100	143,9500	15378,00
Minumum	12951,90	323,0000	23,23000	1639,000
Std.Dev	29767,28	418,0472	28,89175	2360,924
Skewness	0,131419	-0,108938	0,306036	1,324039
Kurtosis	2,352783	1,870957	1,877946	5,052819
Jarque Bera	74,92431	203,0140	250,8313	1723,718
Probability	0,00000	0,000000	0,00000	0,00000

Table 2. Summary Statistics of return series

	XU030	Gold Spot	Brent Oil	Natural gas
Mean	-0,000653	0,000368	0,000228	-0,000148
Median	-0,000860	0,000588	0,000354	-0,000757
Maksimum	0,109019	0,104371	0,181297	0,324354
Minimum	-0,127255	-0,088756	-0,168320	-0,198993
Std.Dev	0,018059	0,011856	0,021999	0,032895
Skewness	0,062663	0,326240	0,071758	-0,777334
Kurtosis	6,407444	9,272974	7,713053	9,731479
Jarque Bera	1784,650	6105,605	3412,836	7326,525
Probability	0.000000	0.000000	0.000000	0.000000
ARCH LM	58,67665	145,3111	39,77587	17,67795
ADF	-61,07679	-60,05891	-58,25491	-61,46372
PP	-61,08128	-60,05663	-58,24363	-61,48292
KPSS	0,052393	0,159194	0,153248	0,037679
Q(20)	7,0920	7,2645	7,1419	6,8666
No. Of Obs	3685	3685	3685	3685

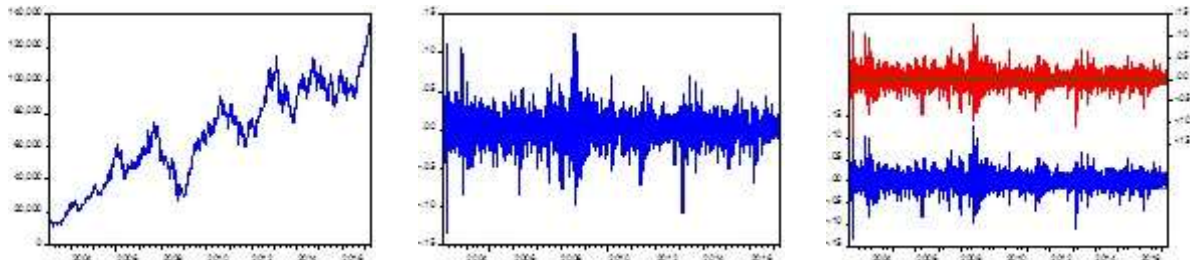


Figure 1: Change over years of XU030 series and XU030 return series and Residual graphs

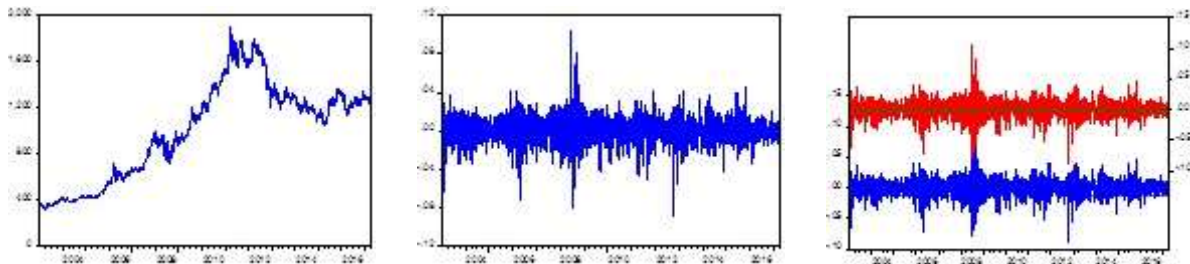


Figure 2: Change over years of Gold Spot series and Gold Spot return series and Residual graphs

