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On a Fuzzy Completely Closed Filter with Respect of Element in a BH-algebra

Mohsin Shaalan Abdalhussein
College of physical planning\Regional Planning Department
University of Kufa, Iraq

Email: mohsins.alakayshee [AT] uokufa.edu.iq

ABSTRACT—*In this paper, we introduce a new notion that we call a fuzzy completely closed filter with respect of an element in a BH-algebra, and we link this notion with notions filter and ideal of BH- algebra, We give some properties of fuzzy completely closed filter with respect of an element and we study properties of it.*

Keywords— fuzzy completely closed filter with respect of an element, BH-algebra, fuzzy filter, fuzzy ideal, fuzzy completely closed filter and fuzzy completely closed filter.

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1. INTRODUCTION

The notion of a BCK-algebras and a BCI-algebras was formulated first in 1966 [6] by (Y.Imai) and (K.Iseki). In 1991, C. S. Hoo introduced the notions of a filter and closed filter of a BCI–algebra[2].In 1996, M. A. Chaudhry and H. F-Ud-Din studied the concepts of filter and closed filter of a BCH-algebra[7]. In the same year, (J.Neggers) introduced the notion of d-algebra[5]. In 1998, Y.B.Jun, E.H.Roh and H.S.Kim introduced a new notion, called a BH-algebra[11]. In 2012, H.H.Abass and H.A.Dahham introduced the notions of a completely closed ideal and completely closed ideal with respect to an element of a BH-algebra[3]. In this paper, we introduced the notions as we mentioned in the abstract. On the other hand, we will mention the development of a fuzzy set, fuzzy subalgebra, fuzzy ideals, fuzzy closed ideals and some other types of fuzzy ideals. In 1965, L. A. Zadeh introduced the notion of a Fuzzy subset of a set as a method for representing uncertainty in real physical world[12]. In 1991, O. G. Xi applied the concept of fuzzy sets to the BCK-algebras[13]. In 1993, Y. B. Jun was the first author who solved the problem of classifying fuzzy ideals by their family of level ideals in BCK(BCI)-algebras[15]. In the same year, Y. B. Jun introduced the notion of closed fuzzy ideals in BCI-algebras[16]. In 1994, Y. B. Jun and J. Meng introduced the notion of fuzzy p-ideals in BCI-algebras[14]. In 1999, Y. B. Jun introduced the notion of Fuzzy closed ideals in BCH-algebras[17]. In 2002, C. Lele, C. Wu and T.Mamadou introduced the notion of Fuzzy filter in BCI-algebras[6]. In 2009, A. B. Saeid and M. A. Rezvani . In 2011, H. H. Abass and H. M. A.Saeed introduced the notion of Fuzzy closed ideals with respect to an element of BH-algebras[10]. In 2012, H. H. Abass and H. A. Dahham, Some Types of Fuzzy Ideal With Respect To an Element Of a BG-Algebra[9] .

2. PRELIMINARIES

In this section, we review some basic definitions and notations of BH-algebras, fuzzy completely closed filter, filter, ideals and other notions, that we need in our work.

Definition (1.1) [12]:

Let X be a non-empty set. A fuzzy set A in X (a fuzzy subset of X) is a function from X into the closed interval [0,1] of the real number.

Definition (1.2) [8]:

Let A and B be two fuzzy sets in X, then :

1. $(A\cap B)(x)=\min\{A(x),B(x)\}$, for all $x\in X$.
2. $(A\cup B)(x)=\max\{A(x),B(x)\}$, for all $x\in X$.

$A\cap B$ and $A\cup B$ are fuzzy sets in X.

In general, if {Aα, α∈Λ} is a family of fuzzy sets in X, the :

$\left(\bigcap_{i\in\Gamma} A_i\right)$(x)=inf{Ai (x),i∈ Γ}, for all x∈X and

$\left(\bigcup_{i\in\Gamma} A_i\right)$(x)=sup{ Ai (x),i∈Γ}, for all x∈X.

which are also fuzzy sets in X.

