On Defensive Location Problem and its Solution Using Heuristic Methods

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ABSTRACT— ***A new approach to the defensive location problem (DLP) is proposed which aims to generalize the problem into a global optimization problem, hence the name "generalized DLP" (GDLP). In DLP, a decision maker locates defensive facilities in a network in order to prevent their enemies from reaching an important site called a core. The fact that whether or not a defensive facility is located on a given vertex of the network leads to formulating DLP as a bi-level 0-1 programming problem. In GDLP, on the other hand, the defensive capacity allocated to vertices of the network can be any real number in a continuous interval. This gives rise to the formulation of GDLP as a global constraint optimization problem. Furthermore, simple but efficient heuristic solution method based on Improved Electromagnetism-like Mechanism tailored for GDLPs is proposed. The efficiency of the proposed solving methods is then shown by applying to some numerical examples of GDLP.***

Keywords— competitive facility location, defensive location problem, constraint global optimization, heuristic methods, improved electromagnetism-like mechanism.

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1. INTRODUCTION

Facility location problems (FLP) are a class of mathematical programming problems which aims to locate some facilities in a plane or on a network under some criteria. For a detailed survey on FLPs, see [10, 25]. In an especial class of FLPs, facilities are located in accordance with the location of other competitive facilities on a network. This problem is called competitive facility location problem. There has been extensive study of this class of FLPs so far. (See e.g., [9, 11, 12, 18, 23] and references therein). This class of problems has also been extended using fuzzy logic. Some interesting fuzzy studies of competitive facility location problems can be found in [20 - 22].

Recently a new form of competitive FLPs was introduced by Uno and Katagiri [19]. They called this new problem "defensive location problem" (DLP). In DLP, a decision maker (the defender) locates defensive facilities in order to prevent their enemies (the invader) from reaching an important site, called a core.

The objective of the defender, therefore, is to stop the invader in a farther distance from the core. It is assumed that the region where the decision maker locates their defensive facilities is represented as a network and the core is a vertex in the network, and that the facility locater and their enemy are an upper and a lower level decision maker, respectively. Moreover, it is assumed that all defensive facilities have equal defensive capacities, so DLP is formulated as bi-level 0-1 programming problems.

The decision variables in DLP are 0 and 1 and also the defender can locates as many defensive facilities as the constrains are met. These features made DLP similar to 0-1 knapsack problem. This contributes to the possibility of using some efficient solution algorithms. The tailored Tabu search algorithm proposed by Uno and Katagiri [19] has been proved to be quite promising for DLP. Furthermore, Uno and Kato [24] developed a stochastic version of DLP in which the location and the energy of the invader lacks certainty. They used an interactive fuzzy method to solve this problem.

In this paper, we present a different approach towards formulating DLP which leads to a global optimization problem. The initial motivation of the new approach is taking into consideration the real-world applications of DLP. In real-world applications, a decision maker may be able to equip their defensive facilities in order to obtain certain defensive capacities. For instance, consider the problem of defending a strategic site as a DLP. A defender may have defensive facilities with different capacities, so they should be able to decide how much defensive capacity to allocate to each of the facilities. In other words, the only different assumption in our approach is that the defensive facilities do not necessarily have equal defensive capacities and can be equipped by the defender to have a capacity which is a real number in a continuous interval. We call our proposed approach generalized defensive location problem (GDLP). Note that with this assumption, DLP is actually a particular case of GDLP.

To approach a more efficient formulation for GDLP, we also assume that the defender has a total defensive capacity, each fraction of which can be allocated to each vertex. This total capacity can be interpreted as the maximum defensive capacity of the defender and/or the maximum financial ability of the defender to equip their facilities to reach certain defensive capacities and etc. Regarding these assumptions, we develop DLP into GDLP which is formulated as a global programming problem.

The new assumptions in a GDLP made it more efficient to formulate and solve defensive location problems. In other words, if the defender can locate facilities with different defensive capacities, they can defend the core more efficiently; that is, stop the invader in a farther distance from the core. This is well demonstrated in the following example which is shown in Figure 1. In this example, we consider a network with 8 vertices and 10 edges and the initial energy of the invader is assumed to be 10 units. Also, the core and the invader are assumed to be located on the 1^{st} and 8^{th} vertices, respectively. The weights of the edges are also shown in the picture.

