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Utilization of Mathematical Frame Work in Bridge Deformation Monitoring

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ABSTRACT--- ***Bridges are widely used in the construction for traffic and transportation solutions. In the long-term process of use, due to the influence of various factors as material aging, load effect and environmental change, a certain degree of deformation will occur to the foundation and superstructure of the bridge, the long-term monitoring for the deformation of the bridge as well as its influencing factors and the establishment of mathematical models for predicting deformation will contribute to the maintenance and management of large bridge. Bridges deformation can be monitored by using geodetic method such as Global Positioning System (GPS) and the conventional method. However, as the technical feasibility of using GPS for recording relative displacements has been proven, the challenge issue for users is to determine how to make use of the relative displacements being recorded. This work proposes a mathematical framework that supports the use of Real Time Kinematics (RTK)-GPS data for structural monitoring and suggests some equation for the prediction of*** **deformation with time.**

Keywords – RTK, Deformation, Monitoring, least squares.

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1. INTRODUCTION

Engineering structures require periodic monitoring to determine the status of their safety under traffic load and wind. With the growth of scientific and technological progress in a geodetic production and technical level of construction of structures there is a need to develop and improve techniques and technology of measurement for such monitoring. GPS has the advantages of high data acquisition rate. Also, using differential techniques can furnish position estimates in real-time with sub centimeter-level accuracy. Our intention is to present a mathematical frame work that is supported by numerical solution of the various equations used.

2. THEORY OF ERRORS IN OBSERVATION

We must not only know the procedures used to make observations, but also know the sources of errors in our observations, how to estimate their size, and how to remove them when possible. No observation is exact, the true value of an observation is never known and the exact error in an observation is never known. The following are the sources and type of errors.

- **Sources of Errors**

1. Natural errors
2. Instrumental errors
3. Personal errors.

- **Types of Errors**

1. Systematic errors
2. Random errors

2.1 PRECISION AND ACCURACY

Precision relates to the quality of an operation (degree of closeness) by which a result is obtained, and is distinguished from accuracy, which relates to the quality of the result (degree of closeness to a true value)

a. The shots are precise but not accurate - this can be caused by the presence of systematic errors.
b. The shots are accurate, but not precise. Note the mean of the shots is at the center of the target, but there are large discrepancies between the shots.
c. The shots are neither precise nor accurate.
d. The shots are both precise and accurate. This is what we try to achieve in measurements.

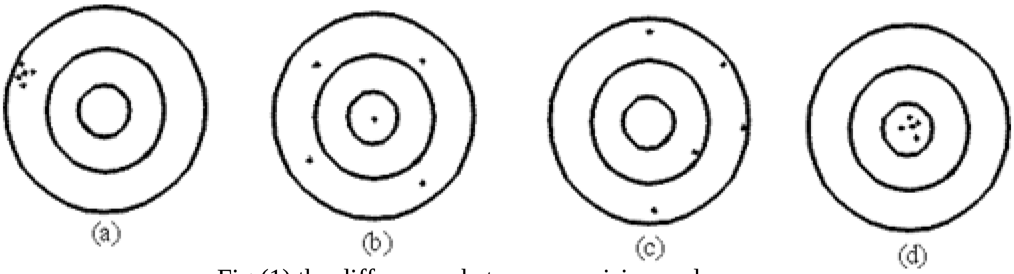


Fig (1) the differences between precision and accuracy

We present here the equations for equal and unequal weight in observation.