Definition (1.3) [5] :

A BH-algebra is a nonempty set X with a constant 0 and a binary operation"*" satisfying the following conditions:

1) x*x=0, ∀ x∈ X.
2) x*y=0 and y*x =0 imply x = y, ∀ x, y∈ X.
3) x*0 =x, ∀ x∈ X.

Example (1.4)[4]:

Let X = {0,1,2} be a set with the following table:

*	0	1	2
0	0	1	2
1	1	0	1
2	2	2	0

Then (X,*,0) is a BH-algebra.

Definition (1.5)[9]:

A BH-algebra X is called an associative BH-algebra if:

(x*y)*z=x*(y*z) , for all x,y,z∈X.

Definition (1.6)[9]:

Let X be a BH-algebra and b∈X, a filter F is called a completely closed filter with respect to b (denoted by b-completely closed filter) if b*(x*y)∈F ∀x, y∈F

Definition (1.7)[19] :

A fuzzy set M in a BH-algebra X is said to be fuzzy normal if it satisfies the inequality M((x*a)*(y*b)) ≥ min{M(x*y),M(a*b)}, for all a, b, x, y ∈X.

Definition (1.8) [14]:

A fuzzy set A in a BH-algebra X is said to be a fuzzy subalgebra of X if it satisfies: A(x*y)≥min{A(x),A(y)}, ∀x,y∈X.

Definition (1.9)[6] :

A non constant fuzzy set A of X is a fuzzy filter if

1- A (x∧y) ≥ min{A (x),A(y)}and A (y∧x) ≥ min{A(x),A(y)}For any x,y∈X.
2- A(y) ≥ A(x) ,when x≤ y.

Definition (1.10)[9]:

Let X be a BH-algebra, A be a fuzzy filter of X and b∈X. Then A is called a fuzzy closed filter with respect to an element b∈X, denoted by a fuzzy b-closed filter of X, if A(b*(0*x)) ≥ A(x), ∀x∈X.

Definition (1.11)[9] :

Let X be a BH-algebra and A be a fuzzy filter of X. Then A is called a fuzzy completely closed filter ,if A(x*y) ≥ min{A(x),A(y)} ∀x,y∈X.

Definition (1.12) [5]:

A fuzzy subset A of a BH-algebra X is said to be a fuzzy ideal if and only if:

1) For any $x \in X$, $A(0) \geq A(x)$.

2) For any $x, y \in X$, $A(x) \geq \min\{A(x*y), A(y)\}$.

Definition (1.13)[9] :

Let X be a BH-algebra and A be a fuzzy ideal of X. Then A is called a fuzzy completely closed ideal ,if $A(x*y) \geq \min\{A(x),A(y)\}, \forall x,y \in X$.

Definition (1.14)[9] :

Let X be a BH-algebra and A be a fuzzy ideal of X. Then A is called a fuzzy completely closed ideal with respect to an element $b \in X$, denoted by a fuzzy b-completely closed ideal of X, if $A(b*(x*y)) \geq \min\{A(x),A(y)\}$ $\forall x,y \in X$.

Theorem (1.15)[9]:

Let X be a BH-algebra and A be a fuzzy set . Then A is a fuzzy filter if and only if $A'(x)=A(x)+1-A(0)$ is a fuzzy filter.

Theorem (1.16)[9]:

Let X be an associative BH-algebra. Then every fuzzy normal set of X is a fuzzy filter.

Proposition (1.17)[9] :

Let X be a BH-algebra and A is a fuzzy completely closed filter then $A(0) \geq A(x)$ $\forall x \in X$.

Proposition(1.18)[9] :

Let X be a BH-algebra. If M is a fuzzy normal set, then $M(0) \geq M(x)$ $\forall x \in X$.

3. MAIN RESULTS

In this section, we define the a fuzzy completely closed filter with respect to an element, and link the notion with another notions in BH-algebra.

Definition (2.1): Let X be a BH-algebra and A be a fuzzy filter of X. Then A is called a fuzzy completely closed Filter with respect to an element $b \in X$, denoted by a fuzzy b-completely closed Filter of X, if $A(b*(x*y)) \geq \min\{A(x),A(y)\}$ $\forall x,y \in F$.