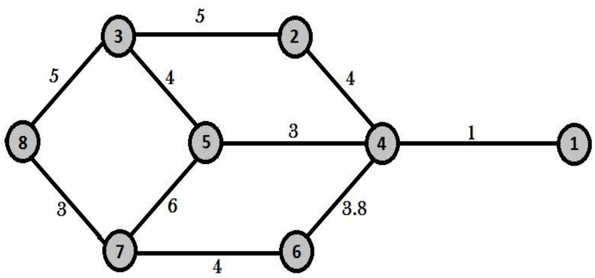

Figure 1 – advantage of GDLP over DLP

We can see that, in this example, with a fixed total defensive capacity, formulating the problem in the form of a GDLP is more efficient than DLP. To show this, we assume that the total defensive capacity of the defender is 6 units. If we model the problem in the form of a DLP (0-1 programming), we have to locate facilities with equal capacities on the network, while in the form of GDLP we can locate facilities with different capacities. Using the same notations for the GDLP problem given in the next section, Table I shows some different locations and their objective function values, including the best function value, for DLP formulation and the best decision of the defender and its corresponding objective function value for the GDLP formulation.

Table 1 – Some decision vectors and their corresponding objective function values for DLP and GDLP formulations

Formulation	Decision Vector of the Defender	Objective Function Value
DLP	$\mathbf{x} = (0,1,1,1,1,1,1)$	$F^{def}(\mathbf{x},\mathbf{y}) = 4.8$
	$\mathbf{x} = (0,2,0,0,2,2,0)$	$F^{def}(\mathbf{x},\mathbf{y}) = 4.8$
	$\mathbf{x} = (0,0,1.5,1.5,0,1.5,1.5)$	$F^{def}(\mathbf{x},\mathbf{y}) = 4.8$
	$\mathbf{x} = (0,0,0,0,6,0,0)$	$F^{def}(\mathbf{x},\mathbf{y}) = 4.8$
GDLP	$\mathbf{x} = (0,0,1.5,0,0,3,1.5)$	$F^{def}(\mathbf{x},\mathbf{y}) = 8.8$

As it is seen in the table, DLP formulation which is limited to the condition of having defensive facilities with equal capacities makes the objective function value worse than that of GDLP model. It is not difficult to investigate that in DLP model the farthest vertex to stop the enemy is 4^{th} vertex, whereas by a GDLP modeling we can stop the enemy on 5^{th} vertex. Therefore, GDLP is more efficient than DLP to formulate the problem. Note that in Table I, some other decision vectors may be considered. However, the best objective value would remain the same in this case.

GDLP can be cast in a similar formulation as DLP. That is, a formulation based on Stackelberg equilibrium [15] can be proposed where two level of decision makers are involved in the model, the upper and the lower level of decision makers. This leads to a bi-level programming formulation for GDLP which will be discussed later in the paper. For details of bi-level programming, see [4, 5].

Then, we propose a simple and efficient solving method for GDLP. It is based on "electromagnetism-like mechanism", first introduced by Birbil and Fang [3]. Since its introduction, EM has been mostly applied to global optimization in many applications [1, 2, 17]. The EM algorithm has been further improved recently for constraint optimization [26]. The efficiency of our proposed algorithm will be verified by applying it to some numerical examples of GDLP.

The remaining structure of this article is organized as follows: in section 2, we represent GDLP as an optimal location problem on a network, and formulate it as a continuous programming problem. In order to solve GDLP, we propose an easy to implement solving method based on improved electromagnetism-like mechanism [26] in section 3. Furthermore, the efficiency of the proposed solution algorithm will be verified by applying to some numerical samples of GDLP and comparison to some other solution algorithms in section 4. Finally, Conclusion and future research is included in section 5.

2. FORMULATION OF GDLP

As GDLP is a generalized model for DLP, we can formulate it in the same manner. However, we need to consider the new assumption added to GDLP. Just like DLP, in this new problem a decision maker locates defensive facilities in order to prevent their enemies from reaching the core. It is also assumed that the region where the decision maker locates their defensive facilities is represented as a network and the core is a vertex in the network, and that the defender and the invader are an upper- and a lower-level of decision maker, respectively.