Equal weight

$$\left.\begin{aligned}
&Observation: L1, L2, L3, L4, ..., Ln\\
&Mean: L = (L1 + L2 + L3 + ... + Ln)/n = \sum Li/n\\
&\mathrm{Re}\,sidual(V): V1 = L1 - L, V2 = L2 - L, V3 = L3 - L, ...Vn = Ln - L\\
&Range(R): R = Iv\max. - v\min.I\\
&Std.deviation(\sigma): \sigma \pm \sqrt{\frac{\sum v_i^2}{n(n-1)}}\\
&Std.deviation.mean(\sigma m): \sigma_m = \pm \sqrt{\frac{\sum v_i^{v2}}{n(n-1)}}
\end{aligned}\right\} \qquad (1)$$

Unequal weight

$$\left.\begin{array}{l} Observation: L1, L2, L3, L4,..., Ln \\ Weight: w1, w2, w3, w4...Wn \\ Mean: L = (L1w1 + L2w2 + L3w3 + ... + LnWn)/\sum wi = \sum LiWi/\sum Wi \\ \mathrm{Re}\,sidual(V): V1 = L1 - L, V2 = L2 - L, V3 = L3 - L, ...Vn = Ln - L \\ Range(R): R = Iv\max. - v\min.I \\ Std.deviation(\sigma): \sigma \pm \sqrt{\dfrac{\sum wiv_i^2}{(n-1)}} \\ Std.deviation.mean(\sigma m): \sigma_m = \pm\sqrt{\dfrac{\sum wiv_i^{v2}}{\sum wi(n-1)}} \end{array}\right\} \quad (2)$$

3. CURVE FITTING TECHNIQUS

Generally, curve fit algorithms determine the best-fit parameters by minimizing a chosen merit function. In order to optimize the merit function, it is necessary to select a set of initial parameter estimates and then iteratively refine the merit parameters until the merit function does not change significantly between iterations. The Levenberg-Marquardt algorithm (LMA) has been used for nonlinear least squares solution in the current implementation.

3.*1 Curve Fit Models*

a)- Linear curve fitting (linear regression)

Given the general form of a straight line; f(x) = ax + b Quantifying error in a curve fit

Assumptions:
1) Positive or negative error has the same value
2) Weight greater errors more heavily.
We can do both of these by squaring the distance denote data values as (x, y), denote points as fitted line as (x, f(x)) sum of error at the four data point with the following;

$$err = \sum (d_i)^2 = (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2 + (y_3 - f(x_3))^2 + (y_4 - f(x_4))^2 \quad (3)$$

Our fit is a straight line, so now substitute f(x) = ax + b

$$err = \sum_{i=1}^{n} (y_i - f(x_i))^2 = \sum_{i=1}^{n} (y_i - (ax_i + b))^2 \quad (4)$$

The 'best' line has **minimum error** between line and data points, this is called the **least squares approach**, since we minimize the square of the error.

$$\min err = \sum_{i=1}^{n} (y_i - (ax_i + b))^2 \quad (5)$$

Take the derivative of the error with respect to (a) and (b), set each to zero, then Solve for the (a) and (b) so that the previous two equations both = 0.

The matrix form of these two equations can be written thus:

$$\begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^{\,2} \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum (x_i y_i) \end{bmatrix} \quad (6)$$

The data points (x_i, y_i) for i= 1…n so we have all the summation terms in the matrix, unknowns are (a) and (b). By solving this problem using Gaussian elimination

$$A = \begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^{\,2} \end{bmatrix} \qquad X = \begin{bmatrix} b \\ a \end{bmatrix} \qquad B = \begin{bmatrix} \sum y_i \\ \sum (x_i y_i) \end{bmatrix} \quad (7)$$

So $AX = B$

b) - Higher order polynomials

There are lots of functions with lots of different shapes that depend on coefficients. We can choose a form based on experience and trial/error. Let's develop a few options for non-linear curve fitting. We'll start with a simple extension to linear regression higher order polynomials. Consider the general form for a polynomial of order j

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^2 + \ldots + a_j x^j = a_0 + \sum_{k=1}^{j} a_k x^k \quad (8)$$

The general expression for any error using the least squares approach is

$$err = \sum (d_i)^2 = (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2 + (y_3 - f(x_3))^2 + (y_4 - f(x_4))^2 \quad (9)$$

Where we want to minimize this error, now substitute the form of our equation into the general least squares error i.e

$$err = \sum_{i=1}^{n} (y_i - (a_0 + a_1 x + a_2 x^2 + a_3 x^2 + \ldots + a_j x^j))^2 \quad (10)$$