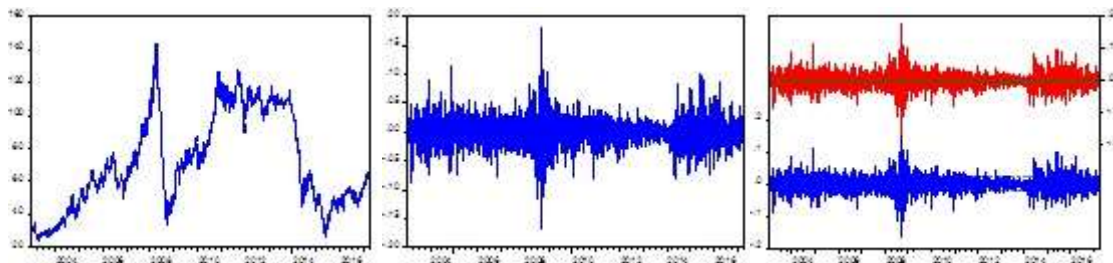


Figure 3: Change over years of Brent Oil series and Brent Oil return series and Residual graphs

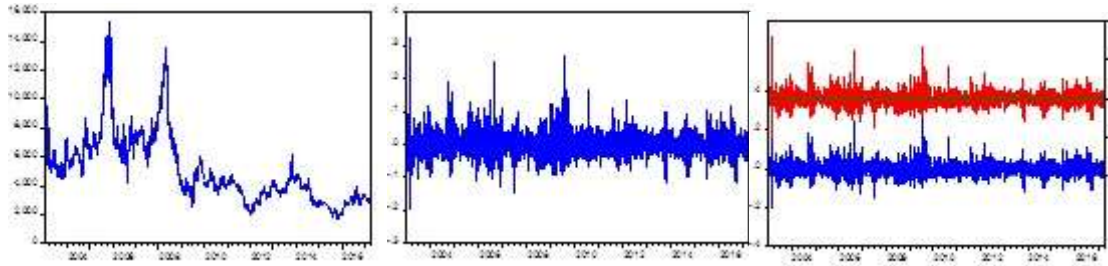


Figure 4: Change over years of Natural gas series and Natural gas return series and Residual graphs

3.1. Results of marginal distribution

In this paper, the ARMA (m,n) GARCH models for financial return series are used. We choose the most suitable with the help of criteria AIC, SIC and HQIC. Table 3 and Table 4 which summarize the best model for all the four marginal distributions: the best models for the marginal; XU030 Gold Spot, Brent Oil and Natural gas; ARMA (3,2)-EGARCH (1,1,1) student t, ARMA (4,4)-GJR-GARCH (1,1) student t, ARMA (4,4)-EGARCH (1,1,1) student t, ARMA (4,4)-EGARCH (1,1,1) student t respectively. Here, the EGARCH model has been chosen to take into account the asymmetric effect. It is seen that γ parameter, which has asymmetrical effect, is positive statistically significant for natural gas return series and for the XU030 and Brent Oil return series is negative. Namely, from table 4, XU030 and Brent Oil have leverage effect, XU030 and Brent Oil have a negative correlation between the past and the future volatility of returns. This state, it shows that for XU030 and Brent Oil asymmetrical effect are stated that according to good news of bad news, the stock has further increased its return volatility. From table 4, Gold Spot has ARMA (4,4)-GJR-GARCH (1,1), namely this model considers that effect of negative shocks and positive shocks is not symmetrical. In table 4, for Gold Spot return series, $\gamma < 0$, it shows that the effect on volatility of positive news indicates more than the effect of negative news. According to table 4 Results of ARCH-LM test (Table 4) show that XU030, Brent Oil and Natural Gas are neither autocorrelation nor ARCH effects exist in the residuals but, in table 4, for Gold Spot, it is seen that the variance problem and the ARCH effect are not completely solved.