Using the same way in which Uno and Katagiri [19] formulated the DLP, let $G=(V,E)$ represent the network, in which $V=\{v_1,...,v_n\}$ and $E=\{e_{ij}\}_{i,j=1}^{n}$ sets of vertices and edges. We show the core with $c\in V$. Without loss of generality, the location of the invader is assumed to be the n^{th} vertex v_n, denoted by η. Let $w_{ij}>0$ be the weight of the edge $e_{ij}\in E$. Then the distance from v to the core can be shown as follows:

$$dis^c(v)\equiv\min_{l\in L_{vc}}\sum_{e_{ij}\in l}w_{ij}, \qquad (1)$$

where L_{vc} is the set of all paths from v to c. According to the new assumption of the GDLP, the defender can locate each proportion of its total defensive capacity, which is a positive real number denoted by $\gamma>0$, on each vertex except η, where the defender cannot locate any facilities. For $i\in\{1,...,n-1\}$, let $x_i\in[0,\gamma]$ denote the defensive capacity of the facility which is located in the i^{th} vertex. We show the decision vector of the defender with $\mathbf{x}=(x_1,...,x_{n-1})$. Then we have:

$$\sum_{1}^{n-1}x_i\le\gamma \ . \qquad (2)$$

On the other hand, the invader decides a path from η to c. To formulate objective functions of the defender and the invader, we use similar assumptions to those of DLP [13]:

- The initial energy of the invader before leaving η is denoted by $Eng_{ini}>0$.
- The invader loses their energy based on the following cases:
  - i. If the invader passes edge e_{ij}, their energy is reduced by w_{ij}.
  - ii. Upon meeting a defensive facility on a vertex, the invader loses energy equal to the capacity of that defensive facility. Therefore, if the invader goes from v_i to v_j, the amount of energy they lose is:

$$Eng_{red}(e_{ij},\mathbf{x})=w_{ij}+x_j \ . \qquad (3)$$

- The invader dies once their energy becomes a negative number.

As it can be seen, just the second assumption is different from DLP due to our new assumption in GDLP.

For the decision vector of the invader $\mathbf{y}$, we use the same notations as in [13]. Let $v^t(\mathbf{y}) \in V$ be the t^{th} vertex in $\mathbf{y}$. Then, the invader can reach the $v^t(\mathbf{y})$ if the following energy of the invader is non-negative:

$$Eng\left(v^t(\mathbf{y})\,|\,\mathbf{x}\right) \equiv Eng_{ini} - \sum_{k=1}^{t} Eng_{red}(e^k(\mathbf{y}),\mathbf{x}) \;, \qquad (4)$$

in which $e^k(\mathbf{y})$ is the k^{th} edge in $\mathbf{y}$. An objective of the invader is to reach the core c, or the nearest possible vertex to it. Thus, the objective function of the invader reads:

$$F^{inv}(\mathbf{x},\mathbf{y}) \equiv \min_{v\in\mathbf{y}} \left\{dis^c(v)\,|\;\; Eng(v\,|\,\mathbf{x}) \geq 0\right\} \;. \qquad (5)$$

Taking GDLP as a zero-sum Stackelberg equilibrium game in the same way results in the following relation between the objective function of the defender, denoted by $F^{def}(\mathbf{x},\mathbf{y})$, and that of the invader:

$$F^{def}(\mathbf{x},\mathbf{y}) + F^{inv}(\mathbf{x},\mathbf{y}) = 0 \;. \qquad (6)$$

Real-world limitations such as limitations of cost, work force and other possible considerations can also be cast into linear constraints for GDLP. Thus, the feasible set for the defender is as follows:

$$D = \left\{\mathbf{x} \in [0,\gamma]^{n-1}\,|\,\mathbf{Ax} \leq \mathbf{b}\right\} \;. \qquad (7)$$

in which $\mathbf{Ax} \leq \mathbf{b}$ denotes the set of constraints. Note that relation (2) can be easily cast into this set of constraints. The feasible set for the invader, denoted by $\mathbf{I}$, is a set of paths from η to c.