Where: n- the of data points given, - I the current data point being summed, - j the polynomial order re-writing eq.

$$err = \sum_{i=1}^{n} (y_i - (a_0 + \sum_{k=1}^{j} a_k x^k))^2 \quad (11)$$

Find the best line = minimize the error (squared distance) between line and data points
Find the set of coefficients a_0, a_k so we can minimize eq.

$$A = \begin{bmatrix} n & \sum x_i & \sum x_i^2 & \cdots & \sum x_i^j \\ \sum x_i & \sum x_i^2 & \sum x_i^3 & \cdots & \sum x_i^{j+1} \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 & \cdots & \sum x_i^{j+2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sum x_i^j & \sum x_i^{j+1} & \sum x_i^{j+2} & \cdots & \sum x_i^{j+j} \end{bmatrix}, X = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_j \end{bmatrix}, B = \begin{bmatrix} \sum y_i \\ \sum (x_i y_i) \\ \sum (x_i^{\,2} y_i) \\ \vdots \\ \sum (x_i^{\,j} y_i) \end{bmatrix} \quad (12)$$

Where all summations above are over *i* = 1…n data points
K=1….j
No matter what the order j, we always get equations **linear** with respect to the coefficients. This means we can use the following solution method.
$AX = B$

4. BRIDGE DESCRRIP

One cable-stayed bridge was constructed by pre-stressed concrete in December 1987, closed in October 2006 because cracks over mid span and opened in August 2007 after rehabilitation. The whole bridge has four lanes with the total length of 510.00 meters, and main span of the bridge is 260.00 meters. For safety assurance, a sophisticated long-term structural health monitoring system has been designed and implemented by the Research Center of Structural Health Monitoring and Control of Harbin Institute of Technology to monitor loads and response of the bridge.