Table 3. Mean Equation

Mean Equation	XU030 (4,4)	Gold Spot (4,4)	Brent Oil (4,4)	Natural gas (4,4)
λ_1	0,091393	-1,278051	-1,091093	-0,511633
λ_2	1,084073	-0,832286	-0,354479	-00,17724
λ_3	-0,055822	-1,238166	-1,083223	-0,545121
λ_4	-0,876281	-0,909126	-0,972216	-0,812788
θ_1	-0,103965	1,277735	1,091562	-0,517096
θ_2	-1,089174	0,808185	0,359669	0,207752
θ_3	0,063884	1,230222	1,105343	0,512102
θ_4	0,847465	0,926261	0,983075	0,809544
AIC	-5,12471	-6,033586	-4,798550	13,38525
SIC	-5,110613	-6,016725	-4,781688	13,40211
HQIC	-5,121470	-6,027584	-4,792548	13,39125

Table 4. Variance Equation

Variance Equation	XU030		Gold Spot		Brent Oil		Natural gas	
	Gaussian	Student t	Gaussian	Student t	Gaussian	Student t	Gaussian	Student t
ARCH(1,0)								
w_0	0,000293	0,000291	0,000120	0,000127	0,000416	0,000424	0,000871	0,000944
β	0,156081	0,171638	0,151878	0,141077	0,281357	0,154450	0,212928	0,119735
AIC	-	-	-	-6,203461	-	-4,938634	-	-4,129189
SIC	5,159615	5,253597	6,062482	-6,196716	4,941518	-4,931889	4,027482	-4,122444
ARCH LM	5,154557	5,246853	6,057423	-6,196716	4,937201	-4,931889	4,022424	-4,122444
Probability	0,009111	0,175994	0,137327	0,000770	0,015181	0,203494	0,585391	0,591041
	0,9240	0,6748	0,7110	0,9779	0,9019	0,6519	0,4442	0,4420
GARCH(1,1)								
w_0	5,78E-06	4,44E-06	1,12E-06	9,27E-07	1,11E-06	1,12E-06	1,56E-05	2,01E-015
α	0,071282	0,062744	0,041681	0,037078	0,042299	0,039089	0,066405	0,059656
β	0,912450	0,924535	0,950134	0,956521	0,956190	0,959349	0,921375	0,920844
AIC	-	-	-	-6,286139	-	-	-	-4,197193
SIC	5,293915	5,339525	6,213366	-6,277709	5,014436	5,048675	4,128564	-4,188762
ARCH LM	5,287170	5,331094	6,206621	-6,277709	5,007691	5,040244	4,121819	-4,188762
Probability	0,527507	1,343747	10,33822	15,84413	0,239000	0,090033	0,073371	0,192731
	0,4677	0,2464	0,0013	0,0001	0,6249	0,7641	0,7865	0,6607
EGARCH(1,0,1)								
w_0	-	-	-	-9,005908	-	-	-	-
α	8,161278	8,157195	9,075447	0,214887	7,827831	7,800080	7,105400	6,9997761
γ	0,225078	0,229057	0,245213	-0,070960	0,230518	0,239340	0,331653	0,195122
AIC	-	-	-	-6,199981	-	-	-	-4,126759
SIC	5,156514	5,252132	6,060198	-6,191550	4,821720	4,936032	4,022348	-4,118328
ARCH LM	5,149770	5,243701	6,053453	-6,191550	4,814975	4,927651	4,015604	-4,118328
Probability	1,098925	0,627135	0,520245	1,820095	2,430359	2,676269	0,122077	3,681254
	0,2945	0,4284	0,4707	0,1773	0,1190	0,1019	0,7268	0,0550
EGARCH(0,1,1)								
w_0	-	-	-	-4,591960	-	-0,038480	-	-10,28621
γ	0,490530	0,573489	4,115993	-0,090243	0,079891	-0,029277	8,923357	-0,017373
β	0,077465	0,094684	0,083775	0,480277	0,044884	0,995098	0,028191	-0,504465
AIC	0,938911	0,928653	0,536373	-6,192302	0,989760	-4,955395	0,306515	-4,120062
SIC	5,178566	-5,26473	6,037769	-6,183871	4,891704	-4,946964	3,989809	-4,111631
ARCH LM	5,171165	-5,25630	6,031051	-6,183871	4,884959	-4,946964	3,983064	-4,111631
Probability	43,82563	38,09718	36,62184	36,81364	20,03207	29,69820	19,15973	18,26494
	0,0000	0,0000	0,00000	0,0000	0,00000	0,00000	0,0000	0,0000
EGARCH(1,1,1)								
w_0	-	-	-	-0,126331	-	-0,086584	-	-0,238740
α	0,320760	0,295327	0,164738	0,090762	7,633746	0,076884	0,215018	0,137357
	0,153358	0,143329	0,106468		0,010000		0,146041	

