# Multiple Regression Model for the Prediction of Flexural Behaviour of FRP Plated Pre-stressed Concrete Beams

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**ABSTRACT---- *This paper deals on the development of multiple regression model for the prediction of flexural behaviour of fibre reinforced polymer plated prestressed concrete beams. In this study, fourteen post-tensioned prestressed concrete beam specimens were constructed and tested in a gradually applied load till the failure prestressed concrete beam. The data such as load and their corresponding deflection at yield and ultimate stages, deflection and energy ductility, maximum crack with were collected from the tested post-tensioned concrete beams were used for regression analysis. The predicted results showed a considerable agreement with the experimental values. Hence, it is suggested that the developed models were best suited for the flexural performance of FRP plated prestresssed concrete beams.***

**Keywords---** Beams, FRP, plate, prestressed concrete, regression, prediction

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## 1. INTRODUCTION

A structure comprises of three major elements namely beam, column and slab. All these elements have importance of their own. Beams must be designed in such a way that it can withstand any type of loads without producing any deformation or cracking to the structures. But at times the beams may experience sudden static loads for which they are not designed. Due to these sudden loads the beams tends to crack. This is mainly due to the tension or compression in the beam. In the bridges, industrial buildings etc the most common type of beams used are pre-stressed beams. The prestressed concrete beams (PSC) are designed to withstand heavy loads. These beams have lesser beam depth that possesses improved resistance to shearing. These beams are mostly used where the span is more. In order to protect the beam from the cracking produced due to the sudden application of the fatigue load additional strengthened should be done. By strengthening the beams it is observed that the crack produced in the beams are greatly reduced. The size and the width of the crack are considerably reduced. There are various kind of strengthening methods are available but every method cannot be suited for all cases, with the proper study of the case we have to decide the proper method which is suited for particular case. Several conventional techniques were available including steel plates bonded at the tension face, steel or concrete jacketing, post-tensioning at the external face of the member. The materials such as carbon, aramid and glass fibres together either with polymeric or epoxy resin, known as fiber-reinforced polymers (FRP), have paid more attention as a new option to conventional materials and techniques. Au and Du [1] examined the methods for the evaluation of ultimate stress in prestressing tendon at flexural failure of unbonded PSC beams. They stated that the proposed design equation would be suitable for the high tensile strength prestressing tendons and materials made from fibre reinforced polymer. Ling-jia et al [2] conducted a study on an existing prestressed concrete bridge and evaluated the permanent prestress and its load capacity. The investigation reported that the errors in deflection errors for rectangular and T-beams were within 10% and 15.71% for hollow slab beam. It indicated that using the tested fundamental frequency and deflection could able to calculate effective prestress in simply supported beam. Almusallam et al [3] investigated experimentally and numerically on the influence of longitudinal steel ratio on the flexural behaviour of concrete beams externally bonded with FRP. Lees [4] studied the present advancements in the use of FRP in reinforced and prestressed concrete structures. The author reported with some illustrations on the realistic performance of FRP technology and dealt with an important durability issues. Dawood and Al-Katib [5] experimentally investigated on the flexural strength of pretensioned concrete beams with openings and reinforced with carbon fibre reinforced polymer sheets. The results showed a considerable increase at the ultimate load for strengthened beams. It also reported that the load-deflection curves for flexural beams reinforced with CFRP sheets were stiffer than the other beams. The studies to predict the compressive strength of high performance concrete by using multiple regression analysis were also investigated [6-7]. Regression model was also done for the performance of reinforced concrete and fibre reinforced beams [8-9]. In this paper an effort was made to develop a mathematical model by regression analysis for the prediction of load, deflection and ductility for fibre reinforced polymer plated prestressed concrete beams.