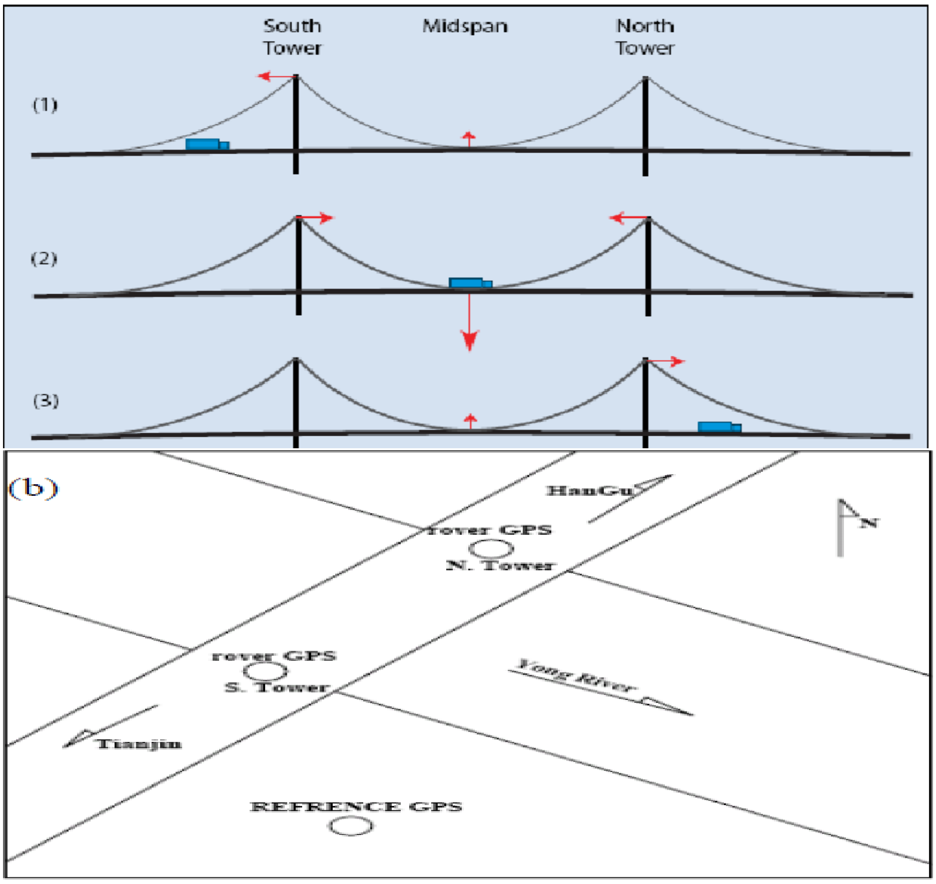


Fig (2): Geometry traffic loads effects on the deck, towers movement and Layout of GPS positions of the bridge

To determine the current operational safety and the cause of bridge cracks, its movements were observed under different stress factors such as wind speed, temperature change and traffic load. The observed X and Y-directions of bridge's towers were studied and described using processing and identification methodology.

4.1 Analysis of Observations

This section discusses the analysis process of the GPS observations and the bridge towers movement. In addition the statistical analyses, below are the steps:

1- Computed the mean and mean standard deviation of all observations.
2- Divide the observations to sessions, which every session contain 5000 observations, then compute the (MPV) most probable value of every session, determine equal weight solution and then compute the (MPV) of session as unequal weight.
3- Take observations observed on the 1st January 2007 (1000 min) from 11.00 AM to 3.40 AM and compute the curve fitting as
    - Linear function
    - Polynomial function
    - Exponential function
    - Logarithmic function
    - Power function
4- Comparison between the rest observations with the computed by curve fitting as prediction solution

Table (1) computed mean and mean standard deviation of all observations

South tower				North tower			
X mean (m)	$\sigma_m\,(cm)$	Y mean (m)	$\sigma_m\,(cm)$	X mean (m)	$\sigma_m\,(cm)$	Y mean(m)	$\sigma_m\,(cm)$
4337438.923	1.086	500401.0239	0.866	4337438.939	0.884	500662.0246	0.766

Divide the observations to session, which every session contain 5000 observations and compute the (MPV)

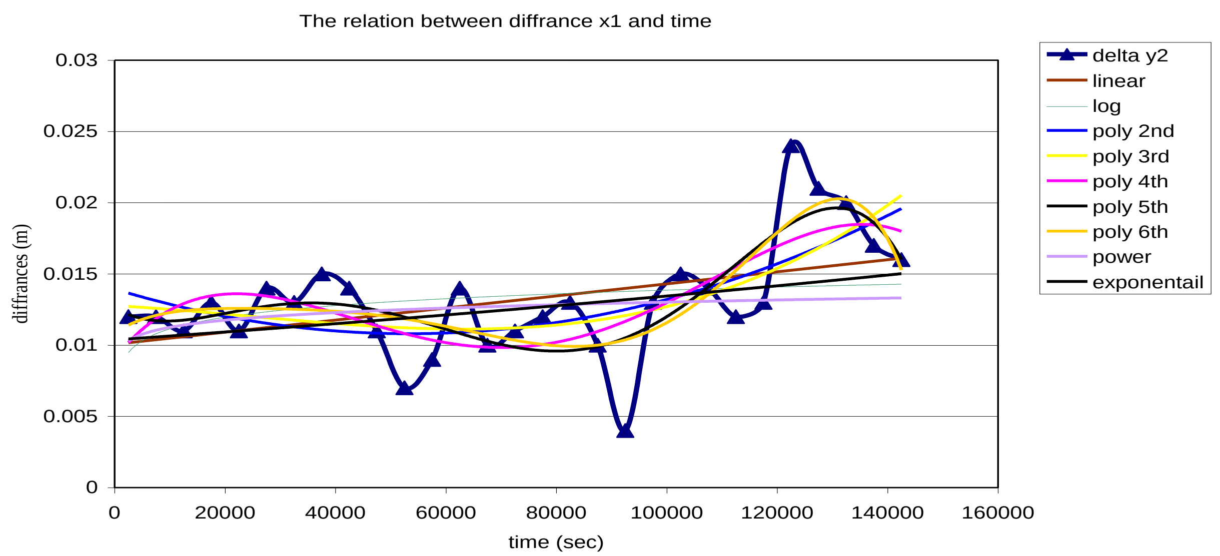
Table (2) computed mean and mean standard deviation of all observations after division