Example (2.2):

Let X = {1, 2, 3 } be a BH-algebra, with the following table:

*	0	1	2
0	0	1	2
1	1	0	1
2	2	2	0

The fuzzy filter A which is defined by :. $A(x)=\begin{cases} 0.6 & x = 0,2 \\ 0.3 & x = 1 \end{cases}$
is a fuzzy 2-completely closed Filter of X, since

$A(2*(0*0)) = A(2*0) = A(2) = 0.6 \geq \min\{A(0), A(0)\} = 0.6$

$A(2*(0*1)) = A(2*1) = A(2) = 0.6 \geq \min\{A(0), A(1)\} = 0.3$

$A(2*(0*2)) = A(2*2) = A(0) = 0.6 \geq \min\{A(0), A(2)\} = 0.6$

$A(2*(1*0)) = A(2*1) = A(2) = 0.6 \geq \min\{A(1), A(0)\} = 0.3$

$A(2*(1*1)) = A(2*0) = A(2) = 0.6 \geq \min\{A(1), A(1)\} = 0.3$

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$A(2*(2*0)) = A(2*2) = A(0) = 0.6 \geq \min\{A(2), A(0)\} = 0.6$

$A(2*(2*1)) = A(2*2) = A(0) = 0.6 \geq \min\{A(2), A(1)\} = 0.3$
$A(2*(2*2)) = A(2*0) = A(2) = 0.6 \geq \min\{A(2), A(2)\} = 0.6$

Theorem (2.3):Let X be BH-algebra such that if x*y=0 implies x=y $\forall x,y\in X$. Then every fuzzy b-completely closed filter is a fuzzy filter .
Proof : Let A be a fuzzy b-completely closed filter and $x,y\in X$.

1)$A(x*(x*y)) \geq \min\{A(x),A(y)\}$ [Since A is a fuzzy b-completely closed filter]

Similarly, $A(y*(y*x)) \geq \min\{A(x),A(y)\}$

2)Let $x\leq y$

$\Rightarrow x*y=0 \Rightarrow x=y$

$\Rightarrow A(y) =A(x) \geq A(x)$

$\therefore$ A is a fuzzy filter

Theorem (2.4):Let X be an associative BH-algebra. Then every fuzzy b- completely closed filter is a fuzzy b- closed filter.

Proof :Let A is fuzzy b- completely closed filter

Now,$0,x\in F, b\in X$

$A(b*(0*x))\geq \min\{A(0),A(x)\}$ [A is fuzzy b-completely closed filter, definition (2.1)]

$A(b*(x*y))\geq A(x)$ [$A(0)\geq A(x)\ \forall\ x\in X$, Proposition (1.17)]

$\therefore$ A is a fuzzy b- closed filter.

Proposition (2.5):Let X be an associative BH-algebra. Then every fuzzy normal set $\forall b\in X$,s.t $(M(b)=M(0))$ is a fuzzy b-completely closed filter.
Proof :Let M be a fuzzy normal set.
$\Rightarrow$ M is a fuzzy filter [(1.16)]

Now,Let $x,y\in M$, $b\in X$

$M(b*(x*y))=M(b*((x*y)*(0*0)))$

$=M((b* (x*y))*(0*0))$

$\geq \min\{M(b),M((x*y)\}$ [Since M is a fuzzy normal set, definition (1.7)]

$\geq \min\{M(0),M((x*y)\}$ [$M(b)=M(0)$]

$= M((x*y)$ [$A(0)\geq A(x)\ \forall\ x\in X$, Proposition (1.18)]

$\geq \min\{M(x),M(y)\}$[Since M is a fuzzy normal set definition (1.7)]

$\therefore$ M is a fuzzy b-completely closed filter.

Proposition (2.6):Let X be a BH-algebra and A is a fuzzy b- completely closed filter. Then $A\alpha$ is a b- completely closed filter $\forall\alpha\in(0,1]$.