γ	-	-0,05370	0,001761	0,025767	0,010000	-0,038812	0,003331	0,009183
β	0,052580	0,97732	0,990527	0,993394	0,010000	0,996479	0,984986	0,980816
AIC	-	-5,34419	-	-	-	-5,055727	-	-4,199140
SIC	-	-5,33407	-	-6,277962	-	-5,045610	-	-4,189023
ARCH LM	0,335602	0,60967	22,85389	60,99098	133,4673	0,000350	0,048564	0,098474
Probability	0,5624	0,4349	0,00000	0,00000	0,00000	0,9851	0,8256	0,7537
GJR-GARCH	Gaussian	Student t	Gaussian	Student t	Gaussian	Student t	Gaussian	Student t
w_0	7,57E-06	6,61E-06	1,08E-06	8,07E-07	7,46E-07	6,49E-07	1,57E-05	2,05E-05
α	0,042067	0,003833	0,050134	0,55406	0,009428	0,011518	0,066787	0,063294
γ	0,066152	0,062373	-	-0,034584	0,051649	0,043492	-	-0,00818
β	0,902220	0,910036	0,950414	0,958233	0,963833	0,965510	0,921371	0,920542
AIC	-	-	-	-6,289445	-	-5,055483	-	-4,196735
SIC	-	-	-	-6,279328	-	-5,045367	-	-4,186618
ARCH LM	0,260751	0,674007	15,28889	36,46513	0,177160	0,108969	0,070893	0,156678
Probability	0,129960	0,014129	0,0001	0,00000	0,6738	0,7413	0,7900	0,6922

3.2. Results for the copula models

In this study, to model dependence of XU030- Gold Spot, XU030- Brent Oil and XU030-Natural Gas pairs, we use copula family in table 5, table 6 and table 7. From Table 5,6 and 7, it is obvious that the for C, XU030- Brent Oil and XU030-Natural Gas pairs, Survival BB8, Rotated Tawn Type 2 180 degrees and Rotated BB8 270 degrees copula best performs according to the AIC, BIC criteria respectively. The obtained tail dependence values for the pairs, while XU030- Gold Spot and B XU030-Natural Gas have symmetric tail dependency, XU030- Brent Oil pair has $\lambda_u = 0$ and $\lambda_l = 0,42$ namely for distribution of this pairs has lower tail dependence.

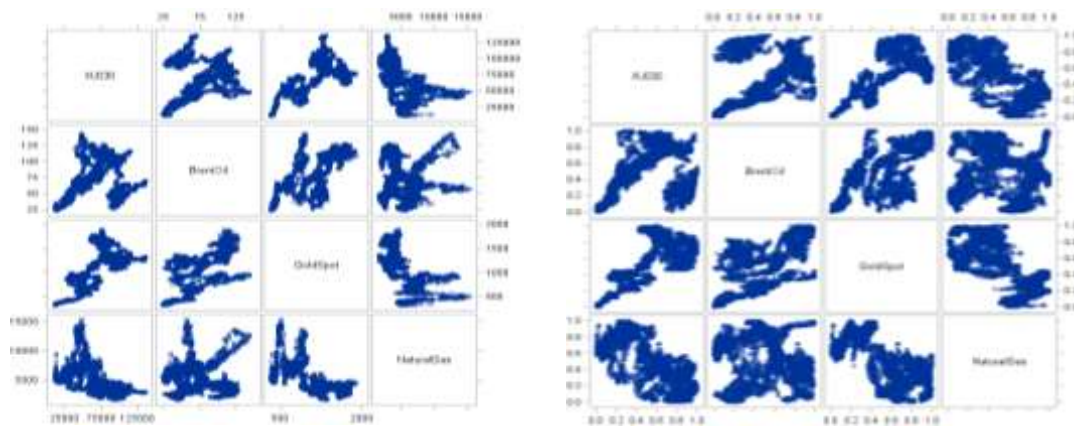


Figure 6. For XU030- Gold Spot, Brent Oil and Natural Gas Raw Data and Transformed Data Scatter Graph