## 2. TEST PROGRAM & PROCEDURE

A total of fourteen unbonded post-tensioned PSC beams were cast for the present study. The beams were 150 mm x 250 mm in cross-section and 3000 mm long. All the beams were cast with M 35 (T- series) and M 60 (S-series) grade of concrete having a designed mix proportion of 1 : 1.3 : 2.35 : 0.42 and 1 : 1.35 : 2.19 : 0.29 respectively. Two numbers of 12mm diameter bars were used for tensile reinforcement and 2 numbers of 10 mm diameter bars were used as hanger bars. 2-legged 8 mm diameter stirrups were provided at 150 mm c/c, in order to avoid any shear failure and ensure flexural action of beams up to failure. Apart from conventional steel reinforcement, 2 numbers of high tensile prestressing wires of 7 mm diameter were provided at an eccentricity 50 mm. One PSC beam was considered as reference specimen and other six PSC beams were strengthened with three different types of glass fibre reinforced polymer (GFRP) plates placed on the soffit of PSC beams with thicknesses of 3 mm and 5 mm. The properties of FRP plate are provided in Table 1

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**Table 1 -** Properties of FRP Plate

| GFRP Material | Thickness (mm) | Tensile Strength (MPa) | Elasticity Modulus (MPa) |
|---|---|---|---|
| Chopped Strand Mat | 3 | 126.20 | 7467.46 |
| | 5 | 156.00 | 11386.86 |
| Woven Rovings | 3 | 147.40 | 6855.81 |
| | 5 | 178.09 | 8994.44 |
| Uni-Directional Cloth | 3 | 446.90 | 13965.63 |
| | 5 | 451.50 | 17365.38 |

The test PSC beam specimens were provided with 10mm thick bearing plates at both ends, wedges were locked with high tensile steel wire at one end and the other end kept unlocked for pre-stressing. Pre-stressing technique was carried out by 10 t hydraulic jack at the unlocked end. High pressure using hand pump was applied gradually until it reaches the predetermined value. The FRP plates were placed on a well prepared concrete substrate of PSC beam specimens. The FRP beams were tested after an interval of 14-days. The PSC beams plated with and without FRP were tested under four point-bending over a simple span of 2800 mm in a loading frame of 100 t capacity. Two - point loads were applied through a spreader beam. The load was applied using a hydraulic jack and load cell arrangement. The load and deflection were collected through a computer operated data logging system.

## 3. MULTIPLE LINEAR REGRESSION ANALYSIS

Multiple linear regression analysis is a mathematical computation of an average functional relationship between two or more variables. In regression analysis, the estimated variable is called as dependent variable and explanatory variable is known as independent variable. The multiple linear regressions are an extension of a simple linear regression where multiple independent variables exist. When there are '*i*' explanatory variables $x_1$, $x_2$,…., $x_i$, the linear multiple regression equation takes up the general form as,

$$y = a_0 + a_1 \ x_1 + a_2 \ x_2 + \ldots\ldots\ldots. + a_i \ x_i \qquad (1)$$

where '*y*' is the estimated variable, $x_1$, $x_2$, …, $x_i$ are the explanatory variables and '*a*' denotes coefficient vector. In this study, multiple linear regression analysis was carried out to develop a relation between the estimated variables including load and their corresponding deflection at yield and ultimate stages, deflection and energy ductility, and crack width. The explanatory variables such as compressive strength of concrete, type of FRP plate and thickness of FRP plate were used.

## 4. RESULTS AND DISCUSSION

Table 2 shows the data collected from the experimental test results of PSC beams are used for the multiple linear regression analysis. Table 3 presents the developed regression equations for various study parameters. The developed regression predictions were also compared with the experimental test results and depicted through Figs 1 to 7.