Proof :Let A be a fuzzy b- completely closed filter.

To prove $A\alpha$ is a filter,

1)Let $x, y \in A\alpha$

$\Rightarrow A(x)\geq\alpha, A(y)\geq\alpha$

$\Rightarrow \min\{A(x),A(y)\}\geq\alpha$.

but $A(x*(x*y)) \geq \min\{A(x),A(y)\}$ [Since A is a fuzzy filter, definition (1.9)]

$\Rightarrow A(x*(x*y)) \geq \alpha$,

$\therefore x*(x*y)\in A\alpha$

Similarly,

$y*(y*x)\in A\alpha$

2)Let $x\in A\alpha$ and $x*y=0$

$\Rightarrow A(x)\geq\alpha$.

But $A(y)\geq A(x)$ [Since A is a fuzzy filter and $x\leq y$., definition (1.9)]

$\Rightarrow A(y)\geq\alpha$,

$\Rightarrow y\in A\alpha$

$\therefore$ $A\alpha$ is a filter

Now,Let $x,y\in F$, $b\in X$, $x,y\in A\alpha$

$\Rightarrow A(x)\geq\alpha$.

$\Rightarrow A(y)\geq\alpha$.

$\Rightarrow\min\{A(x),A(y)\}\geq\alpha$

but $A(b*(x*y))\geq\min\{A(x),A(y)\}$ [Since A is a fuzzy b- completely closed filter, definition (2.1)]

$\Rightarrow A(b*(x*y))\geq\alpha$

$\Rightarrow b*(x*y)\in A\alpha$

$\therefore$ $A\alpha$ is a b- completely closed filter $\forall\alpha\in(0,1]$.■

Proposition(2.7): Let X be a BH-algebra and A be a fuzzy b- completely closed filter. Then the set $X_A=\{x\in X: A(x)=A(0)\}$ is a b- completely closed filter.

Proof :Let A be a fuzzy filter,

Since $A(0)=A(0)$

$\therefore 0\in X_A$

$\therefore X_A$ is a non-empty set

1) Let $x , y\in X_A$

$\Rightarrow A(x)=A(y)=A(0)$

$\Rightarrow\min\{A(x),A(y)\}=A(0)$

But $A(x*(x*y))\geq\min\{A(x),A(y)\}=A(0)$ [Since A is a fuzzy filter. definition (1.9)]

$\therefore$ $A(x*(x*y))\geq A(0)$

But $A(0)\geq A(x*(x*y))$ [Since A is a fuzzy completely closed filter. definition (1.11)]

$\therefore A(x*(x*y))=A(0)$

$\therefore$ $x*(x*y)\in X_A$

Similarly,

$y*(y*x)\in X_A$

2)Let $x\in X_A, x\leq y$

$\Rightarrow A(y)\geq A(x)=A(0)$ [Since A is a fuzzy filter, definition (1.9)]

But $A(0)\geq A(y)$, [Since A is a fuzzy completely closed filter., definition (1.11)]

$\therefore$ $A(y)=A(0)$

$\therefore$ $y\in X_A$

$\therefore X_A$ is a filter.

Now,Let $x, y\in F$, $b\in X_A$

$\Rightarrow A(x)=A(y)=A(0)$

$\Rightarrow\min\{A(x),A(y)\}=A(0)$

But $A(b*(x*y))\geq\min\{A(x),A(y)\}=A(0)$ [Since A is a fuzzy b-completely closed filter definition (2.1)]

$\therefore$ $A(x*y)\geq A(0)$

But $A(0) \geq A(x*y)$ [Since A is a fuzzy completely closed filter, definition (1.11)]

$\therefore A(b*(x*y)) = A(0)$

$\therefore b*(x*y) \in X_A$

$\therefore X_A$ is a b-completely closed filter.■

Proposition (2.8): Let $\{A_i : i \in \Gamma\}$ be a family of fuzzy b-completely closed filter of a BH-algebra X. Then $\left(\bigcap_{i\in\Gamma} A_i\right)$ is a fuzzy b-completely closed filter of X.