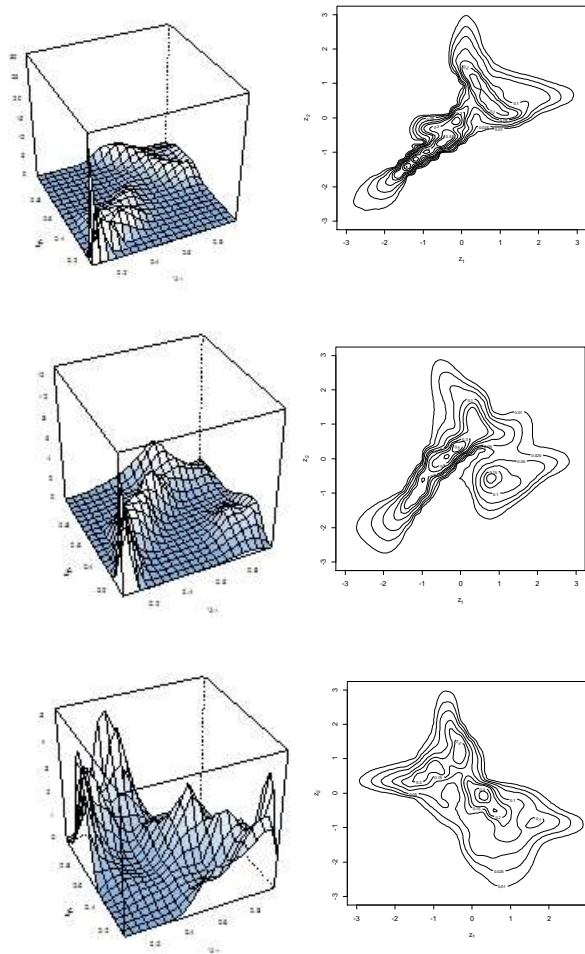


Figure 7. For XU030- Gold Spot, XU030- Brent Oil and XU030-Natural Gas pairs Kernel distribution estimation function graphs.

Table 5. XU030-Gold Spot

Family	θ	ρ	V	λ_u	λ_l	LogL	AIC	BIC
Gaussian								
Student t	-	0,72	6,88	0,29	0,29	1240,53	-2477,05	-2464,63
Clayton	3,08			0	0,8	2356,04	-4710,08	-4703,87
Gumbel	1,66			0,48	0	646,23	-1290,46	-1284,25
Frank	6,71			0	0	1415,47	-2828,93	-2822,72
Joe	1,48			0,4	0	154,91	-307,82	-301,6
BB1		3,08	1	0	0,8	2355,18	-4706,36	4693,94
BB6		1	1,65	0,48	0	645,44	-1286,89	-1274,47
BB7		1	3,08	0	0,8	2355,68	-4707,37	-4694,94
BB8		6	0,57	0	0	1018,83	-2033,65	2021,23
Survival Clayton	0,62			0,33	0	295,88	-589,75	-583,54
Survival Gumbel	2,33			0	0,65	1826,11	-3650,21	-3644
Survival Joe	3,84			0	0,8	2392,48	-4782,95	-4776,74
Survival BB1		0	2,33	0	0,65	1824,83	-3645,66	-3633,24
Survival BB6		3,84	1	0	0,8	2392,04	-4780,08	-4767,66
Survival BB7		3,84	0	0	0,8	2392	-4779,99	-4767,57
Survival BB8		4,18	0,99	0	0	2470,79	-4937,59	4925,16
Tawn type 1		1,7	0,74	0,42	0	546,59	-1089,18	-1076,76
Rotated Tawn Type 1 180 degrees		2,88	0,74	0	0,61	1829	-3655,56	-3643,13
Tawn type 2		1,92	0,74	0,48	0	680,24	-1356,49	-1344,07
Rotated Tawn Type 2 180 degrees		2,47	0,74	0	0,57	1638,03	-3272,06	3259,64

