Asian Journal of Applied Sciences (ISSN: 2321 – 0893)
Volume 05 – Issue 01, February 2017

Computational Test for Convergence of Root-finding of Nonlinear Equations

Jutaporn Neamvonk and Apichat Neamvonk*

Department of Mathematics, Faculty of Science,
Burapha University, Chonburi, Thailand.

**Corresponding author's email: Apichat [AT] buu.ac.th*

ABSTRACT— ***We study a computational test for convergence of iterative methods which are Newton-Raphson method and modified by using Adomian decomposition method for finding root of nonlinear equations in form of*** $f(x)=0$. ***The comparative criterions are number of iteration, computational time, error graph and computational order of convergence (COC).***

Keywords— nonlinear equations, iterative method, Adomian decomposition method

---

1. INTRODUCTION

We now consider finding root of nonlinear equations in form of

$$f(x)=0, \tag{1}$$

where f denote a continuously differentiable function on $[a,b]\subset\Re$. It is found that in some equations, real root cannot be found using analytical method i.e. $f(x)=xe^x-1$ and $f(x)=x\ln x-x^3$. Therefore, it is necessary to find the root by numerical method (iterative method) such as Bisection method, Newton-Raphson method. Although the numerical method do not give the real root but it is close to the real one as possible. In 1982, George Adomian [3],[4] developed a method, so called Adomian decomposition method (ADM) to find approximate roots of nonlinear equations and we call [1],[6] applied basic ADM to solve nonlinear equations.

At present, many researchers develop new computational methods to find a solution or the root of nonlinear equations and compare those to the standard one using the comparative criterions, such as counting the iterative number, total of computational time process and error. Each numerical method provides different number of iteration, total of computational time and error. We find that in some situations the criterions cannot guide the researcher whether which methods is the best. In this paper, we propose two criterions to find computational test for solving roots of nonlinear equations. By comparing with the usual criterions, graphs of the error and computational order of convergence (COC) are applied to find whether the methodologies are suitable.

2. METHOD

We consider nonlinear equation (1) having at least one root in $[a,b]$ and assume that $|f'(x)|>0$. Taylor series formula is

$$f(x-h)=f(x)-hf'(x)+O(h^2), \tag{2}$$

where h is a small number, then

$$f(x-h)=0\approx f(x)-hf'(x). \tag{3}$$

From (3), we have $h=\dfrac{f(x)}{f'(x)}$, $x-h=x-\dfrac{f(x)}{f'(x)}$ then we get Newton-Raphson method.

Algorithm 1 is Newton-Raphson method. Let

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \tag{4}$$

where x_0 is an initial guess point taking any value. Then repeat (4) until sequence of approximate $\{x_n\}$, $n = 1,2,3,...$ converge to real root, α.

We applied Taylor series with 2nd order. Let

$$f(x-h) = 0 \approx f(x) - hf'(x) + \frac{h^2}{2} f''(x). \tag{5}$$

From (5), we have $h = \frac{f(x)}{f'(x)} + \frac{h^2}{2}\frac{f''(x)}{f'(x)}$ then use ADM [2], [5], [6], [7] and [8] for $m = 1$, we get

Algorithm 2
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{f^2(x_n)f''(x_n)}{2f'^3(x_n)}. \tag{6}$$

For $m = 2$, we get

Algorithm 3
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{f^2(x_n)f''(x_n)}{2f'^3(x_n)} - \frac{f^3(x_n)f''^2(x_n)}{2f'^5(x_n)}. \tag{7}$$

For $m = 3$, we get

Algorithm 4
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{f^2(x_n)f''(x_n)}{2f'^3(x_n)} - \frac{f^3(x_n)f''^2(x_n)}{2f'^5(x_n)} - \frac{f^4(x_n)f''^3(x_n)}{2f'^7(x_n)}, \tag{8}$$

where $n = 1,2,3,...$.

Definition 1: Computational order of convergence (COC) of sequence $\{x_n\}$ is $\rho = \frac{\ln\left|e_{n+1}/e_n\right|}{\ln\left|e_n/e_{n-1}\right|}$, where $e_n = x_n - \alpha$ [7].