**Table 2 –** Collected data for regression analysis

| Beam ID | Yield Stage | | Ultimate Stage | | Ductility | | Crack Width at Ultimate Stage (mm) |
|---|---|---|---|---|---|---|---|
| | Load (kN) | Deflection (mm) | Load (kN) | Deflection (mm) | Deflection | Energy | |
| T | 18.62 | 3.08 | 50.52 | 39.25 | 12.74 | 34.42 | 1.58 |
| TC1 | 20.00 | 3.00 | 59.56 | 42.00 | 14.00 | 39.61 | 1.19 |
| TC2 | 22.24 | 3.10 | 63.00 | 44.86 | 14.47 | 49.12 | 0.89 |
| TW1 | 25.60 | 3.24 | 70.20 | 55.20 | 17.04 | 44.96 | 0.81 |
| TW2 | 23.24 | 3.93 | 79.44 | 64.50 | 16.41 | 48.07 | 0.76 |
| TU1 | 33.74 | 4.09 | 85.26 | 68.00 | 16.62 | 45.06 | 0.72 |
| TU2 | 36.12 | 4.53 | 97.58 | 75.50 | 16.67 | 54.95 | 0.7 |
| S | 20.43 | 10.70 | 70.50 | 62.34 | 5.83 | 6.10 | 1.71 |
| SC1 | 30.70 | 11.70 | 78.00 | 68.45 | 5.85 | 10.79 | 1.48 |
| SC2 | 35.86 | 11.70 | 82.30 | 71.30 | 6.09 | 17.30 | 1.26 |
| SW1 | 35.14 | 12.00 | 83.50 | 73.86 | 6.16 | 18.27 | 1.08 |
| SW2 | 42.70 | 12.37 | 87.60 | 77.65 | 6.28 | 22.19 | 0.97 |
| SU1 | 45.00 | 13.03 | 93.50 | 83.98 | 6.45 | 23.20 | 0.82 |
| SU2 | 48.86 | 13.57 | 108.70 | 89.58 | 6.60 | 26.58 | 0.76 |

**Table 3-** Regression Equations for beams

| Parameter | Regression Equation | $R^2$ Value |
|---|---|---|
| Yield Load (kN) | $18.17 - (0.000308\ f_{ck}) + (0.052\ t_f) + (0.047 f_{fu})$ | 0.886 |
| Yield Deflection (mm) | $3.72-(0.00015\ f_{ck}) + (0.3232\ t_f) + (0.0081 f_{fu})$ | 0.947 |
| Ultimate Load (kN) | $48.51-(0.00457\ f_{ck}) + (8.05\ t_f) + (0.22ffu)$ | 0.908 |
| Ultimate Deflection (mm) | $37.07-(0.00164\ f_{ck}) + (9.24\ t_f) + (0.06079\ f_{fu})$ | 0.899 |
| Deflection Ductility | $13.27-(0.000833\ f_{ck}) + (1.38\ t_f) + (0.024 f_{fu})$ | 0.648 |
| Energy Ductility | $33.07-(0.000125\ f_{ck}) + (2.546\ t_f) + (0.0101 f_{fu})$ | 0.801 |
| Maximum Crack Width (mm) | $1.5 + (0.0000814\ f_{ck})-(0.2123\ t_f)-(0.0027 f_{fu})$ | 0.870 |