Table 6. XU030-Brent Oil

Family	θ	ρ	V	λ_u	λ_l	LogL	AIC	BIC
Gaussian								
Student t		0,36		0	0	256,84	-511,68	-505,47
Clayton	-	0,37	4,48	0,17	0,17	301,77	-599,54	-587,11
Gumbel	1,02			0	0,51	719,64	-1437,27	-1431,06
Frank	1,17			0,19	0	66,83	-131,67	-125,46
Joe	2,2			0	0	219,01	-436,01	-429,8
BB1	1			0	0	-0,02	2,05	8,26
BB6		1,01	1	0	0,51	718,59	-1433,17	-1420,75
BB7		1	1,7	0,19	0	66,21	-128,41	-115,99
BB8		1	1,02	0	0,51	719,01	-1434,02	-1421,59
Survival Clayton		6	0,28	0	0	167,17	-330,33	-317,91
Survival Gumbel	0,03			0	0	0,57	0,87	7,08
Survival Joe	1,43			0	0,38	554,83	-1107,65	-1101,44
Survival BB1	1,88			0	0,55	789,09	-1576,18	-1569,97
Survival BB6		0	1,43	0	0,37	553,88	-1103,75	-1091,33
Survival BB7		1,88	1	0	0,55	788,6	-1573,21	-1560,78
Survival BB8		1,88	0	0	0,55	788,42	-1572,84	-1560,42
Tawn type 1		1,93	1	0	0	799,69	-1595,38	-1582,96
Rotated Tawn Type 1 180 degrees		1,67	0,41	0,28	0	152,86	-301,73	-289,3
Tawn type 2		1,13	0,47	0,1	0	26,58	-49,17	-36,74
Rotated Tawn Type 2 180 degrees		2,75	0,47	0	0,42	867,2	-1730,4	-1717,97

Table 7. XU030-Natural Gas

Family	θ	ρ	V	λ_u	λ_l	LogL	AIC	BIC
Gaussian		-0,58		0	0	757,22	-1512,44	-1506,23
Student t	-	-0,58	30	0	0	719,74	-1435,47	-1423,05
Frank	-4,95			0	0	986,94	-1971,88	-1965,67
Rotated Clayton 90 degrees	-0,87			0	0	589,36	-1176,72	-1170,51
Rotated Gumbel 90 degrees	-1,48			0	0	444,58	-887,15	-880,94
Rotated Joe 90 degrees	-1,5			0	0	195,52	-389,05	-382,84
Rotated BB1 90 degrees		-0,79	1,05	0	0	590,21	-1176,42	-1163,99
Rotated BB6 90 degrees		-1	1,48	0	0	444,15	-884,29	-871,87
Rotated BB7 90 degrees		-1	0,87	0	0	589,12	-1174,25	-1161,82
Rotated BB8 90 degrees		-6	0,54	0	0	862,6	-1721,22	-1708,79
Rotated Clayton 270 degrees	-0,63			0	0	339,11	-676,21	-670
Rotated Gumbel 270 degrees	-1,56			0	0	618,77	-1235,55	-1229,34
Rotated Joe 270 degrees	-1,77			0	0	488,19	-974,38	-968,17
Rotated BB1 270 degrees		0	1,56	0	0	618,64	-1233,27	-1220,85
Rotated BB6 270 degrees		-1	1,56	0	0	618,61	-1233,22	-1220,8
Rotated BB7 270 degrees		-1,77	0	0	0	488,19	-972,38	-959,96
Rotated BB8 270 degrees		-6	-0,6	0	0	1014,68	-2025,37	-2012,95
Rotated Tawn Type 1 90 degrees		-1,52	0,66	0	0	363,97	-723,94	-711,51
Rotated Tawn Type 1 270 degrees		-1,77	0,66	0	0	592,44	-1180,88	-1168,46
Rotated Tawn Type 2 90 degrees		-1,64	0,66	0	0	430,1	-856,2	-843,78
Rotated Tawn Type 2 270 degrees		-1,61	0,66	0	0	509,07	-1014,13	-1001,71
Gaussian		-0,58		0	0	757,22	-1512,44	-1506,23

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