South tower				North tower			
X mean (m)	$\sigma_m\,(cm)$	Y mean (m)	$\sigma_m\,(cm)$	X mean (m)	$\sigma_m\,(cm)$	Y mean(m)	$\sigma_m\,(cm)$
4337438.921	1.089	500401.022	0.875	4337438.94	0.884	500662.0239	0.787

Table (3) computed mean and mean standard deviation of every 5000 observations

South Tower				North Tower			
X mean (m)	$\sigma_m\,(cm)$	Y mean (m)	$\sigma_m\,(cm)$	X mean (m)	$\sigma_m\,(cm)$	Y mean(m)	$\sigma_m\,(cm)$
4337438.922	0.1210478	500401.0219	0.1283981	4337438.942	0.1410235	500662.0258	0.1436492
4337438.922	0.0903509	500401.022	0.1716108	4337438.944	0.1118415	500662.0261	0.144649
4337438.921	0.1323943	500401.0212	0.102164	4337438.942	0.1512039	500662.0254	0.1382105
4337438.923	0.5671054	500401.0237	0.4009945	4337438.943	0.8043924	500662.0239	0.6133886
4337438.921	2.323607	500401.0207	1.6988739	4337438.931	1.1738589	500662.0174	1.0126845
4337438.924	1.7903054	500401.0227	1.3387633	4337438.934	1.1823273	500662.0188	0.8831068
4337438.923	0.7704134	500401.0221	0.6689967	4337438.942	0.7904495	500662.0257	0.6135269
4337438.925	0.6146141	500401.0257	0.4686997	4337438.941	0.7704385	500662.0279	0.5461825
4337438.924	1.7060287	500401.0255	1.269873	4337438.936	1.0936967	500662.0219	0.8778658
4337438.921	1.2156757	500401.0227	0.9034515	4337438.937	1.1250775	500662.0211	0.8560577
4337438.917	0.1099445	500401.0197	0.0763166	4337438.94	0.1108388	500662.0226	0.1384791
4337438.919	0.1676974	500401.0188	0.0560212	4337438.939	0.1152355	500662.0211	0.0754895
4337438.924	0.1177275	500401.0162	0.1331214	4337438.942	0.1496545	500662.021	0.1144031
4337438.92	0.1863135	500401.0156	0.1229025	4337438.938	0.214111	500662.024	0.226578
4337438.921	0.5733469	500401.025	0.4230165	4337438.935	0.6819211	500662.0321	0.5385106
4337438.922	0.5839426	500401.0309	0.4854754	4337438.939	0.6097962	500662.0362	0.4520296
4337438.923	0.1425252	500401.0324	0.0780726	4337438.944	0.1436383	500662.0359	0.172687
4337438.92	1.0782979	500401.0272	0.8024664	4337438.936	0.7317473	500662.0242	0.5522056
4337438.914	0.1227704	500401.0233	0.0876888	4337438.934	0.0983335	500662.0223	0.0756657
4337438.923	0.6605022	500401.0265	0.4767247	4337438.939	0.9559023	500662.0228	0.7025255
4337438.925	0.2481653	500401.023	0.1952671	4337438.943	0.3484637	500662.0242	0.2644145
4337438.924	0.6399065	500401.0219	0.4451521	4337438.938	0.7552729	500662.0235	0.5558253