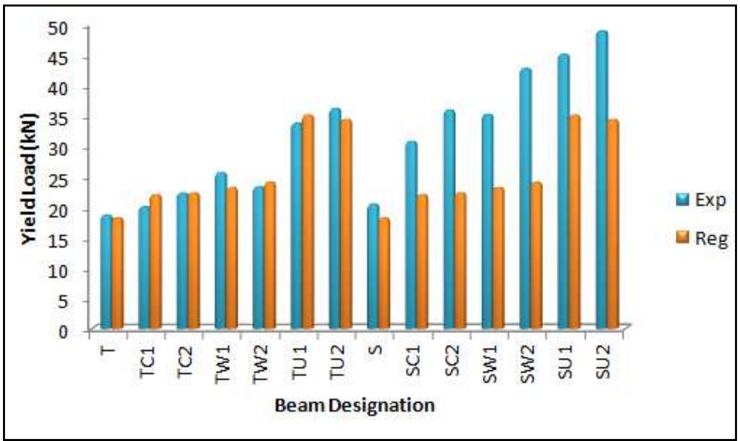


**Fig. 1** Experimental vs Predicted results for Yield Load

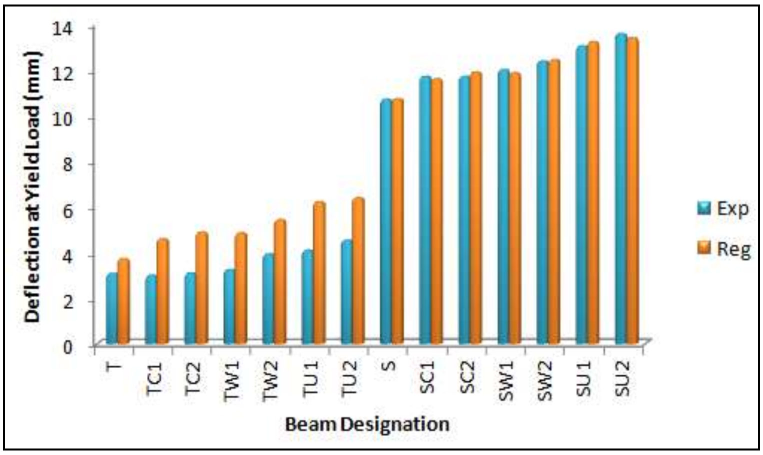


**Fig. 2** Experimental vs Predicted results for Deflection at Yield Load

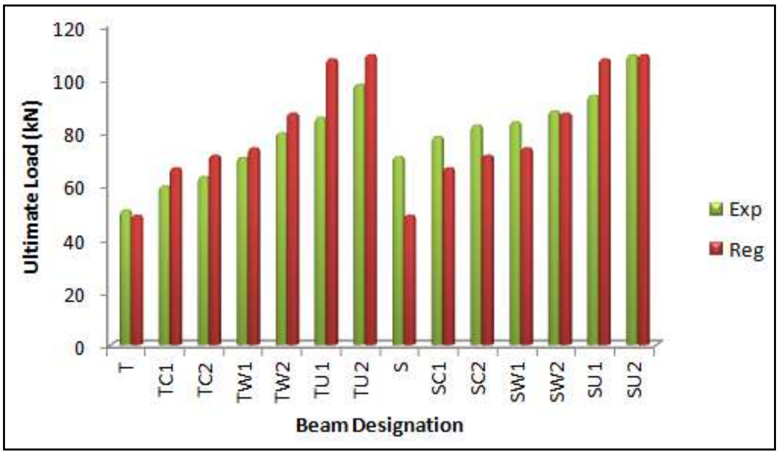


**Fig. 3** Experimental vs Predicted results for Ultimate Load

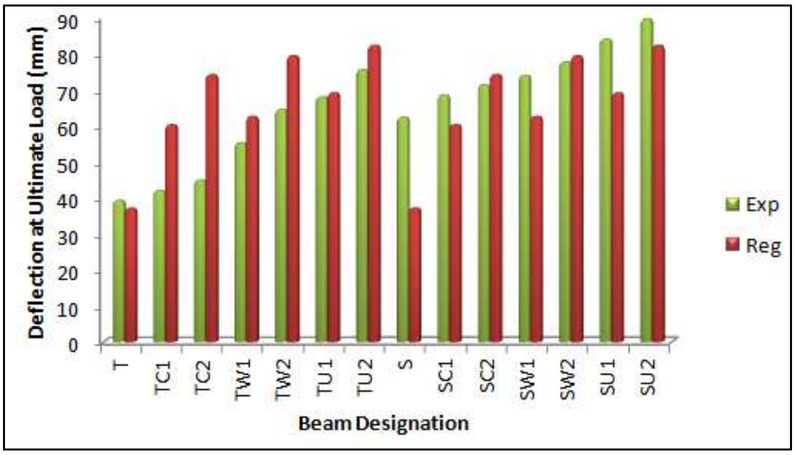


**Fig. 4** Experimental vs Predicted results for Deflection at Ultimate Load

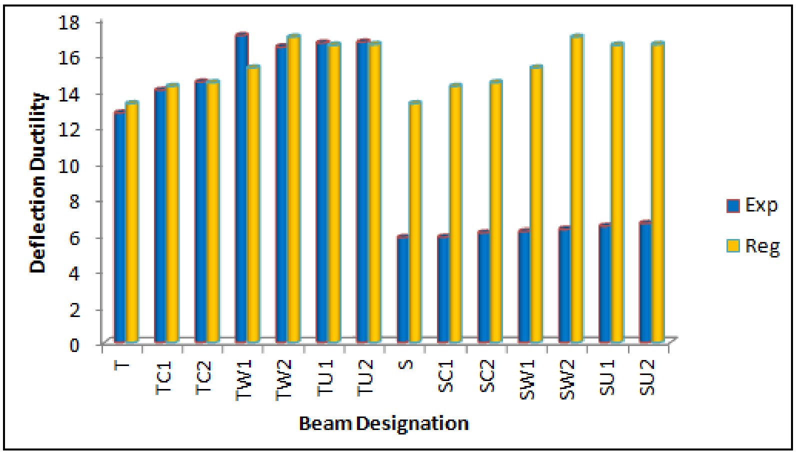


**Fig. 5** Experimental vs Predicted results for Deflection Ductility

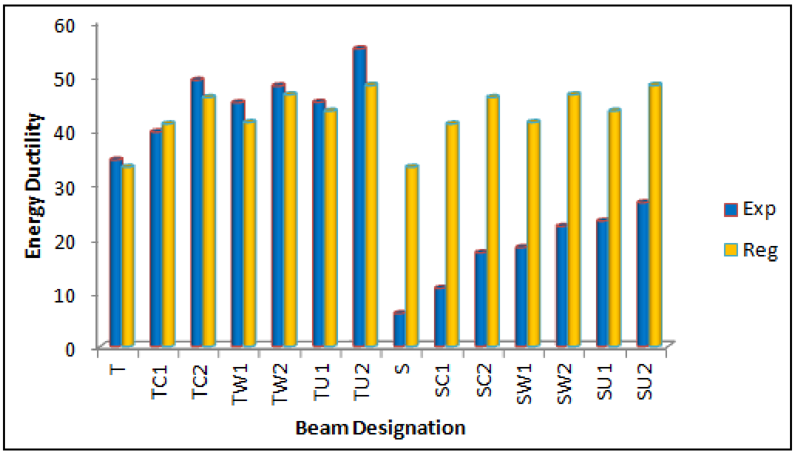


**Fig. 6** Experimental vs Predicted results for Energy Ductility

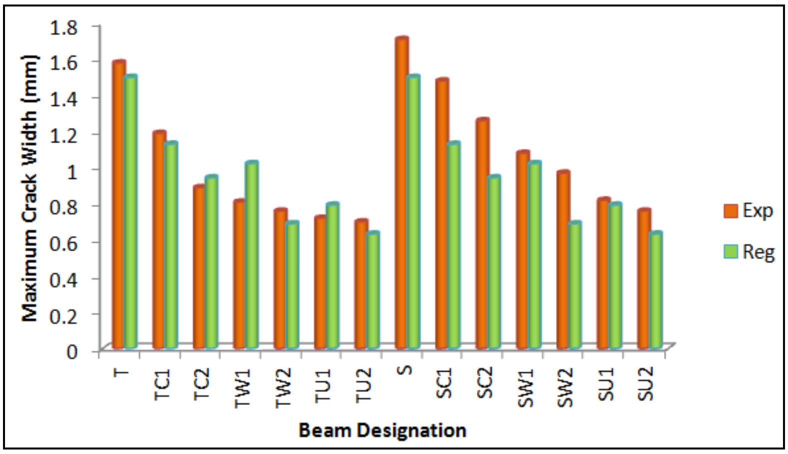


**Fig. 7** Experimental vs Predicted results for Maximum Crack Width

The developed models are applicable for the unbonded post-tensioned concrete beams externally bonded with FRP plated composites. The load and their deflection at yield and ultimate stage, ductility and maximum crack width predicted by the models showed a considerable agreement with the experimental data. The analysis of multiple linear regression models for the flexural behaviour of FRP plated PSC beams indicated that the overall significance of the model was best fitted. The root mean square error values varied from 0.17 to 13.76. The errors were within a reasonable limit and hence the model performance is agreeable for prediction purposes. These errors are also due to the limited number of data provided to the analysis. The accuracy of predictions made by regression analysis can also be improved by providing more number of experimental results.

## 5. CONCLUSIONS

The main conclusions from the study are:

1. Multiple linear regression is a best statistical tool to study the various parameters for the flexural performance of FRP plated prestressed concrete beams.
2. The analysis of multiple linear regression models for the flexural behaviour of FRP plated PSC beams indicated that the overall significance of the model was best fitted.
3. The root mean square error values varied from 0.17 to 13.76. The errors were within a reasonable limit and hence the model performance is agreeable for prediction purposes.

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