Proof :

To prove that $\left(\bigcap_{i\in\Gamma} A_i\right)$ is a fuzzy filter,

(1) Let $x,y \in X$.

$\left(\bigcap_{i\in\Gamma} A_i\right)(x*(x*y)) = \inf\{ A_i((x*(x*y))), i\in\Gamma\}$

$\geq \inf\{ \min\{ A_i(x), A_i(y), i\in\Gamma\}$ [Since A_i is a fuzzy filter, $\forall i\in\Gamma$, definition (1.9)]

$\geq \min\{\left(\bigcap_{i\in\Gamma} A_i\right)(x), \left(\bigcap_{i\in\Gamma} A_i\right)(y) \}$

Similarly $\left(\bigcap_{i\in\Gamma} A_i\right)(y*(y*x))$

(2) Let $x \in X$ and $x \leq y$

$\Rightarrow \left(\bigcap_{i\in\Gamma} A_i\right)(x) = \inf\{A_i(x), i\in\Gamma\}$

$\geq \inf\{ A_i(y)\}, i\in\Gamma \}$ [Since Ai is a fuzzy filter, $\forall i\in\Gamma$, definition (1.9)]

$= \left(\bigcap_{i\in\Gamma} A_i\right)(y)$

$\Rightarrow \left(\bigcap_{i\in\Gamma} A_i\right)$ is a fuzzy filter

To prove that $\left(\bigcap_{i\in\Gamma} A_i\right)$ is a fuzzy b-completely closed filter of X

Let $x,y \in F$, $b \in X$

$\left(\bigcap_{i\in\Gamma} A_i\right)(b*(x*y)) = \inf\{ A_i(b*(x*y)), i\in\Gamma\}$

$\geq \inf\{\min\{ A_i(x), A_i(y)\}, i\in\Gamma \}$

$\geq \min\{\inf A_i(x), \inf A_i(y)\}, i\in\Gamma \}$

$\geq \min\{\left(\bigcap_{i\in\Gamma} A_i\right)(x), \left(\bigcap_{i\in\Gamma} A_i\right)(y) \}$

$\Rightarrow \left(\bigcap_{i\in\Gamma} A_i\right)(b*(x*y)) \geq \min\{\left(\bigcap_{i\in\Gamma} A_i\right)(x), \left(\bigcap_{i\in\Gamma} A_i\right)(y) \}\ \forall x,y\in F$

Therefore, $\left(\bigcap_{i\in\Gamma} A_i\right)$ is a fuzzy b-completely closed filter of X. ■

Proposition (2.9):
Let $\{A_i : i\in\Gamma\}$ be a family of fuzzy b-completely closed filter of a BH-algebra X. Then $\left(\bigcup_{i\in\Gamma} A_i\right)$ is a fuzzy b-completely closed filter of X.

Proof :

To prove that $\left(\bigcup_{i\in\Gamma} A_i\right)$ is a fuzzy filter,

(1) Let $x,y \in X$.

$\left(\bigcup_{i\in\Gamma} A_i\right)(x*(x*y)) = \sup\{ A_i((x*(x*y))), i\in\Gamma\}$

$\geq \sup\{ \min\{ A_i(x), A_i(y), i\in\Gamma\}$ [Since A_i is a fuzzy filter, $\forall i\in\Gamma$, definition (1.9)]

But $\{A_i : i\in\Gamma\}$ is a chain $\Rightarrow$ there exist $j\in\Gamma$ such that sup{ min{Ai(x*y), Ai(y)}, i∈Γ } = min{Aj(x), Aj(y)}

= min{sup{Ai(x), i∈Γ}, sup{Ai(y), i∈Γ}}

$\geq \min\{ \left(\bigcup_{i\in\Gamma} A_i\right)(x), \left(\bigcup_{i\in\Gamma} A_i\right)(y) \}$

Similarly $\left(\bigcup_{i\in\Gamma} A_i\right)$ (y*(y*x))