Therefore, the GDLP is formulated as follows:

$$\begin{aligned}
&\max_{\mathbf{x}} && F^{def}(\mathbf{x},\mathbf{y}) \\
&\text{where } \mathbf{y} \text{ solves} && \\
&\min_{\mathbf{y}} && F^{def}(\mathbf{x},\mathbf{y}) \\
&\text{subject to} && \\
& && \mathbf{x} \in \mathbf{D}, \\
& && \mathbf{y} \in \mathbf{I}.
\end{aligned} \qquad (8)$$

3. SOLVING GDLP

The new assumption for GDLP makes it a continuous optimization problem. There are, indeed, several algorithms which can be employed to solve GDLP. Addressing the real-world application of GDLP, we seek an algorithm which is not only simple and easy to implement but also efficient enough to solve both small and large-scale GDLP with satisfactory computational and memory requirement.

The electromagnetism-like mechanism (EM) is a heuristic method for solving global optimization problems which is inspired by an analogy made between the mechanism of the convergence of the points to the one with the best function value and the attraction-repulsion mechanism of the electromagnetism theory. Birbil and Fang [3] showed that their introduced method is powerful yet easy for global optimization. EM was further improved for constraint optimization by Zhang, Li, Gao and Wu [26]. Dealing with a constraint optimization problem in case of GDLP, our proposed method involves use of improved total force calculation proposed in Improved EM (IEM) algorithm [26]. Moreover, the same feasibility and dominance rules aiming at satisfying the constraints efficiently have been employed here. However, we use a modified update procedure which is specifically tailored for GDLPs.

For the sake of comparison, we have employed two versions of IEM to solve the problem (9). One uses a simple local random search mechanism as suggested in [3], while the other uses a heuristic updating approach tailored for the GDLP. In the following we give a brief summary of the two versions of IEM employed for solving GDLP.

4.1 IEM with local random search

This is similar to method developed in [26]. The only difference is that here the movement probability is not introduced and instead the updating of the points is performed based on a random local search as in the original EM algorithm [3].

In an overall view, IEM operates as follows: First it selects p random points in the feasible region. Then, for each of these points, using a random local search procedure, it searches their local region to find a better value point if there is any, and update the points. Next, it assigns a value to each of the points. This value, which can be interpreted as electromagnetic charge of each point, is calculated using the following formula:

$$q^i = \exp\left(-n \frac{f(x^{best}) - f(x^i)}{\sum_{j=1}^{p} (f(x^{best}) - f(x^j))} \right), \qquad (10)$$

in which q^i is the charge of the i^{th} point which is denoted by x^i and x^{best} is the current best point , n and m are the dimension of the problem and the number of points, respectively and $\exp(x)$ represents the exponential function. The idea is to allocate a larger value of charge to the point with better objective function values. This is clearly not the only way to express the charges of the points, but it was proved to be efficient [3].

After that, the total force exerted on each point is computed. As mentioned before, we use the simplified total force calculation proposed in [26] as follows:

$$F^i = \sum_{j=R1,R2,R3}^{p} \begin{Bmatrix} (x^j - x^i) q^i q^j & if \quad f(x^j) > f(x^i) \\ (x^i - x^j) q^i q^j & if \quad f(x^j) \le f(x^i) \end{Bmatrix}, \qquad (11)$$

with F^i being the total force on the i^{th} point.

And finally, all points, except the one with the best function value, are moved to the direction of the total force exerted on them by a random step length. This procedure will continue until a stopping criterion is met. The procedures of handling the constraints is also the same as that of proposed in [26].

4.2 IEM with Heuristic Update Procedure

This version of EM follows the same principles as the one introduced by Birbil and Fang [1]. The only difference is in the updating procedure which takes place as the first step of the main iterative loop of the EM algorithm. Here we use a heuristic updating approach which is motivated by an important characteristic of the GDLP.

The problem with the local random search is that it updates the points based on their objective function values. In other words, if the procedure finds a local point with a better objective function value, it replaces it with the current point. But computing the objective function value for GDLP means solving shortest path problems using Dijkstra algorithm with the mentioned computational complexity of $O(r + n\log n)$. This can be prohibitively expensive, especially for large-scale GDLP. Therefore, we propose a simple updating approach to replace the local random search process in the second step of the IEM algorithm.