In order to calculate ρ, as the real root, α is unknown, we substitute the real root by the k^{th} approximate root, x_k, which is the final root of the algorithm. Then we have

$$\rho = \frac{\ln\left|(x_k - x_{k-1})/(x_{k-1} - x_{k-2})\right|}{\ln\left|(x_{k-1} - x_{k-2})/(x_{k-2} - x_{k-3})\right|}, \tag{9}$$

3. MAIN RESULT

Table 1-3 show finding root of nonlinear equations which are $f_1(x) = x^3 - 2x + 2$, $f_2(x) = \sin(x) - x^5 + x^3 - 1$ and $f_3(x) = \cos(x) - x^3$ by comparing among number of iteration, computational time, relative error graph and computational order of convergence (COC). The initial guess point, x_0, is set to be the same for all algorithms of a function. The iteration process will run repeatedly and will be stopped when $\left|\frac{x_n - x_{n-1}}{x_{n-1}}\right| < 1\times10^{-12}$.

Table 1: show finding root comparison of $f_1(x) = x^3 - 2x + 2$

method	x_0	Appx. root	no.of it.	Relative error	Time (s)	COC
Algorithm 1	3	1.7692923542386314	7	0.00000e+00	0.5620	2.000508200840178
Algorithm 2	3	1.7692923542386314	5	0.00000e+00	0.5150	2.871332006764140
Algorithm 3	3	1.7692923542386314	4	1.34033e-13	0.7020	2.816483532067318
Algorithm 4	3	1.7692923542386314	5	0.00000e+00	0.8580	2.852374551132790

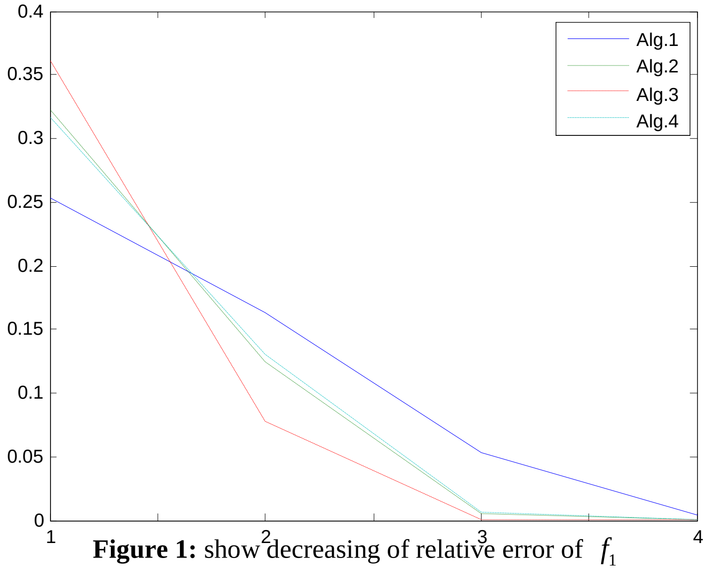

Figure 1: show decreasing of relative error of f_1

Table 2: show finding root comparison of $f_2(x) = \sin(x) - x^5 + x^3 - 1$

method	x_0	Appx. root	no.of it.	Relative error	Time (s)	COC
Algorithm 1	5	1.3455731314024220	12	0.00000e+00	0.7960	2.001143046000341
Algorithm 2	5	1.3455731314024220	8	5.22449e-13	0.8730	2.983893888961246
Algorithm 3	5	1.3455731314024220	7	1.65019e-16	1.2010	2.634366698877359
Algorithm 4	5	1.3455731314024220	9	0.00000e+00	1.5600	2.944418281424580

