Asian Journal of Applied Sciences (ISSN: 2321 – 0893)
Volume 06 – Issue 06, December 2018

On the Problem of Tangency of Ellipse Curve

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ABSTRACT— ***Given a ellipse curve and a point in the exterior region, there are always two tangents from the given point to the curve.***

Keywords— Ellipse curve, Tangents, Exterior region

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1. INTRODUCTION

Finding the points of the tangents from the given point to the ellipse.

In 2002, David R. Duncan and Bonnie H. Litwiller have studied finding the points of tangents to parabola

$$y = kx^2 \text{ with } k > 0$$

through the given point see more detail in]1[.

In 2017, Apisit Pakapongpun has improved to find the formula form of the given point of the tangents from parabola

$$y = Ax^2 + Bx + D$$

where A, B and D are constants and A is not zero, through the given point see more detail in]2[.

In this paper has been found that the form of the points of the tangents from the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

where a and b are constants and neither nor zero through the given point. The set of the points in the exterior region of the ellipse are represented by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} > 1,$$

the set of the points in the interior region of the ellipse are represented by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} < 1$$

and the set of the points on the ellipse are represented by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

2. MAIN RESULTS

Let $P(c,d)$ be a point not on the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

and let $Q(e,f)$ be a point on the ellipse, the line PQ is the tangent to the ellipse,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ at } Q \text{)see Figure 1(.}$$

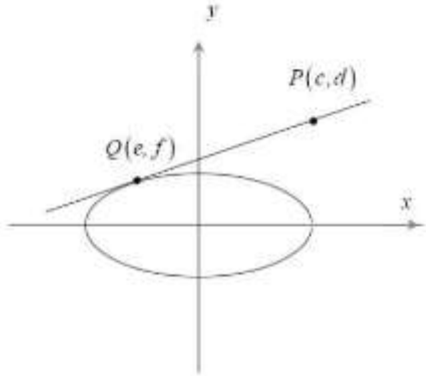

Figure 1: The line PQ.

The slope of the line PQ is

$$\frac{d-f}{c-e}$$

and by the calculus, the slope of the line through Q, tangent to the ellipse, is the derivative of

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$$

evaluated at the point $Q(e,f)$. Since

$$y' = -\frac{b^2x}{a^2y},$$

the slope is $-\frac{b^2e}{a^2f}$. This gives us

$$\frac{d-f}{c-e}=-\frac{b^2e}{a^2f}, \tag{1}$$

thus

$$b^2e^2+a^2f^2=b^2ce+da^2f. \tag{2}$$

From (1) and (2) we get

$$a^2b^2-b^2ce=da^2f$$

square both sides, thus

$$a^4b^4-2a^2b^4ce+b^4c^2e^2=a^4d^2f^2. \tag{3}$$

Since $Q(e,f)$ lies on the ellipse, the equation $\frac{e^2}{a^2}+\frac{f^2}{b^2}=1$ holds.

Hence,

$$b^2e^2+a^2f^2=a^2b^2$$

rewriting

$$f^2=\frac{b^2}{a^2}(a^2-e^2) \tag{4}$$

and putting f^2 into (3), thus

$$(b^4c^2+a^2b^2d^2)e^2-2a^2b^4ce+(a^4b^4-a^4b^2d^2)=0$$

we get the solutions e:

$$e=\frac{-B\pm\sqrt{B^2-4AC}}{2A} \tag{5}$$

where

$$A=b^4c^2+a^2b^2d^2$$
$$B=-2a^2b^4c$$
$$C=a^4b^4-a^4b^2d^2.$$

To evaluate this quadratic equation has solutions for e, consider its discriminant,

$$\begin{aligned}D&=B^2-4AC\\&=(-2a^2b^4c)^2-4(b^4c^2+a^2b^2d^2)(a^4b^4-a^4b^2d^2)\\&=4a^4b^4d^2(b^2c^2+a^2d^2-a^2b^2).\end{aligned}$$

Case I, if $P(c,d)$ is in the interior region of ellipse curve then

$$\frac{c^2}{a^2}+\frac{d^2}{b^2}<1 \text{ thus, } b^2c^2+a^2d^2-a^2b^2<0 \text{ and } D\geq 0.$$

Hence, there are no tangents to the ellipse curve pass through the point $P(c,d)$.

Case II, if $P(c,d)$ is in the exterior region of ellipse curve then

$$\frac{c^2}{a^2}+\frac{d^2}{b^2}>1 \text{ thus, } b^2c^2+a^2d^2-a^2b^2>0 \text{ and } D\geq 0.$$

Hence, there are two tangents to the ellipse curve pass through the point.

In conclusion, let $P(c,d)$ is in the exterior region of the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$$

there are always two tangents to the ellipse curve pass through the point $P(c,d)$.