4337438.922	0.9769177	500401.0192	0.7531368	4337438.936	0.7449821	500662.0222	0.5152297
4337438.923	1.7605532	500401.0207	1.3150618	4337438.93	1.2226407	500662.0156	0.876977
4337438.934	1.6021766	500401.0294	1.1768468	4337438.942	1.4700074	500662.024	1.1101328
4337438.931	0.1824654	500401.0297	1.691383	4337438.949	0.2471159	500662.0342	0.1763435
4337438.93	0.9864913	500401.0275	0.7238103	4337438.942	0.8909028	500662.0279	0.757612
4337438.927	1.4414826	500401.0281	1.111428	4337438.933	0.9694583	500662.0233	0.6734645



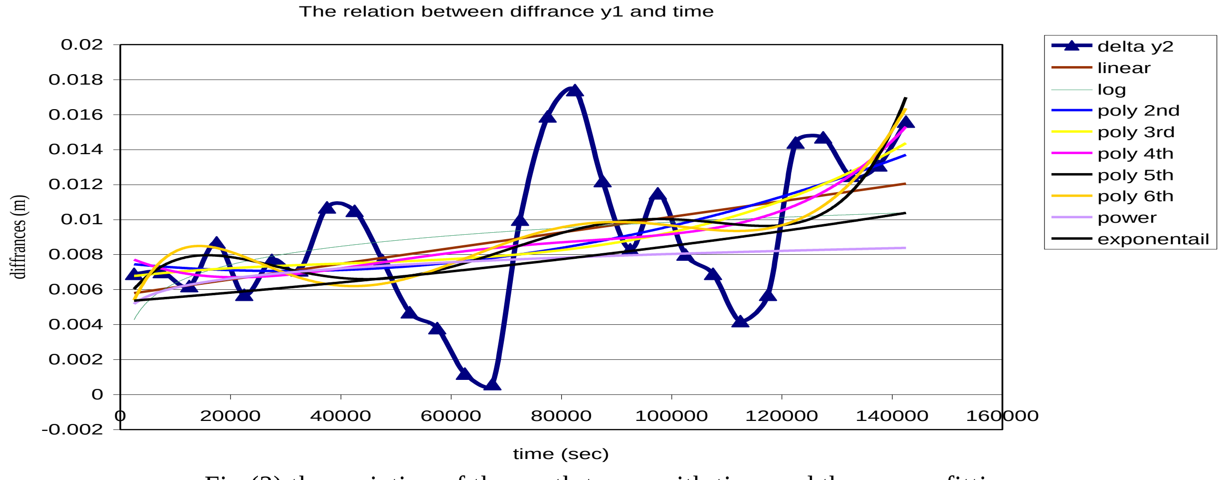


Fig (3) the variation of the south tower with time and the curves fitting

3- Take observations at 1st January 2007 (1000 min) from 11.00 AM to 3.40 AM and compute the curve fitting.

R^2-value measures how much of the variation in the data points can be explained by the selected regression model:

$$R^2 = \frac{SSR}{SST} = 1.0 - \frac{SSE}{SST} \qquad 0 \le R^2 \le 1$$

Where

$$SSR = \sum_{i=1}^{n} (\hat{Y}_i - \bar{Y})^2$$

(The regression sum of squares)

$$SSE = \sum_{i=1}^{n} (Y_i - \hat{Y})^2$$

(The residual or error sum of squares)

$$SST = \sum_{i=1}^{n} (Y_i - \bar{Y})^2$$

(The total sum of squares, SST= SSE+SSR)

and $\hat{Y}_i$ represents the ith fitted value (calculated using the selected model) of the dependent variable Y.

5. CONCLUSION

Monitoring the reactions of engineering structures under the effect of loads using appropriate observation devices are essential for maintaining safety and extending the lifetime of such structures. General information about the movements of a structure can be obtained using frame work of mathematical models that will provide information about action and reaction quantities over time. This information will be even more useful when the data is collected from very precise observations. However, in certain cases, although the quantities affecting the structures are well known, they cannot be measured, therefore, the structure can only be evaluated on the basis of the analysis performed for reaction quantities and in this situation frame work of mathematical analysis presented above become a useful tool.

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