The idea is based on the fact that the more defensive capacity allocated to the network by the defender, the larger would be the value of the objective function. In fact, let $\mathbf{x}^1 = (x_1^{\ 1}, ..., x_{n-1}^{\ \ 1}) \in \mathbf{D}$ and $\mathbf{x}^2 = (x_1^{\ 2}, ..., x_{n-1}^{\ \ 2}) \in \mathbf{D}$ be two decision vectors of the defender such that $\sum_{i=1}^{n-1} x_i^{\,j} \le \gamma, \quad (j=1,2)$ and $x_i^{\,2} - x_i^{\,1} \ge 0, \quad (i=1,...,n-1)$. Then, we have the following relation:

$$F^{def}(\mathbf{x}^1, \mathbf{y}) \le F^{def}(\mathbf{x}^2, \mathbf{y}). \qquad (12)$$

Having this observation in mind, instead of doing a local random search, we add equal proportions of the unallocated defensive capacity to each of the vertices of the network. In other words, let $\mathbf{x}^j = (x_1^{\,j}, ..., x_{n-1}^{\,j}), \ j=1,...,p$ be the p points at the beginning of the current iteration of EM. For each $j=1,..., p$, let

$$\sigma_j = \frac{\gamma - \sum_{i=1}^{n-1} x_i^{\,j}}{n-1} \ge 0 .$$

Then, we update the point $\mathbf{x}^j = (x_1^{\,j}, ..., x_{n-1}^{\,j})$ to point $\mathbf{x}^j = (x_1^{\,j} + \sigma_j, ..., x_{n-1}^{\,j} + \sigma_j)$. Relation (12) shows that this approach leads to points with larger or equal objective function values. Our computational results given in the next section prove that this simple updating heuristic improves the efficiency of the EM algorithm in terms of both objective function values and the speed of computations.

5. NUMERICAL RESULTS AND DISCUSSION

In this section, the computational results of applying two versions of the IEM algorithm on the relaxed form of GDLP (9) are presented. In order to investigate the efficiency of IEM algorithm, we employed two other methods to solve the GDLP. One is a continuous genetic algorithm (CGA) [6] and the other is a continuous variable neighborhood search method (CVNS) [16]. The methods are applied to 6 samples.

We generated our sample problems in three sizes, small, medium and large, regarding the number of vertices of the network. The number of vertices (NoV) and edges (NoE), the initial energy of the invader (EoI), the total defensive capacity of the defender (CoD), the number of linear constraints (m) are given in Table II.

Table 2 – Samples GDLP

Problem	NoV	NoE	EoI	CoD	m
S_1	200	1970	200	100	50
S_2	350	2450	300	150	50
M_1	4000	24500	3000	500	50
M_2	6500	32750	4000	600	50
L_1	25000	42500	20000	1500	50
L_2	80000	895000	30000	3000	50

For other requirements of the problem, we have employed a prescription similar to that of Uno and Katagiri [13]. That is, the weights of edges are selected randomly from $\{1, 2, ..., 1000\}$. In addition, the elements of the left-hand-side matrix $\mathbf{A}$ were randomly chosen from $\{1, 2, ..., 1000\}$ and the elements of the right-hand-side vector $\mathbf{b}$ are defined as follows:

$$b_i = \beta_i . \sum_{j=1}^{n-1} a_{ij}$$

in which β_i , $i = 1,...,m$, are random numbers in $(0,1)$.

Each of the methods are applied on each sample problem 10 times and then best (BOFV), worse (WOFV) and average (AOFV) objective function values, together with the average CPU times (AT), are reported in the following tables. Also all the solving algorithms stop if there are 5 consecutive iterations with the optimal objective function value unchanged.

The results of applying two versions of IEM on GDLP are presented in Table 3 and Table 4. The IEM method using local random search and IEM featuring our proposed updating approach are represented in the Tables as IEM-LRS and IEM-HU, respectively. Finally, the maximum number of local searches for IEM-LRS is set according our best experimental results in each sample.

It is well observed from Table 3 and 4 that using our proposed updating procedure with the IEM algorithm results in the improvement of both the objective function values and the CPU time, with these improvements being much more significant for larger problems.