(2) Let x ∈X and x≤ y

$\Rightarrow \left(\bigcup_{i\in\Gamma} A_i\right)(x)$ = sup{Ai(x), i∈Γ}

≥ sup{ Ai(y)}, i∈Γ } [Since Ai is a fuzzy filter, ∀i∈Γ, definition (1.9)]

$= \left(\bigcup_{i\in\Gamma} A_i\right)(y)$

$\Rightarrow \left(\bigcup_{i\in\Gamma} A_i\right)$ is a fuzzy filter

To prove that $\left(\bigcup_{i\in\Gamma} A_i\right)$ is a fuzzy b-completely closed filter of X

Let x,y∈F ,b ∈X

$\left(\bigcup_{i\in\Gamma} A_i\right)$(b*(x*y)) = sup{ Ai(b*(x*y)) , i∈Γ}

≥ sup{min{ Ai(x),Ai(y)}, i∈Γ }

≥ min{sup Ai(x),inf Ai(y)}, i∈Γ }

$\geq \min\{ \left(\bigcup_{i\in\Gamma} A_i\right)(x), \left(\bigcup_{i\in\Gamma} A_i\right)(y) \}$

$\Rightarrow \left(\bigcup_{i\in\Gamma} A_i\right)(b*(x*y)) \geq \min\{\left(\bigcup_{i\in\Gamma} A_i\right)(x), \left(\bigcup_{i\in\Gamma} A_i\right)(y)\}$ ∀x,y∈F

Therefore, $\left(\bigcup_{i\in\Gamma} A_i\right)$ is a fuzzy b-completely closed filter of X. ■

Proposition (2.10):

Let {Ai:i∈Γ}be a family of fuzzy b-closed filter of a BH-algebra X. Then $\left(\bigcap_{i\in\Gamma} A_i\right)$ is a fuzzy b-closed filter of X.

Proof :

$\left(\bigcap_{i\in\Gamma} A_i\right)$ is a fuzzy filter [proposition (2.8)]

$\left(\bigcap_{i\in\Gamma} A_i\right)$ to prove is a fuzzy b-closed filter

let b∈X, x∈F

$\left(\bigcap_{i\in\Gamma} A_i\right)$(b*(0*x))= inf{ Ai(b*(0*x)) , i∈Γ}

≥ inf { Ai(x), i∈Γ }

$\geq \left(\bigcap_{i\in\Gamma} A_i\right)(x)$,

$\Rightarrow \left(\bigcap_{i\in\Gamma} A_i\right)(b*(0*x)) \geq \left(\bigcap_{i\in\Gamma} A_i\right)(x)$

Therefore, $\left(\bigcap_{i\in\Gamma} A_i\right)$ is a fuzzy b- closed filter of X. ■

Proposition (2.11):

Let {Ai:i∈Γ}be a family of fuzzy b-closed filter of a BH-algebra X. Then $\left(\bigcup_{i\in\Gamma} A_i\right)$ is a fuzzy b-closed filter of X.

Proof :

$\left(\bigcup_{i\in\Gamma} A_i\right)$ is a fuzzy filter [proposition (2.9)]

$\left(\bigcup_{i\in\Gamma} A_i\right)$ to prove is a fuzzy b-closed filter

$\left(\bigcup_{i\in\Gamma} A_i\right)$ (b*(0*x)) = sup{ Ai(b*(0*x)) , i∈Γ}

≥ sup{ Ai(x), i∈Γ }

≥ $\left(\bigcup_{i\in\Gamma} A_i\right)$ (x)

⇒ $\left(\bigcup_{i\in\Gamma} A_i\right)$(b*(0*x)) ≥ $\left(\bigcup_{i\in\Gamma} A_i\right)$(x)

Therefore, $\left(\bigcup_{i\in\Gamma} A_i\right)$ is a fuzzy b-closed filter of X. ■

Proposition (2.12):

Let X be BH-algebra and A be a fuzzy set of X. Then A is a fuzzy b-completely closed filter if and only if A'(x)=A(x)+1-A(0) is a fuzzy b-completely closed filter.