3. NUMERICAL EXAMPLE

Example 3.1. Find the points of the tangents from the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ through the point $P(0,2)$.

Solution: The ellipse

$$\frac{x^2}{4}+\frac{y^2}{1}=1$$

we know that $a^2=4, b^2=1, c=0, d=2,$ we get

$$A=b^4c^2+a^2b^2d^2=16$$

$$B=-2a^2b^4c=0$$

$$C=a^4b^4-a^4b^2d^2=-48$$

so,

$$e=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$=\frac{\pm\sqrt{-4(16)(-48)}}{2(16)}=\pm\sqrt{3}.$$

We find f from (4), $f^2=\frac{b^2}{a^2}(a^2-e^2)$

hence $f=\pm\frac{1}{2}$

therefore, $Q_1(\sqrt{3},\frac{1}{2})$ and $Q_2(-\sqrt{3},\frac{1}{2})$ are two points of the tangents to the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ through the point $P(0,2)$ see figure 2.

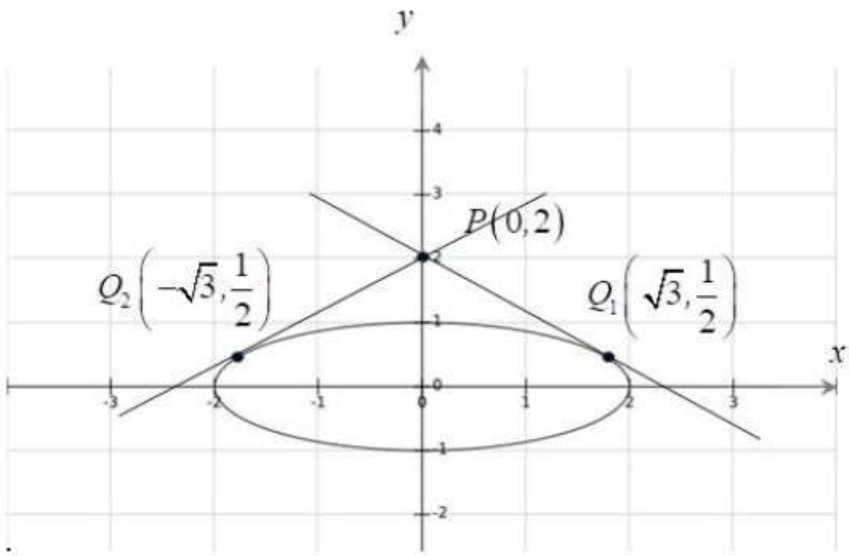

Figure 2: picture of example 3.1.

Example 3.2. Find the points of the tangents from the ellipse $\frac{x^2}{1}+\frac{y^2}{4}=1$ through the point $P(2,0)$.

Solution: The ellipse

$$\frac{x^2}{1}+\frac{y^2}{4}=1$$

we know that $a^2=1, b^2=4, c=2, d=0,$ we get

$$A=b^4c^2+a^2b^2d^2=64$$

$$B=-2a^2b^4c=-64$$

$$C=a^4b^4+a^4b^2d^2=16$$

so,

$$e=\frac{-B\pm\sqrt{B^2-4AC}}{2A}=\frac{1}{2}.$$

We find f from (4), $\quad f^2=\frac{b^2}{a^2}(a^2-e^2)=3$

hence $\quad f=\pm\sqrt{3}$

therefore, $Q_1(\frac{1}{2},\sqrt{3})$ and $Q_2(\frac{1}{2},-\sqrt{3})$ are two points of the tangents to the ellipse $\frac{x^2}{1}+\frac{y^2}{4}=1$ through the point $P(2,0)$ see figure 3

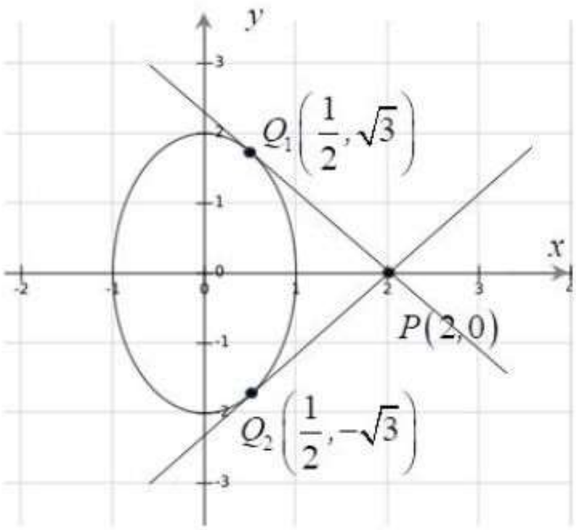

Figure 3: picture of example 3.2.

4. ACKNOWLEDGEMENT

I would like to give a thanks to Jaktupat who gave some advice and this work is supported by Faculty of Science, Burapha University, Thailand.

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