Table 3 – Results of applying IEM with random local search to samples of GDLP

Problem	BOFV	WOFV	AOFV	AT
S_1	185	128	151.4	592.4
S_2	315	123	228.6	726.1
M_1	842	445	623.1	1262.1
M_2	971	556	780.9	2112.8
L_1	1748	832	1334.2	4389.6
L_2	2533	1135	1893.8	10927.3

Table 4 – Results of applying IEM heuristic update to samples of GDLP

Problem	BOFV	WOFV	AOFV	AT
S_1	192	165	185.5	356.3
S_2	342	142	274.1	402.5
M_1	911	552	796.2	869.1
M_2	1294	629	990.2	1562.9
L_1	2412	1280	2126.4	3426.5
L_2	3302	1485	2863.8	8672.8

The results of applying CVNS on GDLP are shown in Table 5. By our own experience from solving the samples more than 1000 times for each sample problem, in the reported results the number of neighborhood structures, $k_{\max}$ and the number of local searches in each neighborhood is set according our best experimental results in each sample during the 100 final reported runs.

Table 5 — Results of applying CVNS to samples of GDLP

Problem	BOFV	WOFV	AOFV	AT
S_1	185	119	164.5	762.1
S_2	322	128	231.1	843.8
M_1	914	494	685.2	1619.7
M_2	1002	512	712.4	2613.1
L_1	2141	912	1436.8	6104.6
L_2	2362	1098	1783.2	13311.6

Although the CVNS algorithm works similar to IEM-LRS with regards to the objective function values, its time expense is much more than that of IEM-LRS. In addition, the CVNS performance is dramatically outperformed by the proposed IEM-HU.

The results of applying CGA on GDLP are shown in Table 6. The necessary parameters for genetic algorithm, including the terminal generation, the probabilities of inversion, permutation and crossover, the generation gap and scaling parameters, are set based on our experience from solving the each sample more than 1000 times during the 100 final reported runs.

Table 6 – Results of applying CGA to samples of GDLP

Problem	BOFV	WOFV	AOFV	AT
S_1	173	108	120.5	863.1
S_2	302	116	195.2	981.8
M_1	801	312	517.8	1789.7
M_2	901	512	603.5	3121.5
L_1	1650	783	920.3	6632.8
L_2	3156	1598	1208.5	150362.6

The CGA algorithm is comparable with IEM-LRS in terms of best objective function values. However, a deeper look at the average objective function values column suggests that in most of the runs, it leads to a smaller value for the objective function. Furthermore, it is much more time consuming than both versions of IEM and CVNS.

Overall, the best algorithm is IEM-HU. It provides promising results in terms of both the solution speed and the final objective function value, particularly for large scale problems. While taking more time, IEM-LRS does not work comparable to IEM-HU in the sense of objective function value, again particularly for large scale problems. Although CVNS seems to be comparable to, and in some cases better than, IEM_LRS in terms of objective function values, it is much more time consuming than both IEM algorithms, a factor which may take it out of the menu for medium and large scale problems. However, CVNS is outperformed by IEM-HU in all aspects. Finally, the CGA, with not much improvement to the objective function value yet being too slow, seems to be a good candidate to be labeled the most inefficient solving method among all four.

6. CONCLUSION AND FUTURE RESEARCH

The new development of defensive location problem is introduced and formulated as a continuous optimization problem, hence its name as "continuous defensive location problem" (GDLP). The key feature of this new development making it much more practical for real-world problems is that the capacity of the defensive facilities can be set to be any fraction of a total defensive facility. Using this new assumption one could use the convenience of continuous optimization in terms of simplicity and computational requirements. A very simple yet efficient heuristic optimization algorithm based on improved EM is also proposed which uses a tailored updating procedure for GDLP. The comparison of the results of employing this simple algorithm with some famous and widely used optimization algorithms for global optimization, such as continuous versions of Variable Neighborhood Search and Genetic algorithms, shows the efficiency of the method, particularly when dealing with large scale GDLP.

The future research in this field will be devoted to finding much more efficient algorithms for very large scale GDLP by employing more characteristics of the nature of the problem into the solution procedure. Developing the idea to multi-objective defensive problem and finding efficient solving methods for that is also another open future research in this field.

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