Proof :

Let A be a fuzzy b-completely closed filter,

⇒ A is a fuzzy filter. [Proposition (2.3)]

⇒ A' is a fuzzy filter. [By theorem(1.15)]

Now,Let x,y∈A, b∈X

A'(b*(x*y))=A(b*(x*y))+1-A(0)

≥ min{A(x),A(y)}+1-A(0) [Since A is a fuzzy b-completely closed filter]

≥ min{A(x) +1-A(0) ,A(y) +1-A(0)}

≥ min{A'(x),A'(y)}

∴ A'(x)≥ min{A'(x),A'(y)}

∴ A' is a fuzzy b-completely closed filter

Conversely

Let A' be a fuzzy b-completely closed filter,

⇒ A' is a fuzzy filter. [By theorem(1.15)]

⇒ A is a fuzzy filter. [By theorem(1.15)]

Now,Let x,y∈A, b∈X

A(b*(x*y))=A'(b*(x*y))-1+A(0)

≥ min{A'(x),A'(y)}-1+A(0) [Since A' is a fuzzy filter, definition (1.9)]

≥ min{A'(x) -1+A(0) ,A'(y) -1+A(0)}

≥ min{A(x),A(y)}

∴ A(b*(x*y))≥ min{A(x),A(y)}

∴ A is a fuzzy b-completely closed filter.■

Proposition (2.13): Let X be BH-algebra and A be a fuzzy set of X. Then every fuzzy completely closed filter is a fuzzy b-completely closed ideal, ∀b∈X,A(b)=A(0).

Proof : Let A be a fuzzy completely closed filter.

To prove A is a fuzzy ideal,

1) A(0) ≥ A(x) ∀x∈X [Proposition (1.16)]
2) A(x)=A(x*0) =A(x*(y*y))
   =A((x*y)*y)) [Since X is an associative. By definition(1.5)]

$\geq$ min {A(x*y),A(y)} [Since A is a fuzzy completely closed filter By definition(1.11)]

$\therefore$ A is a fuzzy ideal.

A(b*(x*y))$\geq$ min{A(b),A(x*y)} [A is a fuzzy completely closed filter. By definition(1.11)]

$\geq$ min{A(0),A(x*y)}

$\geq$ A(x*y)

$\geq$ min{A(x),A(y)}

$\therefore$ A is a fuzzy b-completely closed ideal.

Proposition (2.14):

Let X be BH-algebra such that if x*y=0 implies x=y $\forall$x,y$\in$X. Then every fuzzy sub algebra is a fuzzy filter.

Proof :Let A be a fuzzy sub algebra and x,y$\in$X.

1)A(x*(x*y)) $\geq$ min{A(x),A(x*y)} [Since A is a fuzzy sub algebra, definition (1.8)]

If min{A(x),A(x*y)}= A(x) $\geq$ min{A(x),A(y)}

If min{A(x),A(x*y)}= A(x*y) $\geq$ min{A(x),A(y)}

$\therefore$ A(x*(x*y)) $\geq$ min{A(x),A(y)}

Similarly, A(y*(y*x)) $\geq$ min{A(x),A(y)}

2)Let x$\leq$ y

$\Rightarrow$x*y=0$\Rightarrow$x=y

$\Rightarrow$ A(y) =A(x) $\geq$A(x)

$\therefore$A is a fuzzy filter

Theorem (2.15):

Let X be BH-algebra such that x*y=0 implies x=y $\forall$x,y$\in$X. Then every fuzzy sub algebra is a fuzzy b-completely closed filter , $\forall$b$\in$X such that A(b) =A(0) .

Proof :

A is a fuzzy filter [proposition(2.14)]

A(b*(x*y))$\geq$ min{A(b),A(x*y)} [A is a fuzzy sub algebra, definition (1.8)]

$\geq$ min{A(0),A(x*y)}

$\geq$ A(x*y) [A is a fuzzy sub algebra, definition (1.8)]

$\geq$ min{A(x),A(y)}

$\therefore$ A is a fuzzy b-completely closed filter.

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