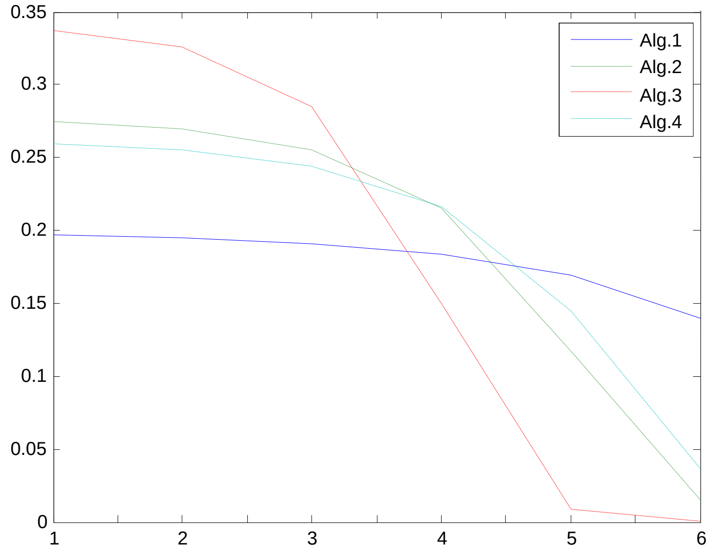

Figure 2: show decreasing of relative error of f_2

Table 3: show finding root comparison of $f_3(x) = \cos(x) - x^3$

method	x_0	Appx. root	no.of it.	Relative error	Time (s)	COC
Algorithm 1	4	0.8654740331016144	9	0.00000e+00	0.7800	2.000515362225596
Algorithm 2	4	0.8654740331016144	6	1.84722e-13	0.6400	2.986274993986881
Algorithm 3	4	0.8654740331016144	6	0.00000e+00	1.0290	2.800631871536915
Algorithm 4	4	0.8654740331016144	7	0.00000e+00	1.2170	2.975431520247845

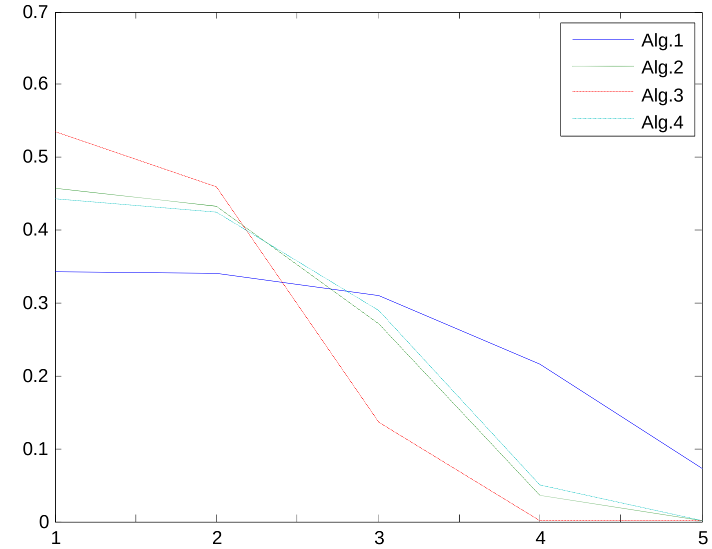

Figure 3: show decreasing of relative error of f_3

4. CONCLUSION

Four algorithms which are Newton-Raphson method and applied Taylor series with higher order and ADM for finding root of nonlinear equations; f_1 , f_2 and f_3, have been compared in table 1-3. It is clearly that Algorithm 3 has the lowest numbers of iteration. However, Algorithm 1 and 2 take small computational process. These two criterions do not give the same solution. The COC of Algorithm 2 for all functions are the largest value. This solution corresponds to the error graph in figure 1-3. Therefore, COC is effective to be a comparison criterion.

5. ACKNOWLEDGEMENT

We would like to thank the Faculty of Science, Burapha University support through funds for research.

6. REFERENCES

[1] Abbaoui, K., Cherruault, Y. "Convergence of Adomians method Applied to Differential Equations", Applied Mathematics and Computational, vol. 28, pp. 103-109, 1994.

[2] Abbasbandy S., "Improving Newton-Raphson method for nonlinear equations by modified Adomian decomposition method", Applied Mathematics and Computational, vol. 145, pp. 887-893, 2003.

[3] Adomian, G. Nonlinear Stochastic Systems and Applications to Physics, Kluwer Academic Publishers, Dordrecht, 1989.

[4] Adomian, G. Solving Frontier Problems of Physics: The Decomposition Method, Kluwer Academic Publishers, Dordrecht, 1994.

[5] Adomian, G., Rach, R. "On the solution of algebraic equations by the decomposition method", Mathematical Analysis Applications, vol. 105, no. 1, pp. 141-166, 1985.

[6] Babolian, E., Biazar, J. "Solution of nonlinear equations by modified Adomian decomposition method", Applied Mathematics and Computational, vol. 132, pp. 167-172, 2002.

[7] Babolian, E., Biazar, J. "On the order of convergence of Adomian Method", Applied Mathematics and Computation, vol. 130, pp. 383-387, 2002.

[8] Cherruault, Y., Adomian, G. "Decomposition method: a new proof of convergence", Mathematics Computational Modelling, vol. 18, no. 12, pp. 103-106, 1993.